SoLID Tracking Reconstruction

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Previously on Tracking Resolution

- Using Kalman Filter algorithm to reconstruct vertex variables

- Using non-digitized signal track, no process noise simulated (Coulomb multiple scattering and energy loss)

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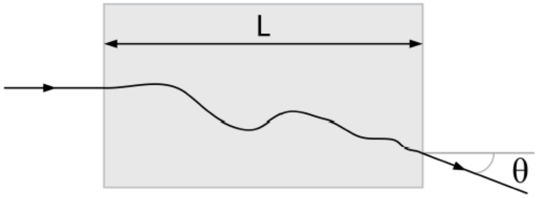
- GEM resolution - 90um, beam spot resolution - 300um.

	θ (mrad)	φ (mrad)	z (cm)	p (%)
SIDIS FA	0.30	0.77	0.54	0.44
SIDIS LA	0.67	0.57	0.47	0.57
PVDIS	0.37	0.77	0.24	0.60

Process Noise

- Coulomb multiple scattering
  - RMS of the scattering angle can be described by:

$$\Theta_{CMS} = \frac{13.6\boldsymbol{MeV}}{\boldsymbol{\beta pc}} z\sqrt{\boldsymbol{t}}\left(1+0.038\ln \boldsymbol{t}\right) \quad \text{where} \quad \boldsymbol{t} = \frac{\boldsymbol{l}}{\boldsymbol{X}_0}$$



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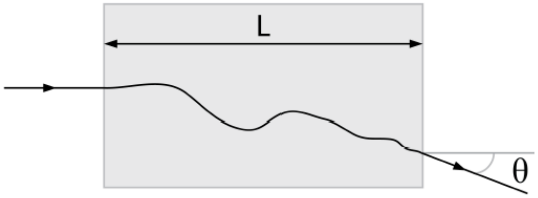
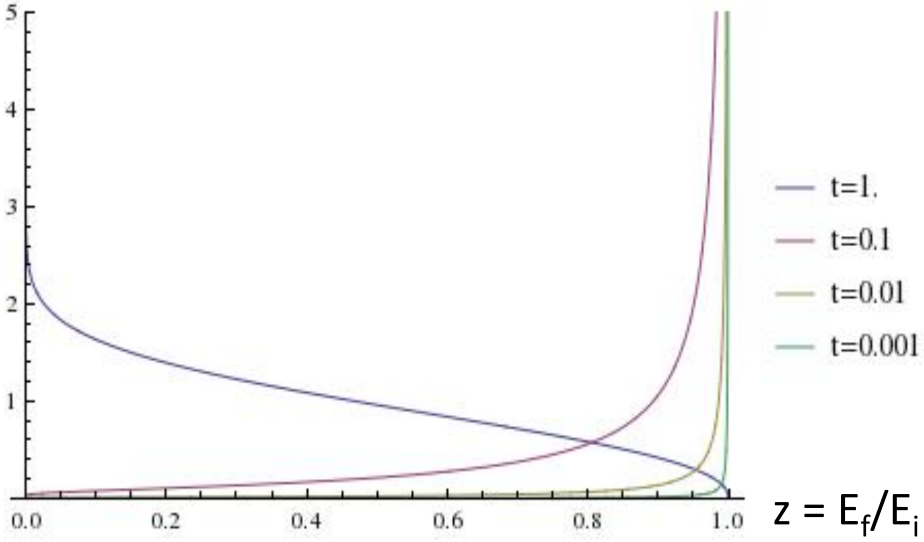


- Bremsstrahlung radiation for electrons
  - Mean value and probability density function given by the Bethe-Heitler model:

$$\left\langle \frac{\boldsymbol{E}_f}{\boldsymbol{E}_i} \right\rangle = \boldsymbol{e}^{-t}$$

$$\boldsymbol{f}\left(z = \frac{\boldsymbol{E}_f}{\boldsymbol{E}_i}\right) = \frac{[-\ln z]^{t/\ln 2 - 1}}{\Gamma(\boldsymbol{t}/\ln 2)}$$



Material Properties

For everyone

	L/X_0 (%)
GEM	0.52
Air	~1%

SIDIS

	L/X_0 (%)
SIDIS Cell	1.43

J/ψ

	L/X_0 (%)
J/ψ Cell	0.20
LH2	0.21

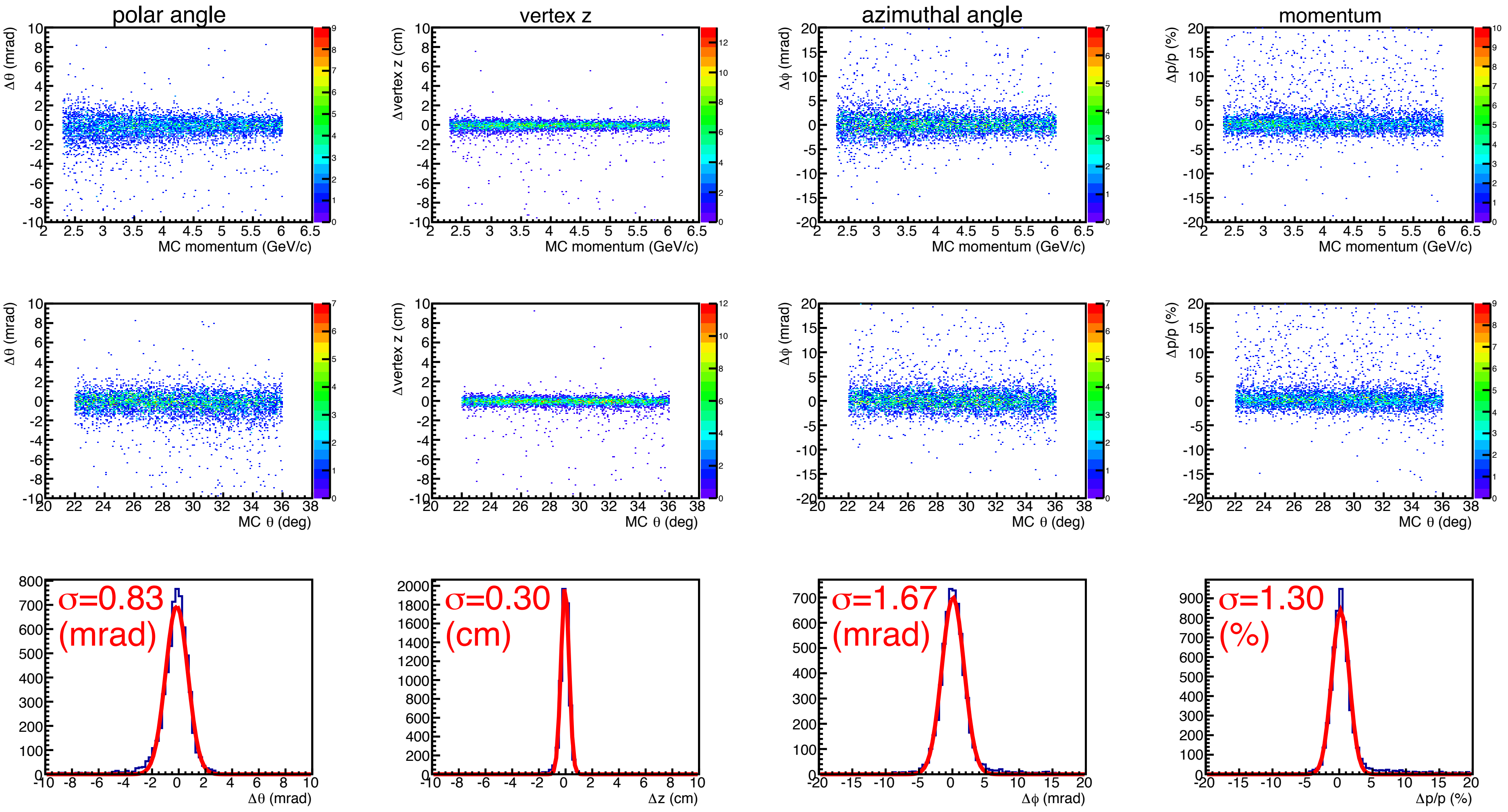
PVDIS

	L/X_0 (%)
PVDIS Cell	0.42
LD2	0.33

PVDIS Vertex Resolution

PVDIS electron. GEM resolution 90um, beam spot resolution 300um

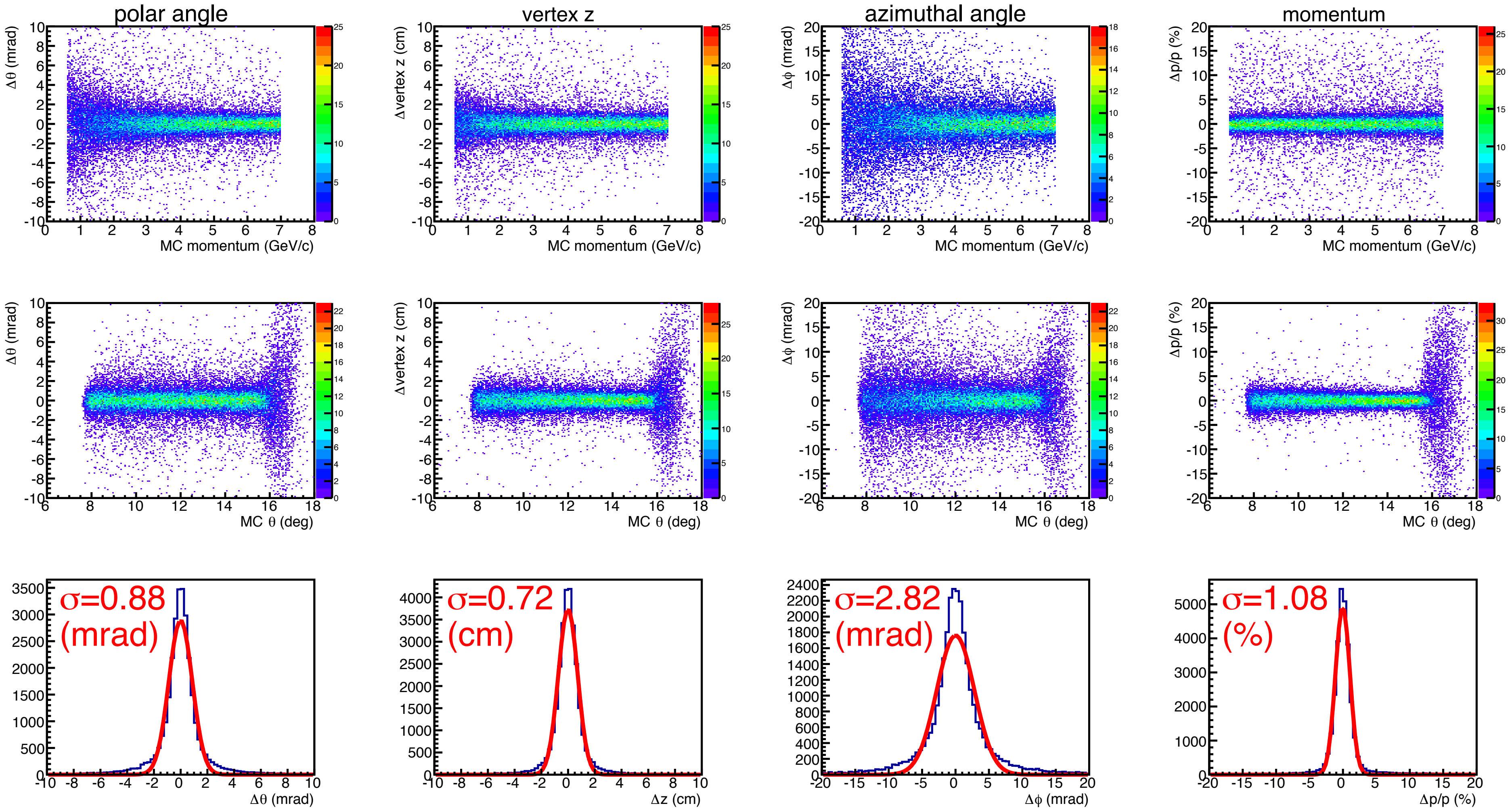
Requirement: 1mrad for polar angle and 2% for momentum



J/ψ FA Vertex Resolution

Pion in J/ψ FA. GEM resolution 90um, beam spot resolution 300um

Requirement: 0.6mrad for polar angle, 5mrad for azimuthal angle and 2% for momentum



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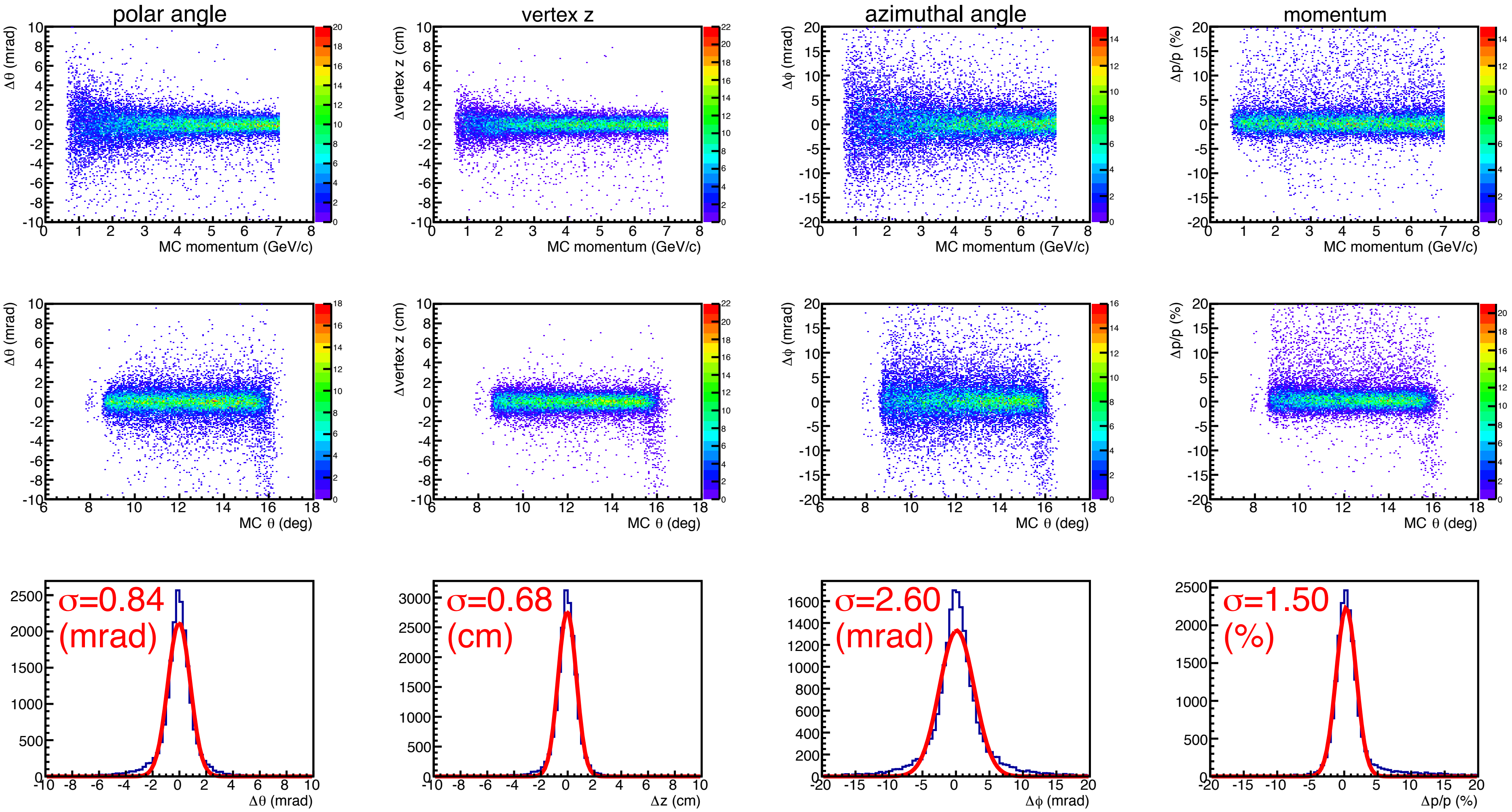


polar angle

vertex z

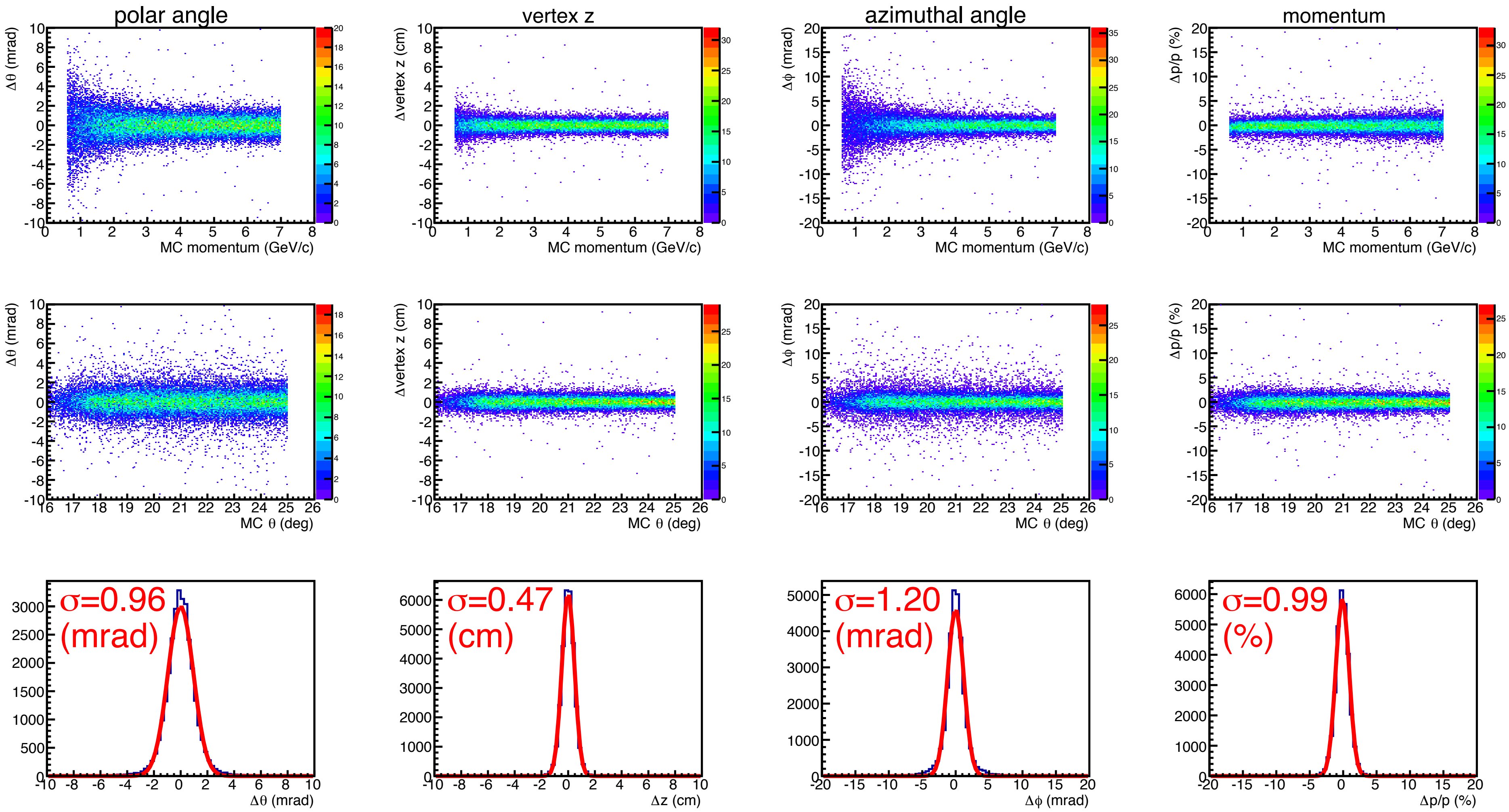
azimuthal angle

momentum

J/ψ LA Vertex Resolution

Pion in J/ψ LA. GEM resolution 90um, beam spot resolution 300um

Requirement: 0.6mrad for polar angle, 5mrad for azimuthal angle and 2% for momentum



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Electron in J/ψ LA. GEM resolution 90um, beam spot resolution 300um

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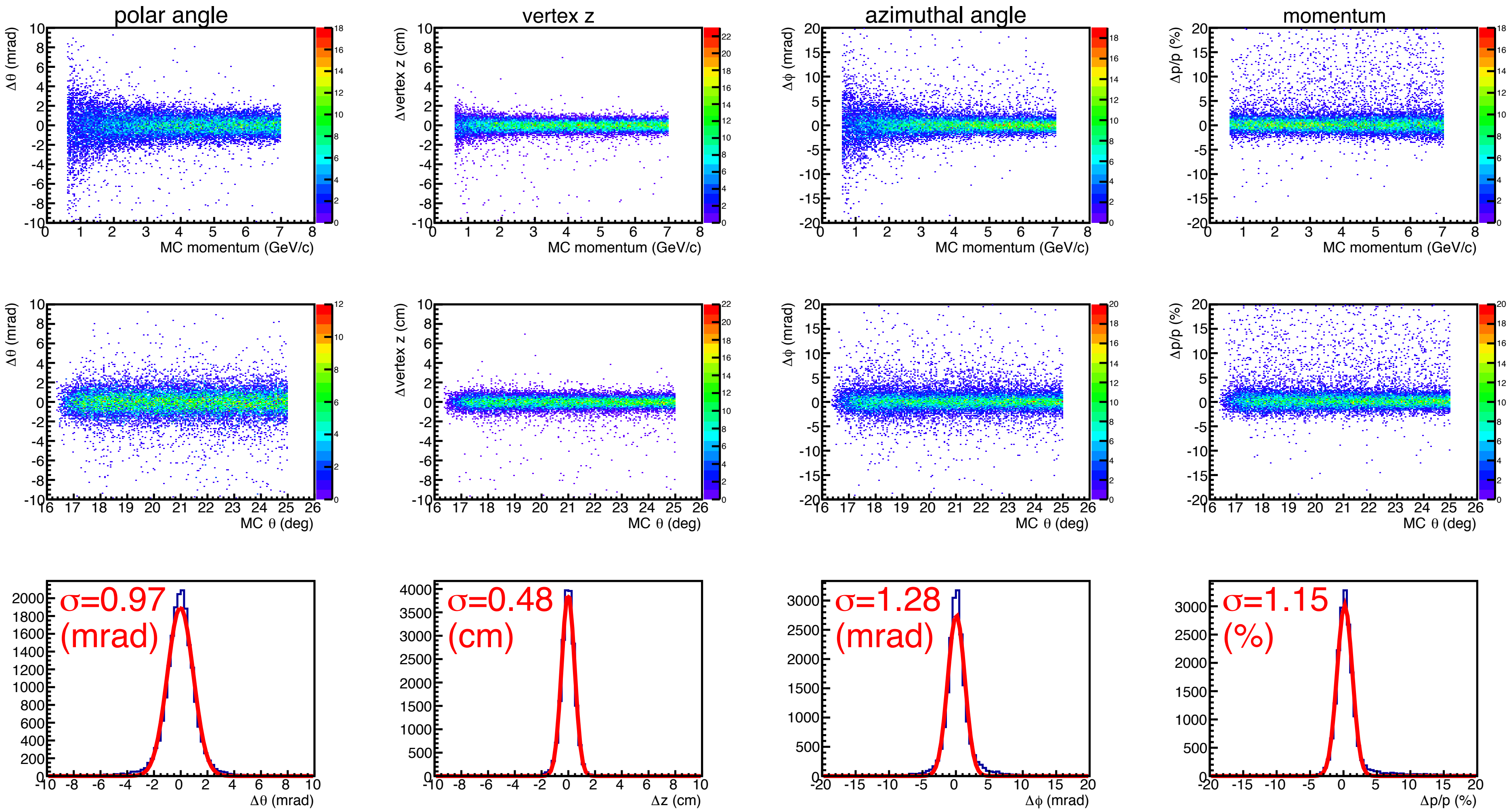

SIDIS FA Vertex Resolution

- Two collimators used for SIDIS, made of tungsten (X_0 = 0.3504cm)
- Effective in blocking electrons, but not as much for hadron

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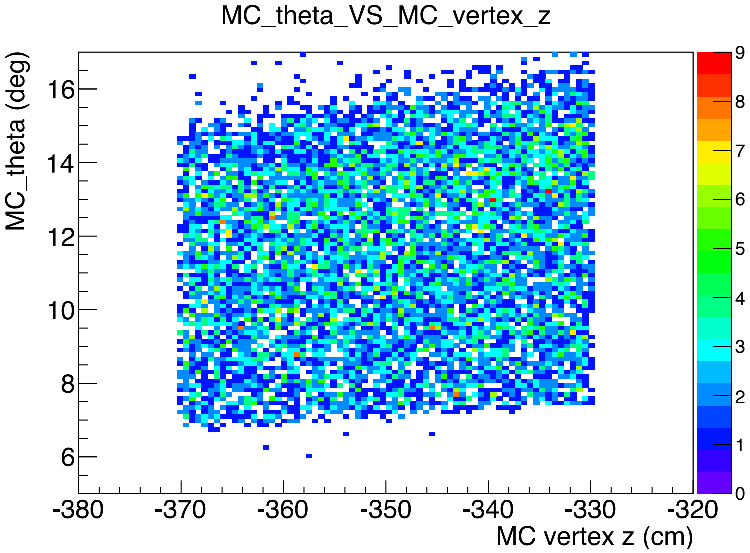
FA pion without collimators



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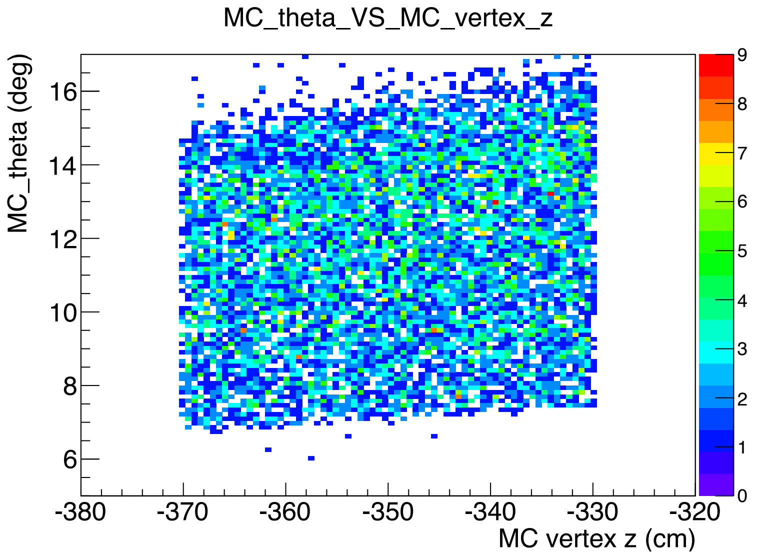
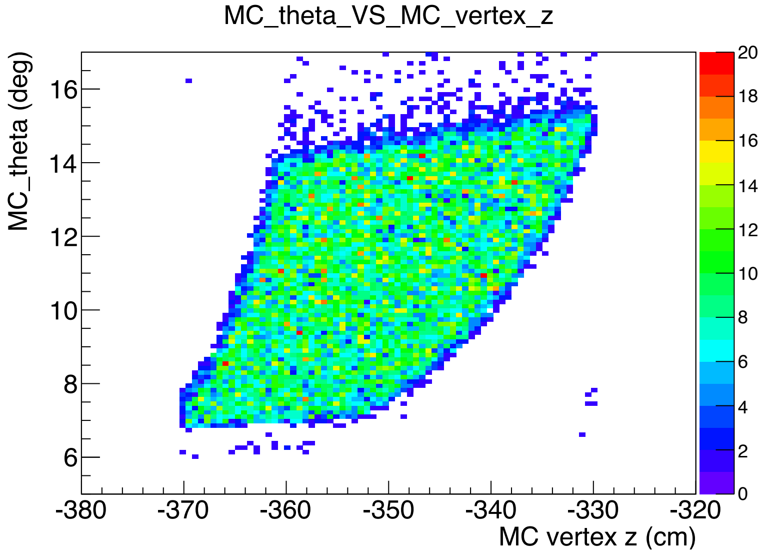


FA electron with collimators



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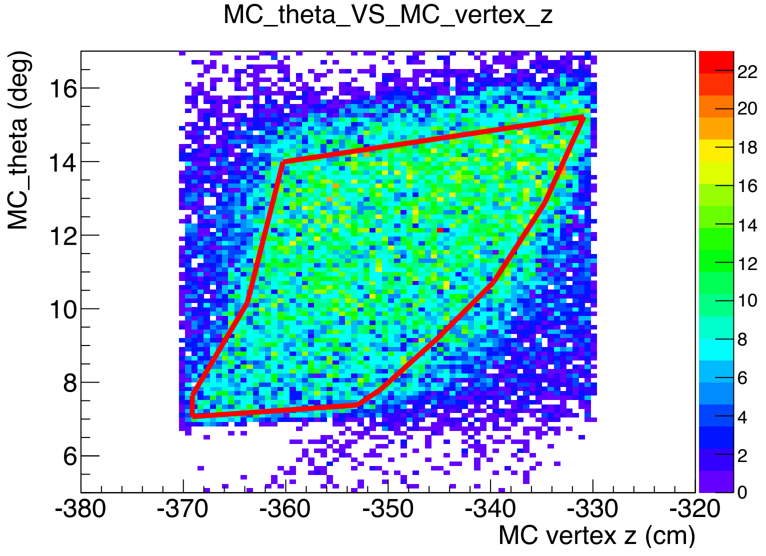
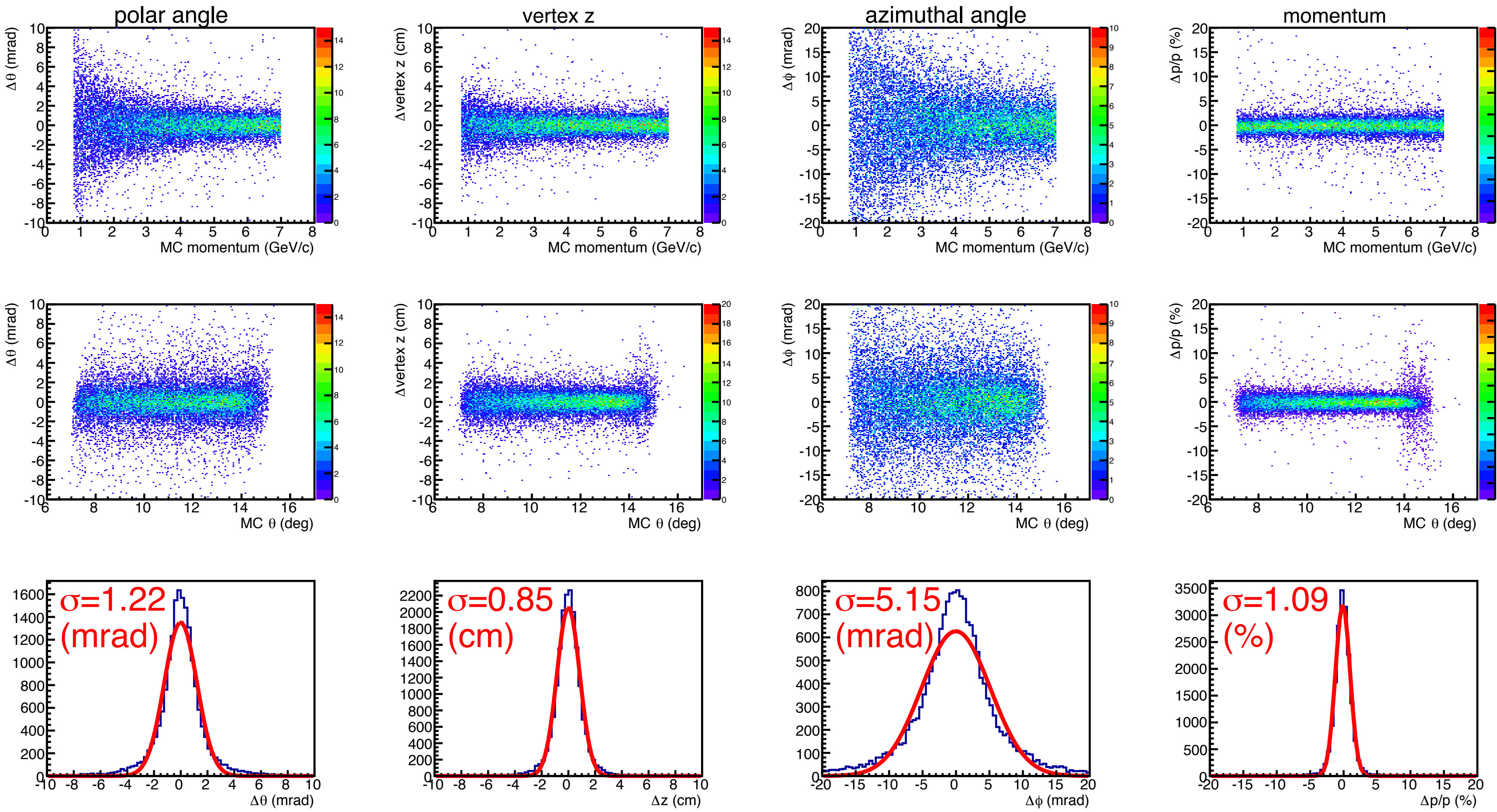


SIDIS FA Vertex Resolution

Pion in SIDIS FA. GEM resolution 90um, beam spot resolution 300um

Requirement: 0.6mrad for polar angle, 5mrad for azimuthal angle and 2% for momentum



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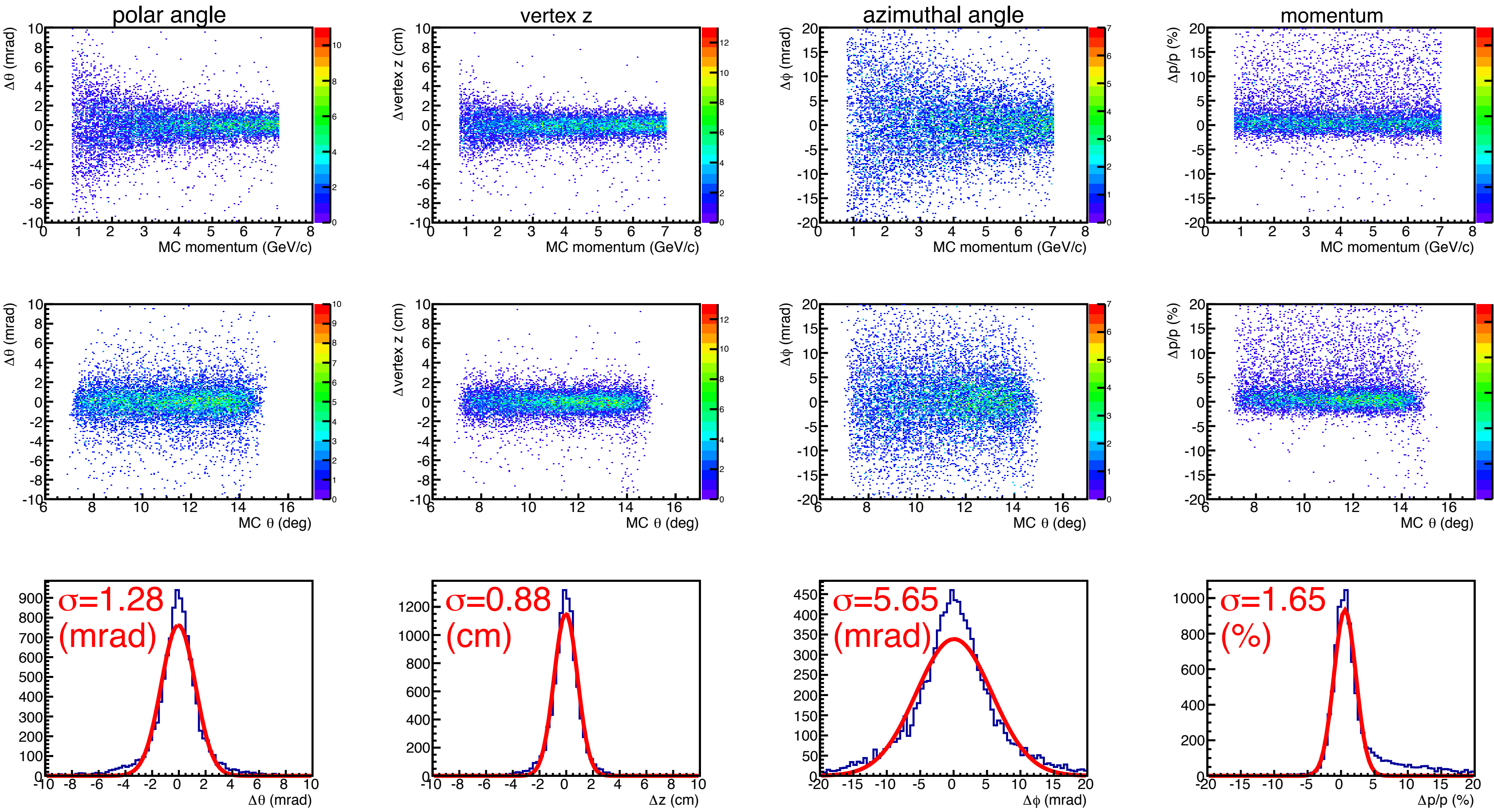
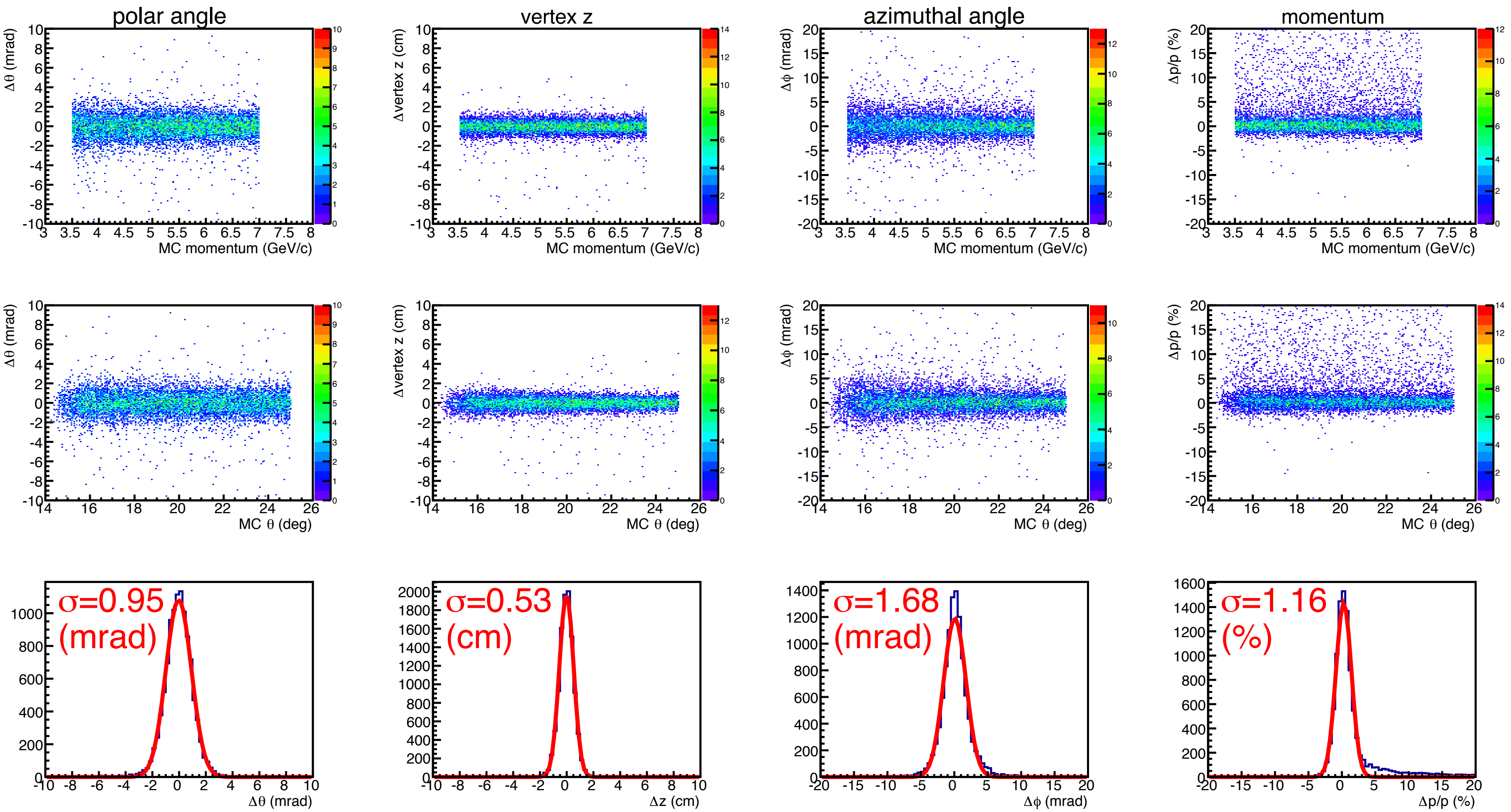


SIDIS LA Vertex Resolution

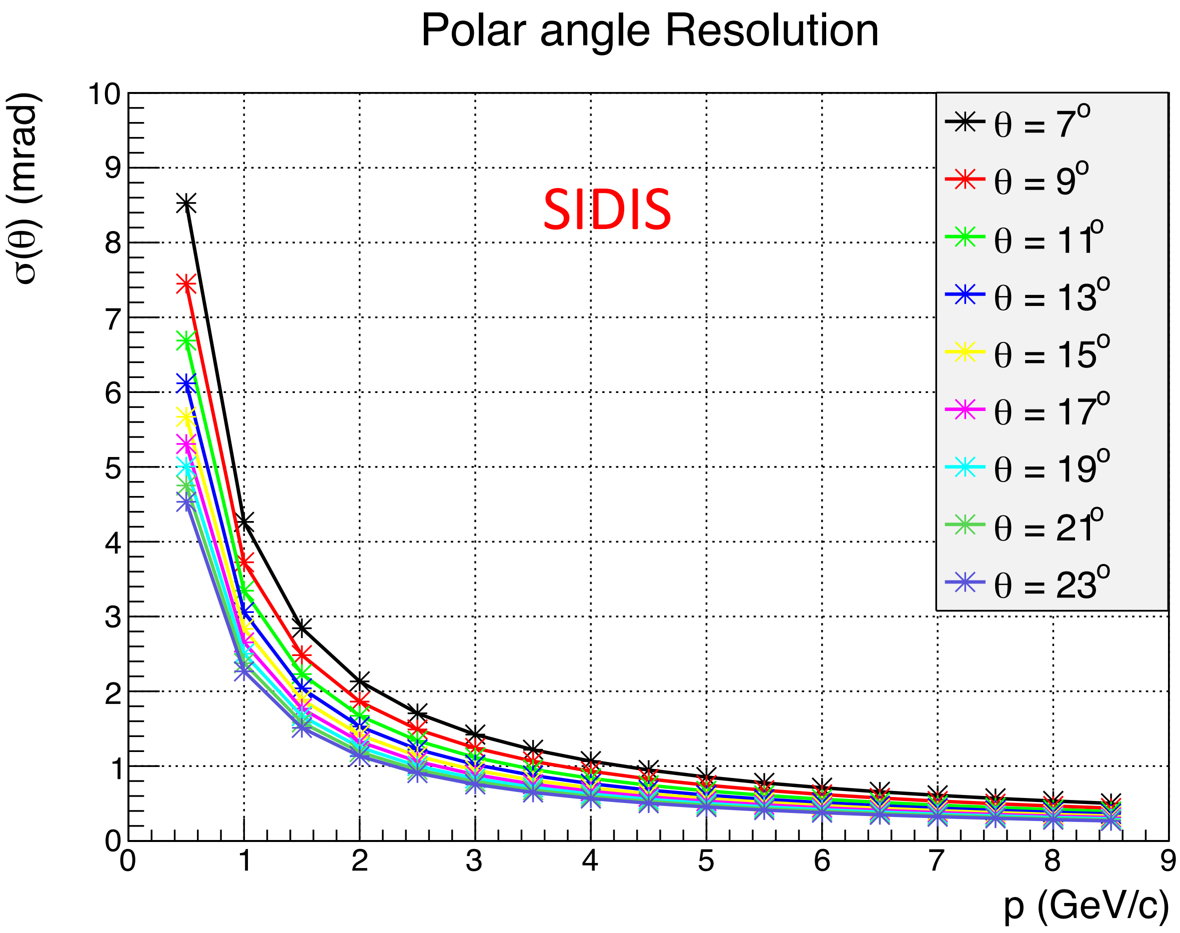
Electron in SIDIS LA. GEM resolution 90um, beam spot resolution 300um

Requirement: 0.6mrad for polar angle, 5mrad for azimuthal angle and 2% for momentum



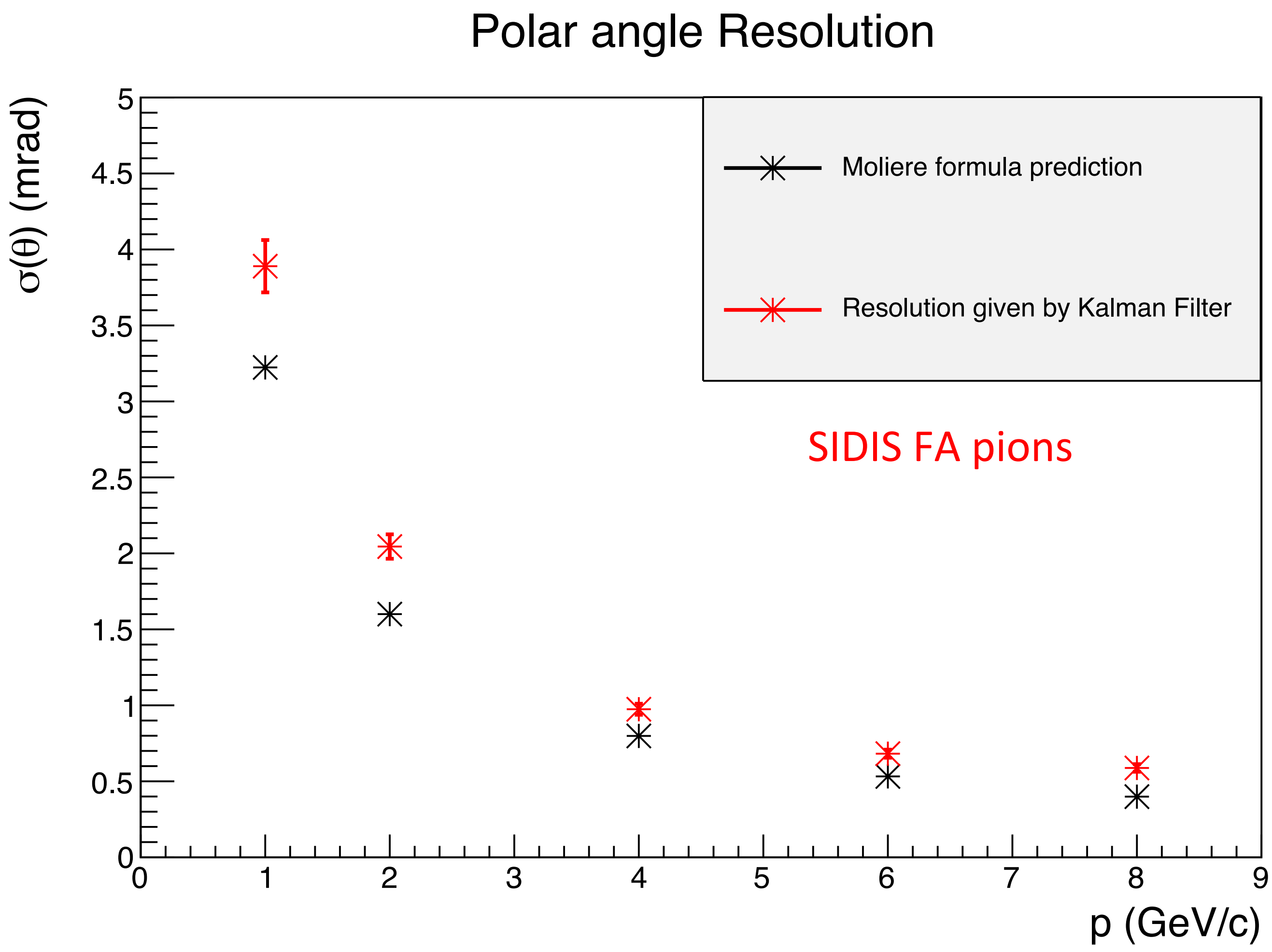
Polar Angle Resolution Limit

- Best guess for polar angle (for now): the polar angle inside the cell is the same as the polar angle outside
- In this case, polar angle resolution cannot be better than results given by the Moliere formula for particles pass though target cell



Molière VS Kalman Filter

MC theta fixed at 12 deg. MC momentum fixed at 1, 2, 4, 6, and 8 GeV.



SIDIS FA pions

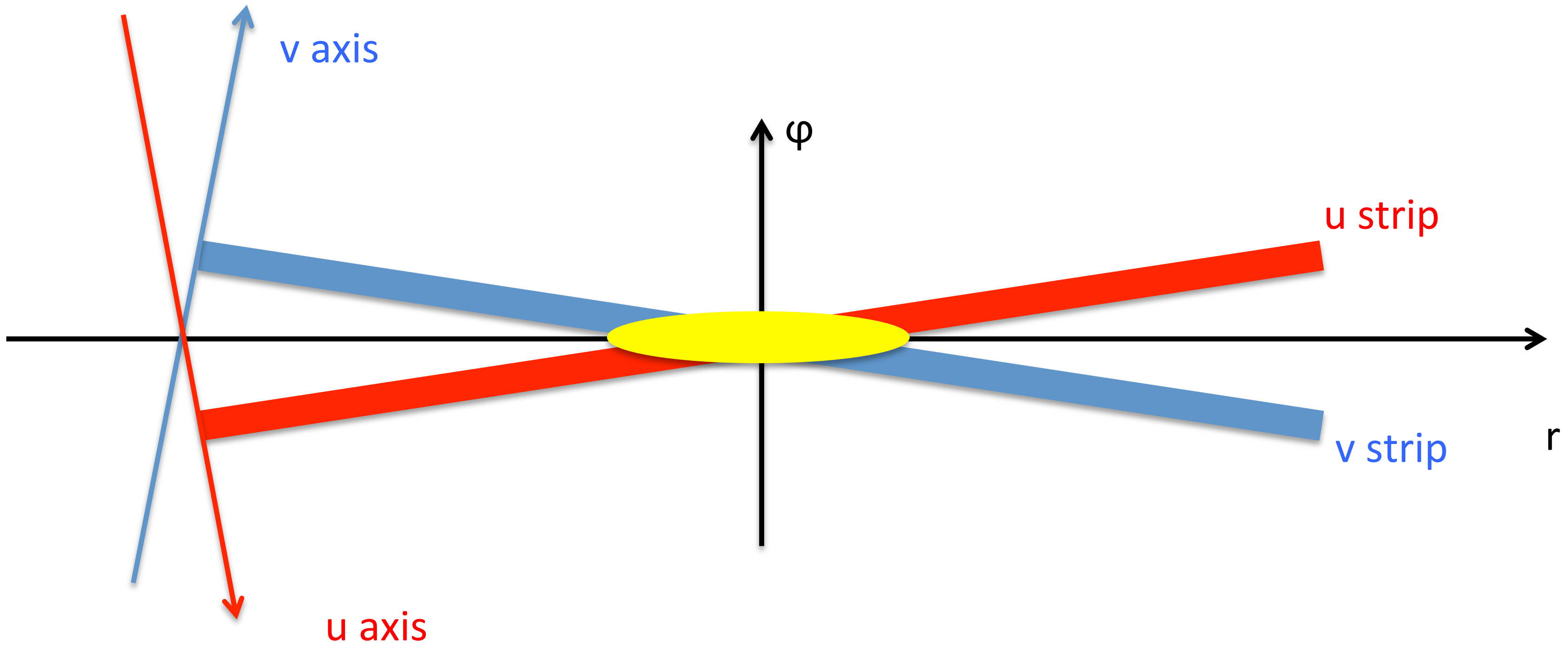
Conclusion

- Obtained preliminary resolution results for SIDIS, J/ψ and PVDIS
- Most of the resolutions satisfy requirements, except for polar angle in the SIDIS and J/ψ configuration
- Improvement quite possible by more careful calculation of the error matrices
- Main challenge is the electron radiation energy loss

Backup Slides

General Info about the Simulation

- Generator is uniform, reconstruct trajectory as long as the particle hits all the GEMs and the calorimeter
- GEM resolution is added according to:



Introduction on Kalman Filter

- Basic steps for Kalman Filter[1]:
  - The optimized state vector a_{k-1} on detector k-1 is extrapolated to detector k by means of a propagation method

$$a_k^{k-1} = f_{k-1}(a_{k-1})$$

  - The covariance matrix of the predicted state vector, and also the covariance matrix of the process noise between detector k-1 and k are computed by error propagation

$$C_k^{k-1} = F_{k-1} C_{k-1} F_{k-1}^T + Q_{k-1}$$

$$F_{k-1} = \frac{\partial f_{k-1}(a_{k-1})}{\partial a_{k-1}}$$

Introduction on Kalman Filter

- Basic steps for Kalman Filter:
  - The predicted state vector and its covariance matrix are projected into the measurement space by means of a projector H, thus obtain the predicted measurement vector
  - The weighted mean of the extrapolated and the actual measurement vector m_k of detector k is computed, yielding an optimal estimate of the state vector at k

$$C_k = (I - K_k H_k) C_k^{k-1}$$

$$a_k = a_k^{k-1} + K_k (m_k - H_k a_k^{k-1})$$

$$K_k = C_k^{k-1} (H_k)^T (V_k + H_k C_k^{k-1} (H_k)^T)^{-1}$$

In the simplest case, assume 1d state vector x, and we directly measure this quality, then the Kalman filter formulae can be reduced to a simple form

$$\frac{x_o}{\sigma_o^2} = \frac{x_p}{\sigma_p^2} + \frac{x_m}{\sigma_m^2}$$

Classical 4th Order Runge-Kutta

First of all, take the derivative of the state vector with respect to z, this is the differential equation that is going to be solved by the Runge-Kutta method:

$$\frac{d\vec{a}}{dz} = \begin{pmatrix} dx/dz \\ dy/dz \\ dt_x/dz \\ dt_y/dz \\ d(q/p)/dz \end{pmatrix} = \begin{pmatrix} t_x \\ t_y \\ t'_x \\ t'_y \\ 0 \end{pmatrix} = f(\vec{a}, z)$$

The solution of the 4th order Runge-Kutta method is:

$$\Delta\vec{a}_1 = h \cdot f(\vec{a}(z_0), z_0)$$

$$\Delta\vec{a}_2 = h \cdot f(\vec{a}(z_0) + \frac{1}{2}\Delta\vec{a}_1, z_0 + \frac{1}{2}h)$$

$$\Delta\vec{a}_3 = h \cdot f(\vec{a}(z_0) + \frac{1}{2}\Delta\vec{a}_2, z_0 + \frac{1}{2}h)$$

$$\Delta\vec{a}_4 = h \cdot f(\vec{a}(z_0) + \Delta\vec{a}_3, z_0 + h)$$

$$\vec{a}_f = \vec{a}_0 + \frac{1}{6}\Delta\vec{a}_1 + \frac{1}{3}\Delta\vec{a}_2 + \frac{1}{3}\Delta\vec{a}_3 + \frac{1}{6}\Delta\vec{a}_4 + \mathrm{O}(h^5)$$

Classical 4th Order Runge-Kutta

- The propagator matrix of this process is:

$$F = \frac{d\vec{a}_f}{d\vec{a}_0} = I + \frac{1}{6}F_1 + \frac{1}{3}F_2 + \frac{1}{3}F_3 + \frac{1}{6}F_4$$

$$F_i = \frac{d\Delta\vec{a}_i}{d\vec{a}_0}$$

- Using this method, there is no need for a pre-defined geometric track. To initialize the fit, x_0 and y_0 can be taken from the last GEM tracker along the beam direction, tx_0 and ty_0 can be calculated using the last two GEM trackers (where field is low and track is almost straight), finally p is given by the calorimeter (we always have a EC hit for PVDIS)

Material Properties

	Radiation Length (cm)	Thickness (cm)
GEM	~302.2	1.55
SIDIS Cell (GE180)	7.038	0.1
J/ψ Cell (Al)	8.897	0.0178
PVDIS Cell (Al)	8.897	0.038
LH2	890.4	1.9
LD2	769.1	2.5

- Some simple calculations for t:

	SIDIS Cell	J/ψ Cell	PVDIS Cell	GEM
10°	0.0818	0.0115	-	0.0052
20°	0.0415	0.0058	-	0.0055
30°	-	-	0.0085	0.0059

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