

Precision  
Measurement of  
the neutron  $d_2$ :  
Towards the  
Electric  $\chi_E$  and  
Magnetic  $\chi_B$  Color  
Polarizabilities

X. Zheng

*Argonne National Laboratory, Argonne, IL 60439, USA*

P. Bertin

*Université Blaise Pascal De Clermont-Ferrand, Aubiere 63177, France*

J.-P. Chen, E. Chudakov, C. W. de Jager, R. Feuerbach, J. Gomez, J. -O. Hansen,  
D.W. Higinbotham, J. LeRose, W. Melnitchouk, R. Michaels, S. Nanda, A. Saha,  
B. Wojtsekhowski

*Jefferson Lab, Newport News, VA 23606, USA*

S. Frullani, F. Garibaldi, M. Iodice, G. Urciuoli, F. Cusanno  
*Istituto Nazionale di Fisica Nucleare, Sezione Sanità, 00161 Roma, Italy*

R. DeLeo, L. Lagamba

*Istituto Nazionale di Fisica Nucleare, Bari, Italy*

A.T. Katramatou, G.G. Petratos

*Kent State University, Kent, OH 44242*

W. Korsch

*University of Kentucky, Lexington, KY 40506, USA*

W. Bertozzi, Z. Chai, S. Gilad, M. Rvachev, Y. Xiao  
*Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

L. Gamberg

*Penn State Berks, Reading, PA, 19610 USA*

F. Benmokhtar, R. Gilman, C. Glashausser, E. Kuchina,  
X. Jiang (co-spokesperson), G. Kumbartzki, R. Ransome  
*Rutgers University, Piscataway, NJ 08855, USA*

Seonho Choi (co-spokesperson)

*University of Seoul, Seoul, South Korea*

B. Sawatzky (co-spokesperson), F. Butaru, A. Lukhanin,  
Z.-E. Meziani (co-spokesperson), P. Solvignon, H. Yao  
*Temple University, Philadelphia, PA 19122, USA*

S. Binet, G. Cates, N. Liyanage, J. Singh, A. Tobias  
*University of Virginia, Charlottesville, VA 22901, USA*

T. Averett, J. M. Finn, D. Armstrong, K. Griffioen, V. Sulkosky  
*College of William and Mary, Williamsburg, VA 23185, USA*

# Precision Measurement of the neutron $d_2$ : Towards the Electric $\chi_E$ and Magnetic $\chi_B$ Color Polarizabilities

## Hall A collaboration proposal

- **Goal:**  
Determine the neutron  $d_2^n$  at  $\langle Q^2 \rangle = 3 \text{ GeV}^2$

$$d_2^n(Q^2) = \int_0^1 x^2 [2g_1^n(x, Q^2) + 3g_2^n(x, Q^2)] dx$$

- **An Experiment in Hall A:**
  - ➔ A polarized electron beam of 5.7 GeV and polarized  $^3\text{He}$  target
  - ➔ Measure unpolarized cross section for  $^3\vec{\text{He}}(\vec{e}, e')$  reaction  $\sigma_0^{^3\text{He}}$  in conjunction with the parallel asymmetry  $A_{\parallel}^{^3\text{He}}$  and the transverse asymmetry  $A_{\perp}^{^3\text{He}}$  for  $0.2 < x < 0.65$  with  $2 < Q^2 < 5 \text{ GeV}^2$ .
- **Beam Request:**
  - ➔ **13** days to achieve a statistical uncertainty of  $\Delta d_2^n = 5 \times 10^{-4}$

## Individual Proposal Report

**Proposal:** PR-01-111

**Title:** Measurement of the Neutron  $d_2^n$  Matrix Element: a Linear Combination of the Electric  $\chi_E$  and Magnetic  $\chi_B$  Color Polarizabilities.

**Spokespersons:** X. Jiang and Z.-E. Meiziani

**Motivation:** The motivation is to test model calculations (in particular, lattice models) of the  $d_2^n$  matrix element, which in the framework of the Operator Product Expansion is sensitive to twist-three quark-gluon correlations, and to electric and magnetic color polarizabilities.

**Measurement and Feasibility:** The measurement uses longitudinally polarized electron scattering from polarized  $^3\text{He}$  to measure the spin structure functions  $g_1(x)$  and  $g_2(x)$  at fixed  $Q^2=2$  (GeV/c) $^2$ . The spin asymmetry for inclusive electron scattering is measured in the standard Hall A spectrometers. The  $d_2^n$  matrix element is evaluated by integrating  $x^2(2g_1+3g_2)$  over the measured region, with extrapolations to  $x=0$  and  $x=1$  to cover the unmeasured regions. Models are used to correct for the difference between a polarized  $^3\text{He}$  nucleus and a free polarized neutron. **The experiment appears to be feasible, building on a foundation of several similar experiments in Hall A.** The experiment could provide a factor-of-four smaller error than existing data for the neutron  $d_2^n$  matrix element.

**Issues:** **The principal issue is that the integral will be strongly affected by contributions in the resonance region, which may introduce significant higher-twist contributions, clouding the interpretation in terms of quark-gluon correlations. A secondary concern is the reliability of the extraction of the value of  $d_2^n$  of the neutron from the data on  $^3\text{He}$ .** Since the nuclear effects are largest at high  $x$ , it is not obvious that these are negligible in an integral that is weighted by  $x^2$ .

**Recommendation:** Defer

**Scientific Rating:** N/A

## Individual Proposal Report

**Proposal:** PR 03-107

**Scientific Rating:** N/A

**Title:** Measurement of the Neutron  $d_2$ : Towards the Electric  $\chi_E$  and Magnetic  $\chi_B$  Color Polarizabilities

**Spokespersons:** Z.-E. Meziani, S. Choi, and X. Jiang

**Motivation:** The aim of the experiment is a precise determination of moments of the neutron spin structure functions, namely the integral of  $g_2^n$  and the  $x^2$ -moment of a particular combination of  $g_1$  and  $g_2$ , called  $d_2$ . In the framework of the Operator Product Expansion, the latter is sensitive to twist-3 quark-gluon correlations, and to electric and magnetic color polarizabilities. This quantity would be compared to lattice QCD predictions and to other model calculations.

**Measurement and Feasibility:** The measurement uses scattering of longitudinally polarized electrons from a polarized  $^3\text{He}$  target, to measure longitudinal and transverse asymmetries, together with the unpolarized cross section, from which the spin structure functions  $g_1(x)$  and  $g_2(x)$  are extracted at fixed  $Q^2 = 2 \text{ (GeV/c)}^2$ . The experiment is optimized to minimize the uncertainty on  $d_2$ , obtained by integrating  $x^2(2g_1 + 3g_2)$  over the measured region ( $x$  from 0.24 to 0.8). Extrapolations to  $x=0$  and  $x=1$  are applied to cover the unmeasured regions. A correction to account for the difference between a polarized  $^3\text{He}$  and a free polarized neutron is applied. The experiment uses existing equipment and proven techniques. It is judged feasible. It would complement the low  $Q^2$  results obtained by E-94-010 and the (significantly less precise) SLAC determination of  $d_2^n$  at  $5 \text{ (GeV/c)}^2$ . Lattice QCD calculations performed at  $Q^2 = 2 \text{ (GeV/c)}^2$  should be available in the near future, enhancing the interest in the measurement.

**Issues:** The principal issue is that the measured integral is dominated by contributions in the resonance region, thus making the interpretation in terms of color polarizabilities doubtful. However, investigating whether the twist expansion breaks down in this region is of interest, and connected to the question of quark-hadron duality. Nuclear corrections seem to be under control for the moment being addressed,  $d_2^n$ .

The PAC would have liked to see this experiment performed, but due to limitations in the available beam time, the proposal cannot be accepted at this time.

**Recommendation:** Defer with Regret.

# Polarized deep inelastic cross sections

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\uparrow - \uparrow\uparrow) = \frac{4\alpha^2}{MQ^2} \frac{E'}{\nu E} \left[ (E + E' \cos \theta) g_1(x, Q^2) - \frac{Q^2}{\nu} g_2(x, Q^2) \right] = \Delta\sigma_{\parallel}$$

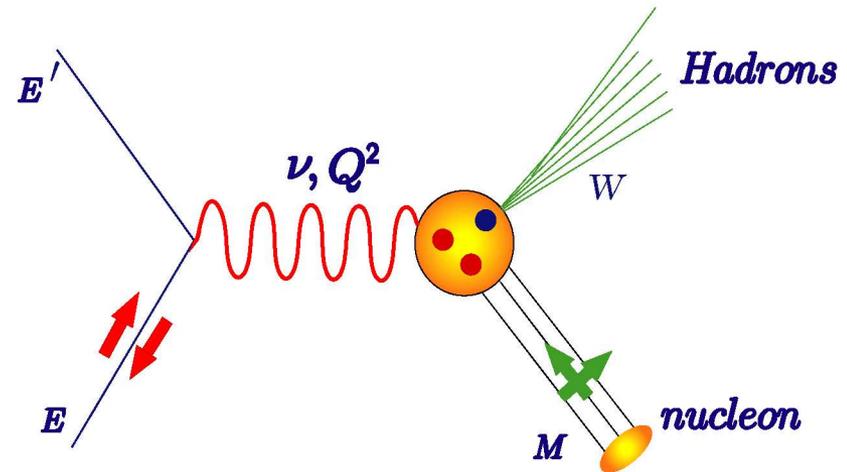
$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2 \sin \theta}{MQ^2} \frac{E'^2}{\nu^2 E} \left[ \nu g_1(x, Q^2) + 2E g_2(x, Q^2) \right] = \Delta\sigma_{\perp}$$

$Q^2$  = 4-momentum transfer squared of the virtual photon.

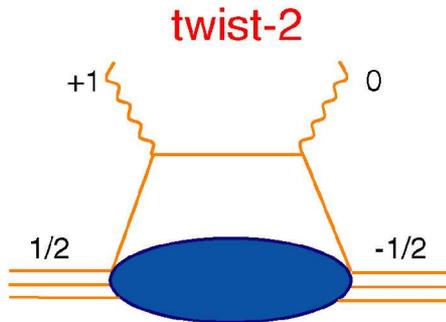
$\nu$  = energy transfer.

$\theta$  = scattering angle.

$x = \frac{Q^2}{2M\nu}$  fraction of nucleon momentum carried by the struck quark.

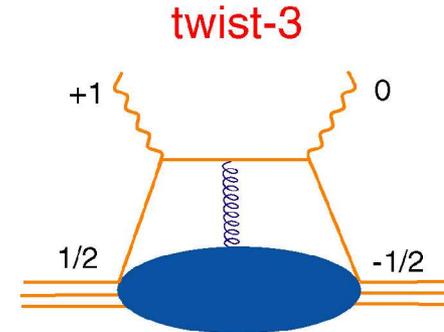


# $g_2$ and Quark-Gluon Correlations



Carry one unit of orbital angular momentum

QCD allows the helicity exchange to occur in two ways



Couple to a gluon

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

- a twist-2 term (Wandzura & Wilczek, 1977):

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(y, Q^2) \frac{dy}{y}$$

- a twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston, 92):

$$\bar{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y} \left( \frac{m_q}{M} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y}$$

transversity

quark-gluon correlation

# Moments of Structure Functions

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2) dx = \underbrace{\mu_2}_{\text{leading twist}} + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \dots$$

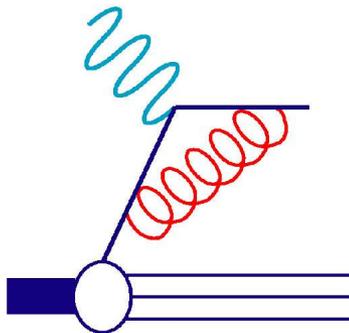
higher twist

$$\mu_2^{p,n}(Q^2) = \left( \pm \frac{1}{12} g_A + \frac{1}{36} a_8 \right) + \frac{1}{9} \Delta\Sigma + \text{pQCD corrections}$$

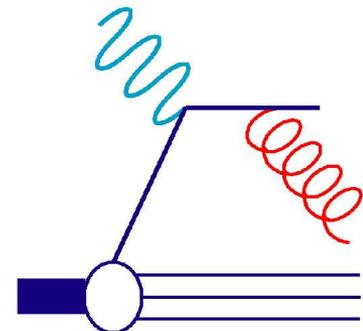
$g_A = 1.257$  and  $a_8 = 0.579$  are the triplet and octet axial charge, respectively

$\Delta\Sigma$  = singlet axial charge

(Extracted from neutron and hyperon weak decay measurements)



$$\begin{aligned} g_A &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ \Delta\Sigma &= \Delta u + \Delta d + \Delta s \end{aligned}$$



pQCD radiative corrections

# Moments of Structure Functions (continued)

$$\mu_4(Q^2) = \frac{M^2}{9} [a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)]$$

Twist - 2    Twist - 3    Twist - 4  
(TMC)

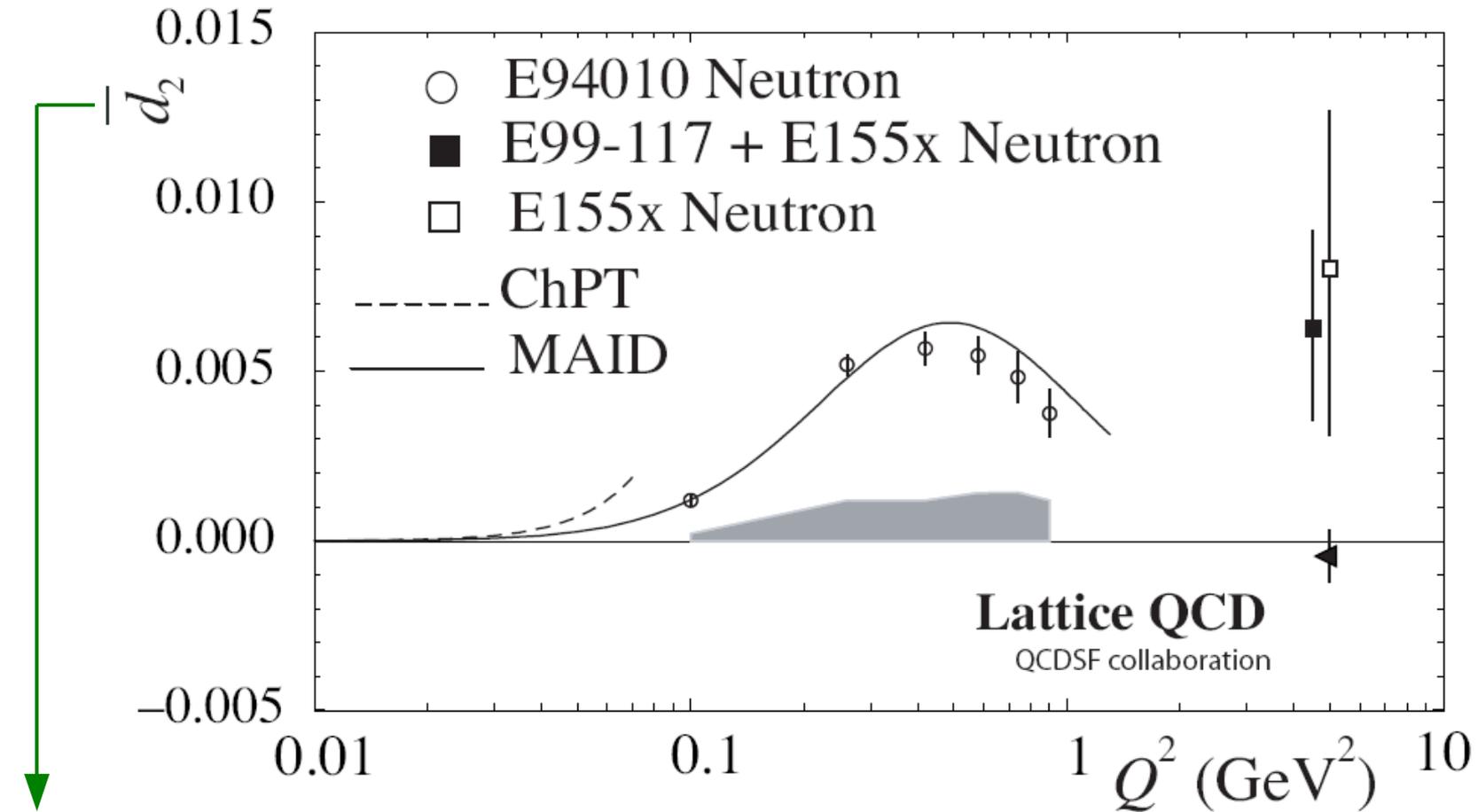
where  $a_2$ ,  $d_2$  and  $f_2$  are higher moments of  $g_1$  and  $g_2$

e.g.  $d_2(Q^2) = \int_0^1 x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] dx = \int_0^1 x^2 \overline{g_2}(x, Q^2) dx$

$$a_2(Q^2) = \int_0^1 x^2 g_1(x, Q^2) dx$$

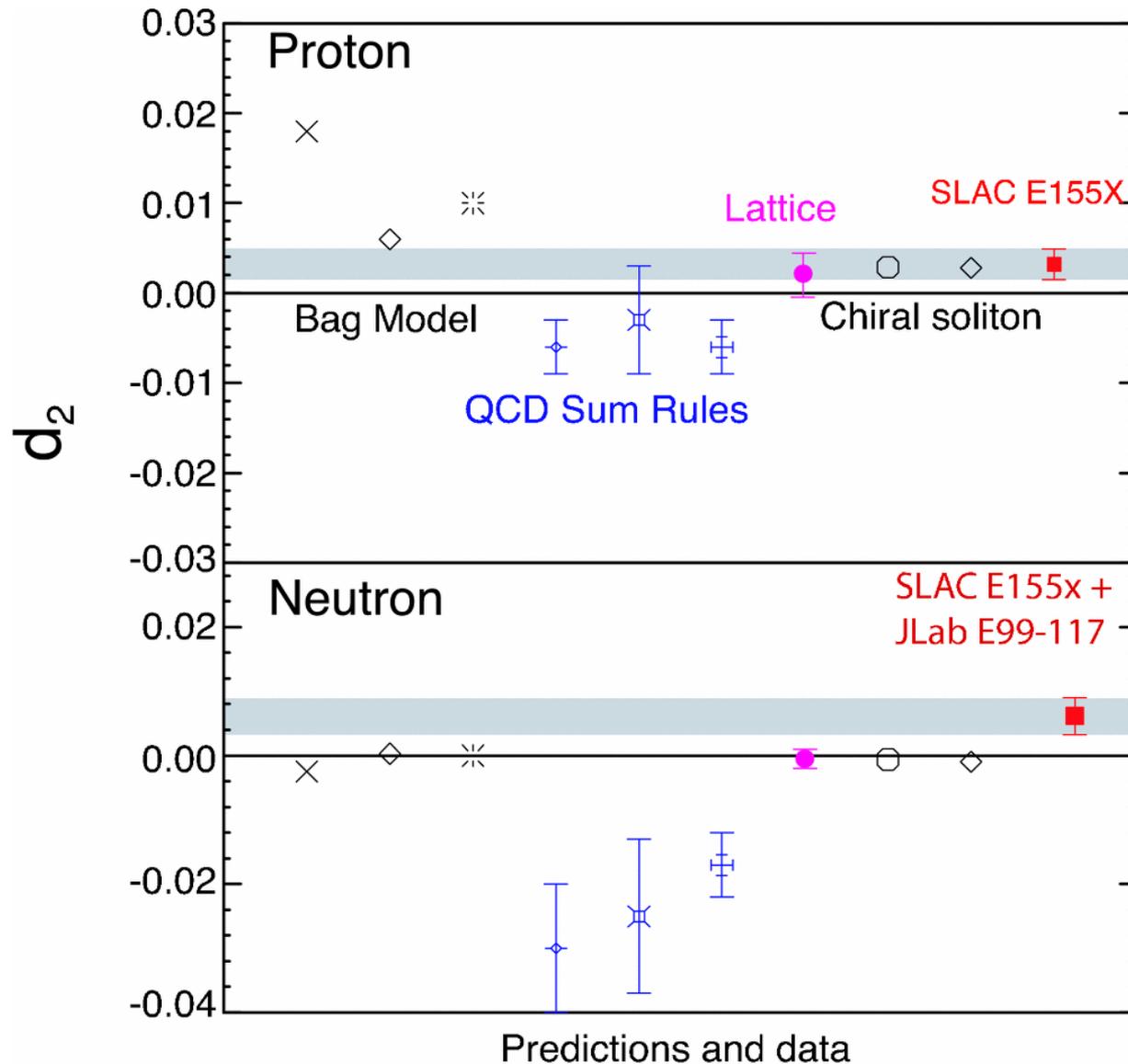
- To extract  $f_2$ ,  $d_2$  needs to be determined first.
- Both  $d_2$  and  $f_2$  are required to determine the color polarizabilities

# World Data on $\bar{d}_2$



(nucleon elastic contribution suppressed)

# Model evaluations of $d_2$



# Color "polarizabilities"

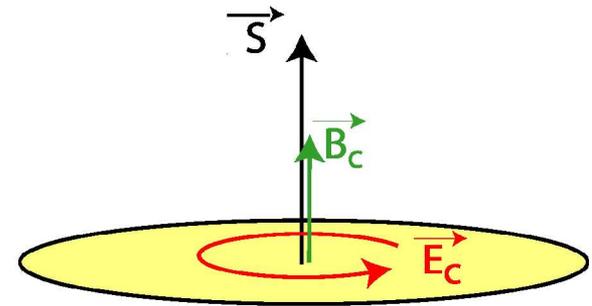
How does the gluon field respond when a nucleon is polarized ?

Define color magnetic and electric polarizabilities (in nucleon rest frame):

$$\chi_{B,E} 2M^2 \vec{S} = \langle PS | \vec{O}_{B,E} | PS \rangle$$

where  $\vec{O}_B = \psi^\dagger g \vec{B} \psi$

$$\vec{O}_E = \psi^\dagger \vec{\alpha} \times g \vec{E} \psi$$



$$\chi_E^n = (4d_2^n + 2f_2^n)/3$$

$$\chi_B^n = (4d_2^n - f_2^n)/3$$

$\chi_E$  and  $\chi_B$  represent the response of the color  $\vec{B}$  &  $\vec{E}$  fields to the nucleon polarization

# The proposal

- A 5.7 GeV polarized electron beam scattering off a polarized  $^3\text{He}$  target
- Measure unpolarized cross section for  $^3\text{He}(\vec{e}, e')$  reaction  $\sigma_0^{^3\text{He}}$  in conjunction with the parallel asymmetry  $A_{\parallel}^{^3\text{He}}$  and the transverse asymmetry  $A_{\perp}^{^3\text{He}}$  for  $0.23 < x < 0.65$  with  $2 < Q^2 < 5 \text{ GeV}^2$ .
  - ➔ Asymmetries measured by BigBite at a single angle:  $\theta = 45^\circ$
  - ➔ Absolute cross sections measured by L-HRS
- Determine  $d_2^n$  using the relation

$$\begin{aligned} \tilde{d}_2(x, Q^2) &= x^2[2g_1(x, Q^2) + 3g_2(x, Q^2)] \\ &= \frac{MQ^2}{4\alpha^2} \frac{x^2 y^2}{(1-y)(2-y)} \sigma_0 \left[ \left( 3 \frac{1 + (1-y)\cos\theta}{(1-y)\sin\theta} + \frac{4}{y} \tan\frac{\theta}{2} \right) A_{\perp} + \left( \frac{4}{y} - 3 \right) A_{\parallel} \right] \end{aligned}$$

where,

$$\begin{aligned} A_{\parallel} &= \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{2\sigma_0}, & A_{\perp} &= \frac{\sigma^{\downarrow\Rightarrow} - \sigma^{\uparrow\Rightarrow}}{2\sigma_0} \\ A_{\perp}^{^3\text{He}} &= \frac{\Delta_{\perp}}{P_b P_t \cos\phi}, & A_{\parallel}^{^3\text{He}} &= \frac{\Delta_{\parallel}}{P_b P_t} \\ \Delta_{\perp} &= \frac{(N^{\downarrow\Rightarrow} - N^{\uparrow\Rightarrow})}{(N^{\downarrow\uparrow} + N^{\uparrow\uparrow})}, & \Delta_{\parallel} &= \frac{(N^{\downarrow\uparrow} - N^{\uparrow\uparrow})}{(N^{\downarrow\uparrow} + N^{\uparrow\uparrow})} \end{aligned}$$

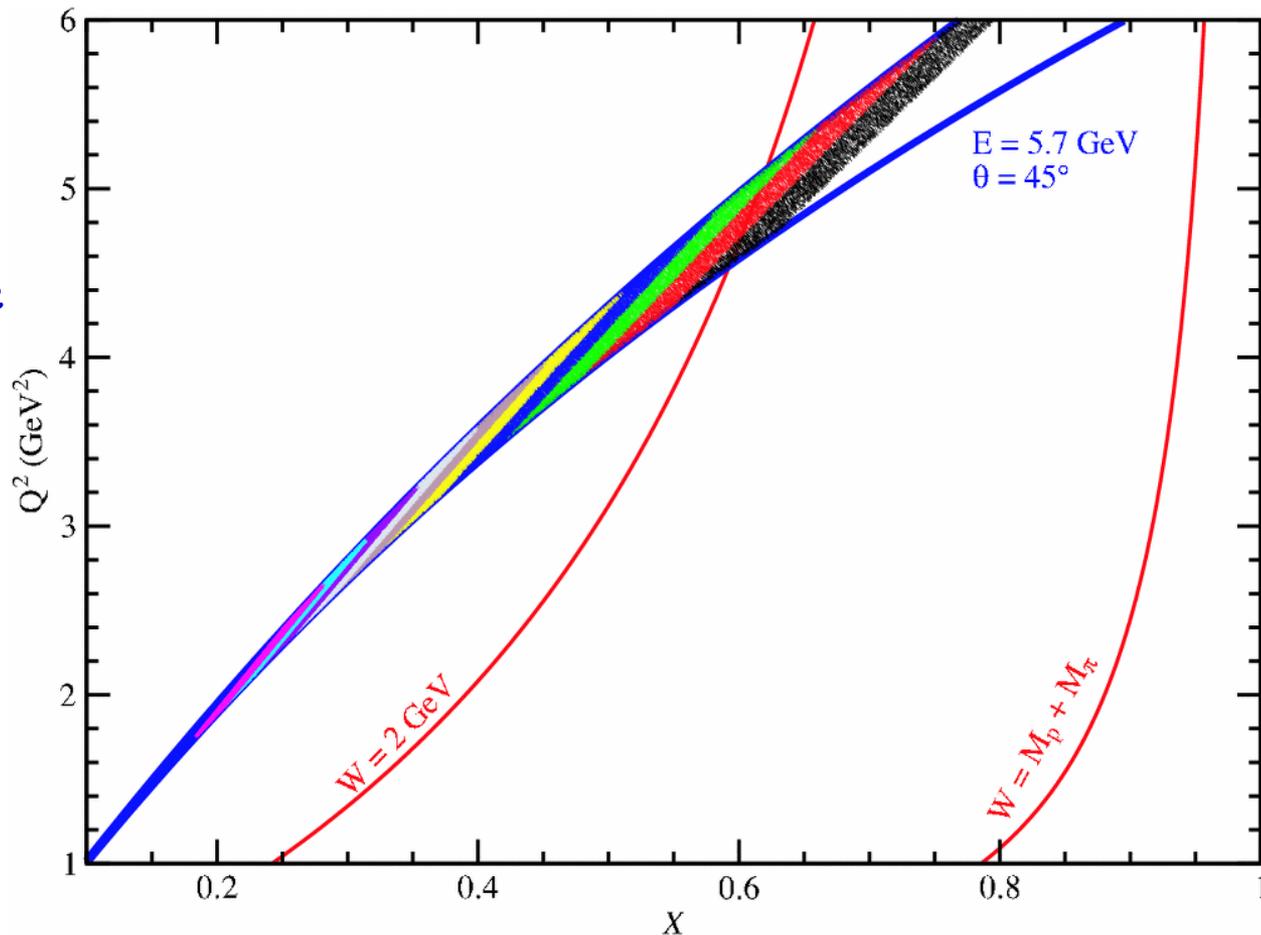
# Kinematics of the proposed measurement

$E_i$ (GeV)	bin central $p$ (GeV)	$x$	$\Delta x$	$Q^2$ (GeV <sup>2</sup> )	$W$ (GeV)	Rate (Hz)	
5.700	1.603	.696	.969E-01	5.35	1.79	0.90	
5.700	1.450	.607	.814E-01	4.84	2.00	1.5	
5.700	1.312	.532	.691E-01	4.38	2.18	2.1	
5.700	1.187	.468	.591E-01	3.96	2.32	2.7	
5.700	1.074	.413	.509E-01	3.59	2.44	3.1	Single
5.700	0.971	.324	.440E-01	3.24	2.55	3.5	Spectrometer
5.700	0.878	.365	.383E-01	2.93	2.65	3.8	Setting
5.700	0.794	.288	.335E-01	2.65	2.73	3.9	
5.700	0.718	.256	.293E-01	2.40	2.80	4.1	
5.700	0.650	.229	.259E-01	2.17	2.86	4.1	
						Time <sub>⊥</sub>	Time <sub>∥</sub>
						hours	hours
<b>Total</b>						257	10

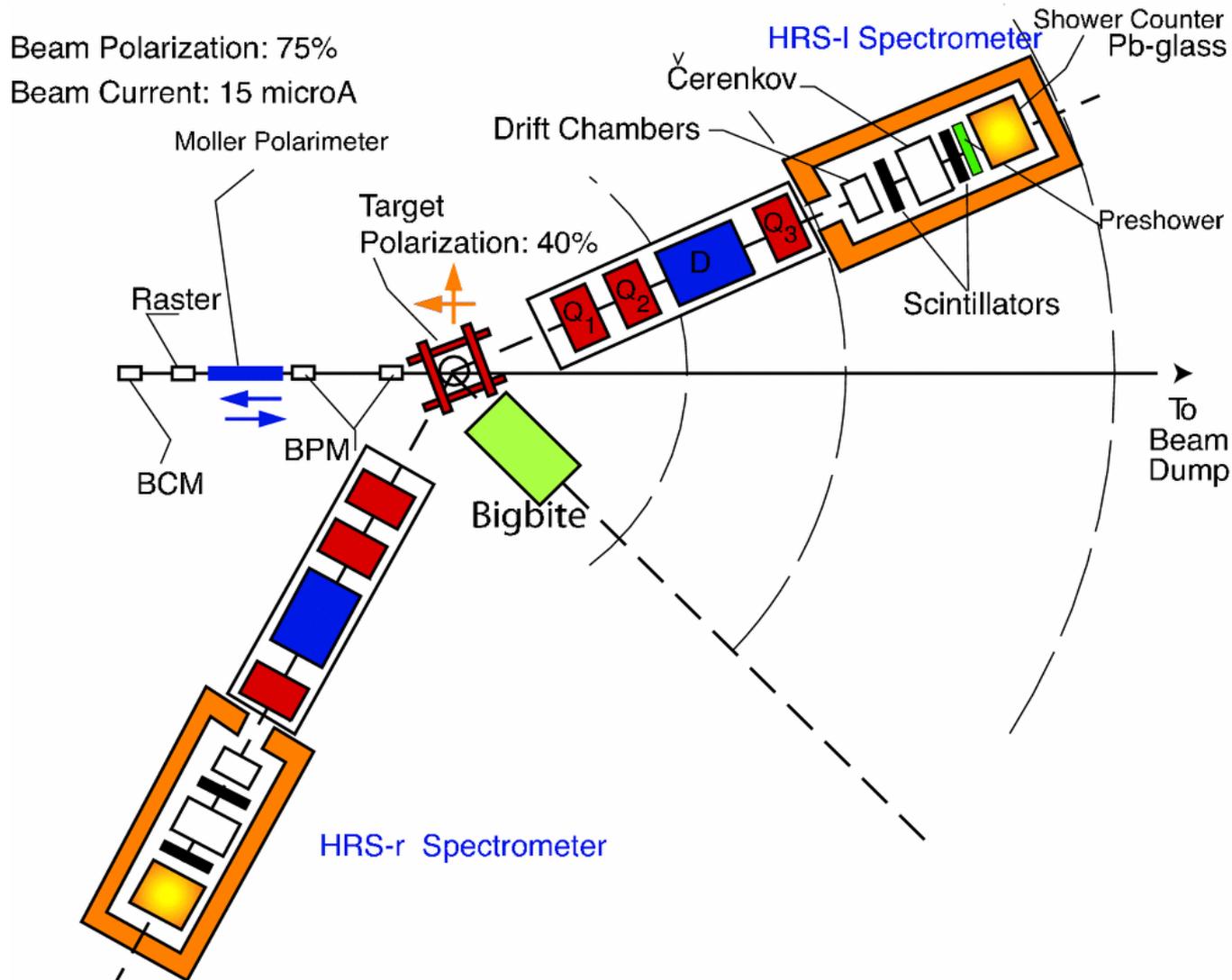
- **L-HRS** used to measure total cross section at **10 momentum settings**.  
 → will also reverse the field to monitor  $\pi^-/\pi^+$  and  $e^-/e^+$  asymmetries
- **BigBite** takes all asymmetry data with **single setting**.  
 → Data divided into 10 bins during offline analysis.
- **267 + 48 hours** for calibration and configuration changes: **13 days**

# Kinematics of the proposed measurement

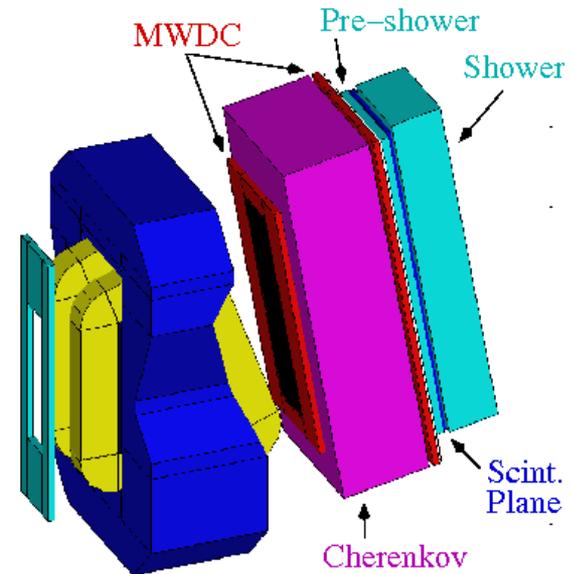
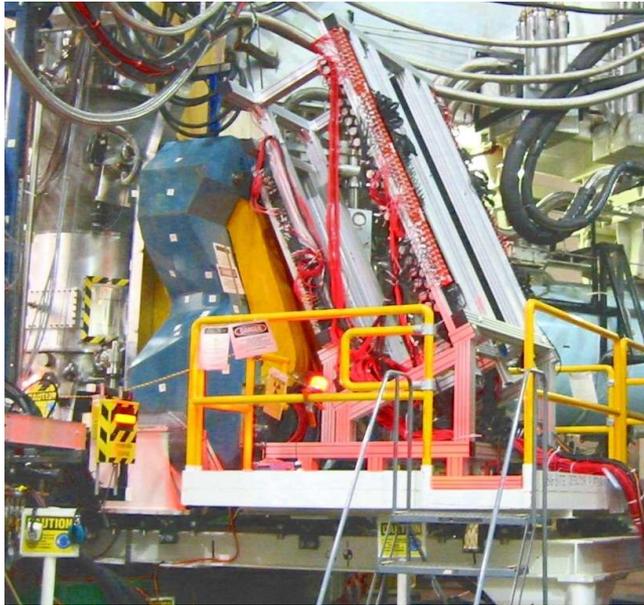
- Single beam energy  
 $5.7 \text{ GeV}$
- **BigBite** fixed at single  
scattering angle  
(data divided into 10  
bins during analysis)
- Avoid resonance  
region as much as  
possible.



# Floor configuration for this proposal



# BigBite Configuration



- non-focusing, large acceptance, open geometry
- $\Delta p/p = 1 - 1.5\%$  (@ 1.2 T)  $\sigma(W) = 50$  MeV
- angular resolution 1.5 mr, extended target resolution 6 mm
- large solid angle: 64 msr
- detector package
  - ➔ 2 MWDCs, segmented trigger, Pb-glass shower
  - ➔ Gas Cherenkov (new)

# Cherenkov Design Parameters

- Dimensions: 200cm x 60cm x 60cm
  - located in gap between first and second wire chamber with minimal modifications to BigBite frame
- Radiator gas:  $C_4F_{10}$  (or Freon12)
  - $n = 1.0015$  (1.0011)
  - $\pi$  threshold: 2.51 GeV/c (2.98 GeV/c)
  - ~28 (18) photo-electrons / 40 cm electron track
    - ↳ Quartz PMT (Photonis XP4318 or equiv.)
    - ↳ mirror reflectivity: ~90%, 10% loss at PMT-gas interface
  - >99% efficient with 4-5 p.e. threshold
    - ↳ negl. pion contamination
    - ↳ **minimum**  $\pi/e$  rejection ratio 1000:1 online

# Background Rates

- MC simulation by Degtyarenko et al. (tested in Halls A and C)
- Online cuts include:
  - BB magnet sweeps particles with  $p < 200 \text{ MeV}/c$
  - GEN BB trigger: shower+pre-shower+scint
    - ↳ provide  $\sim 10:1$  online hadron rejection (or better)
  - $\sim 550\text{--}600 \text{ MeV}$  threshold on shower
  - 4–5 p.e. threshold on Cherenkov
    - ↳ heavily suppress random background
    - ↳ negl. pion contamination ( $\sim 100 \text{ Hz}$  knock-ons)
- Total estimated trigger rate (GEN trig + Cherenkov): 2–5 kHz

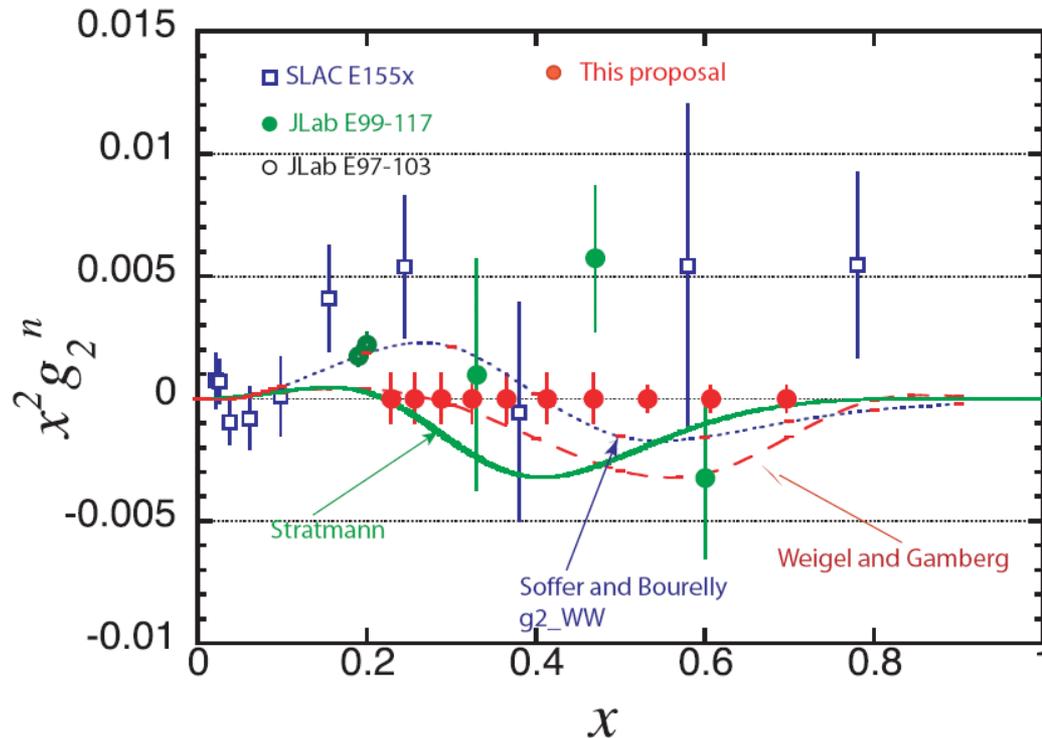
Online  
triggers

$e^-$	2-5 kHz
$e^+$	<1 kHz

$\pi^-$	90 kHz
$\pi^+$	90 kHz
p	50 kHz
n	50 kHz

Removed via  
online cuts

# Projected $x^2 g_2(x, Q^2)$ results

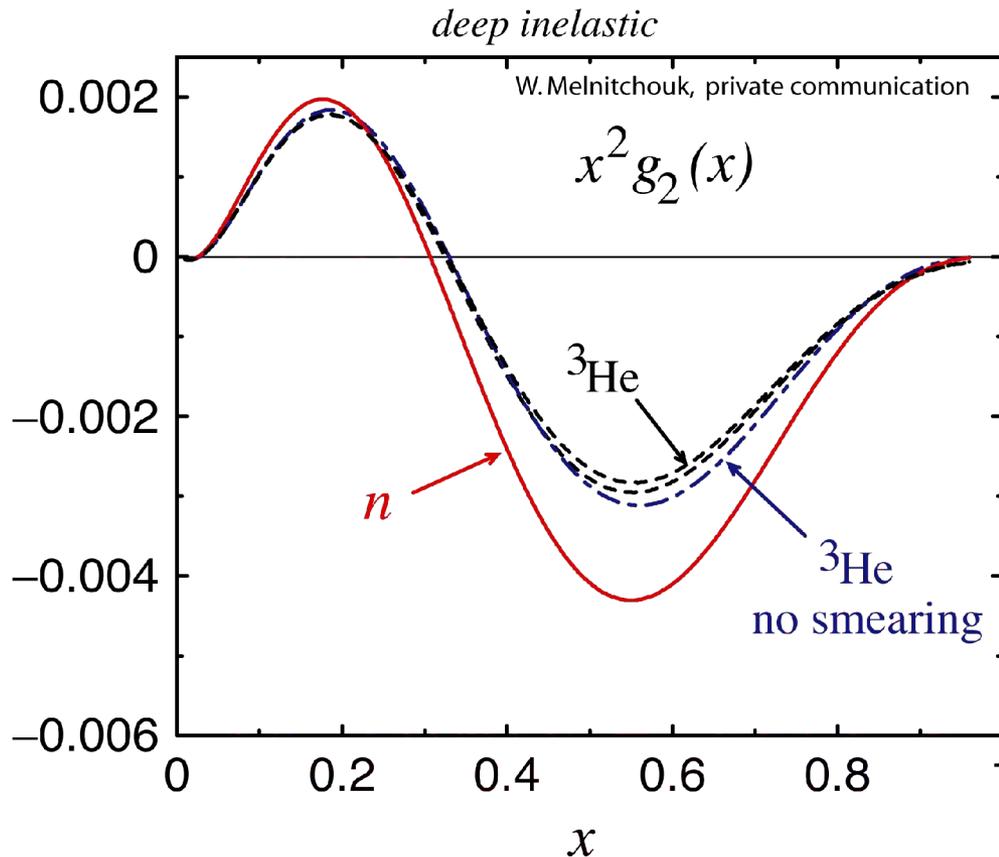


- $g_2$  for  $^3\text{He}$  is extracted directly from  $L$  and  $T$  spin-dependent cross sections measurements within the same experiment.
- The nuclear corrections will be applied to the moments not to the structure functions.
- SLAC E155x  $g_2$  data points at high  $x$  are evolved from  $Q^2$  as large as  $16 \text{ GeV}^2$  to  $5 \text{ GeV}^2$

# Nuclear corrections

- Convolution method using the impulse approximation and realistic ground state wave functions of  ${}^3\text{He}$  (in Bjorken limit:  $g_1^{3\text{He}}$  related to  $g_1^{\text{N}}$ ).
  - Variational Method,
    - ↳ C. Ciofi degli Atti & S. Scopetta, *Phys. Lett. B* 404 (1997) 223, for  $g_1$ ,  
for  $g_2$ , S. Scopetta. private communication
  - Faddeev
    - ↳ F. Bissey et al. *Phys. Rev. C* 64 (2001) 024004
- Finite  $Q^2$  effects (both  $g_1^{\text{N}}$  and  $g_2^{\text{N}}$  contribute to  $g_1^{3\text{He}}$  and to  $g_2^{3\text{He}}$ )
  - S.A. Kulagin and W. Melnitchouk

# From $^3\text{He}$ to Neutron



- ✓ Correction large for  $g_2$  but much smaller for  $d_2$
- ✓ About 5% difference between additive or convolution methods or between potential models

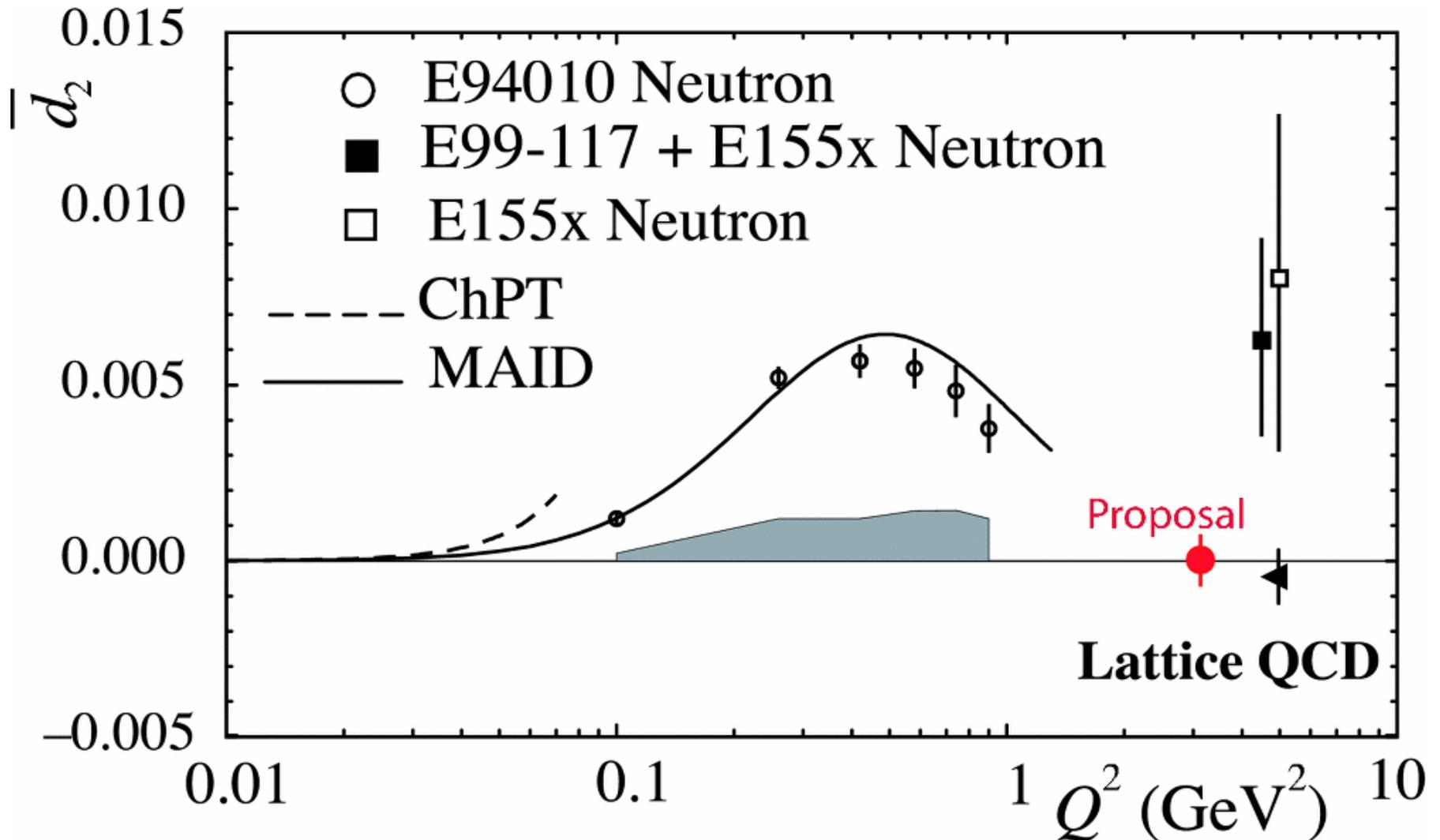
$$d_2^n = d_2^{^3\text{He}} / (1 - \delta^c) \quad \text{with} \quad \delta^c \approx 0.35$$

$$\Delta\delta^c \approx 0.15\delta^c \approx 0.05 \quad \Rightarrow \quad \Delta d_2^n / d_2^n \approx 5\%$$

# Systematic Error Contributions to $d_2^n$

Target polarization		4 %
Beam polarization		3 %
Asymmetry (raw)	<ul style="list-style-type: none"> <li>• Target spin direction (<math>0.5^\circ</math>)</li> <li>• Beam charge asymmetry</li> </ul>	$\approx 1.5 \times 10^{-3}$ 200 ppm
Cross section (raw)	<ul style="list-style-type: none"> <li>• PID efficiency</li> <li>• Background Rejection efficiency</li> <li>• Beam charge</li> <li>• Beam position</li> <li>• Acceptance cut</li> <li>• Target density</li> <li>• Nitrogen dilution</li> <li>• Dead time</li> <li>• Finite Acceptance cut</li> </ul>	$\approx 1$ % $\approx 1$ % $< 1$ % $< 1$ % 2-3 % 2-3 % 2-3 % $< 1$ % $< 1$ %
Radiative corrections		$\leq 10$ %
From $^3\text{He}$ to Neutron correction		5 %
Total effect		$\leq 10$ %
Estimate of contributions	$\int_{0.003}^{0.23} \tilde{d}_2^n dx$	$4.8 \times 10^{-4}$
from unmeasured regions	$\int_{0.70}^{0.999} \tilde{d}_2^n dx$	$5.0 \times 10^{-5}$
Projected absolute statistical uncertainty		$\Delta d_2 \approx 5.4 \times 10^{-4}$
Projected absolute systematic uncertainty assuming $d_2 = 5 \times 10^{-3}$		$\Delta d_2 \approx 5 \times 10^{-4}$

# Expected Error on $d_2$



# Summary

- We propose to precisely measure the neutron  $d_2^n$  at  $Q^2 \approx 3.0 \text{ GeV}^2$ .
  - ➔ Determine asymmetries in conjunction with an absolute cross section measurement over the region ( $0.23 < x < 0.65$ )
- Provide a **benchmark test** for theory (lattice QCD).
  - ➔ we can achieve a statistical uncertainty of  $\Delta d_2^n = 5 \times 10^{-4}$ 
    - ↳ **four** times better than existing world average!
- Utilize standard Hall A equipment **with one addition**:
  - ➔ **new Gas Cerenkov** detector for BigBite

## We request

- **13 days** of polarized beam at 5.7 GeV.
  - ➔ **257 hours of transverse** settings, **10 hours of longitudinal** settings, and 48 hours of overhead and calibration.

Proposal: <<http://www.jlab.org/~brads/PAC29/>>

# Nuclear corrections (continued)

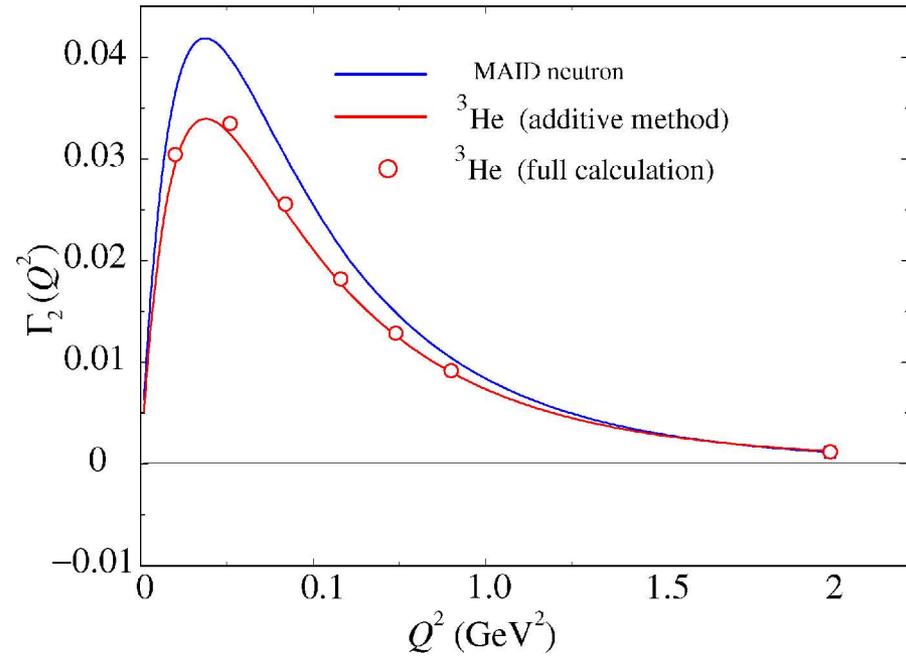
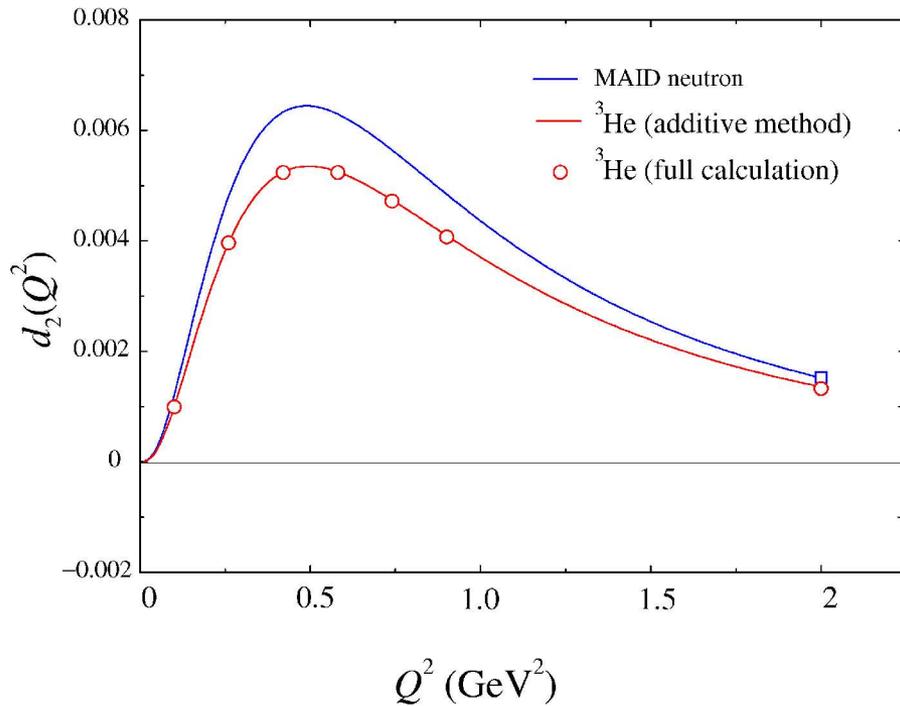
$$S(\vec{p}, E) = \frac{1}{2} \left( f_0 + f_1 \vec{\sigma}_N \cdot \vec{\sigma}_A + f_2 \left[ \vec{\sigma}_N \cdot \hat{p} \vec{\sigma}_A \cdot \hat{p} - \frac{1}{3} \vec{\sigma}_N \cdot \vec{\sigma}_A \right] \right)$$

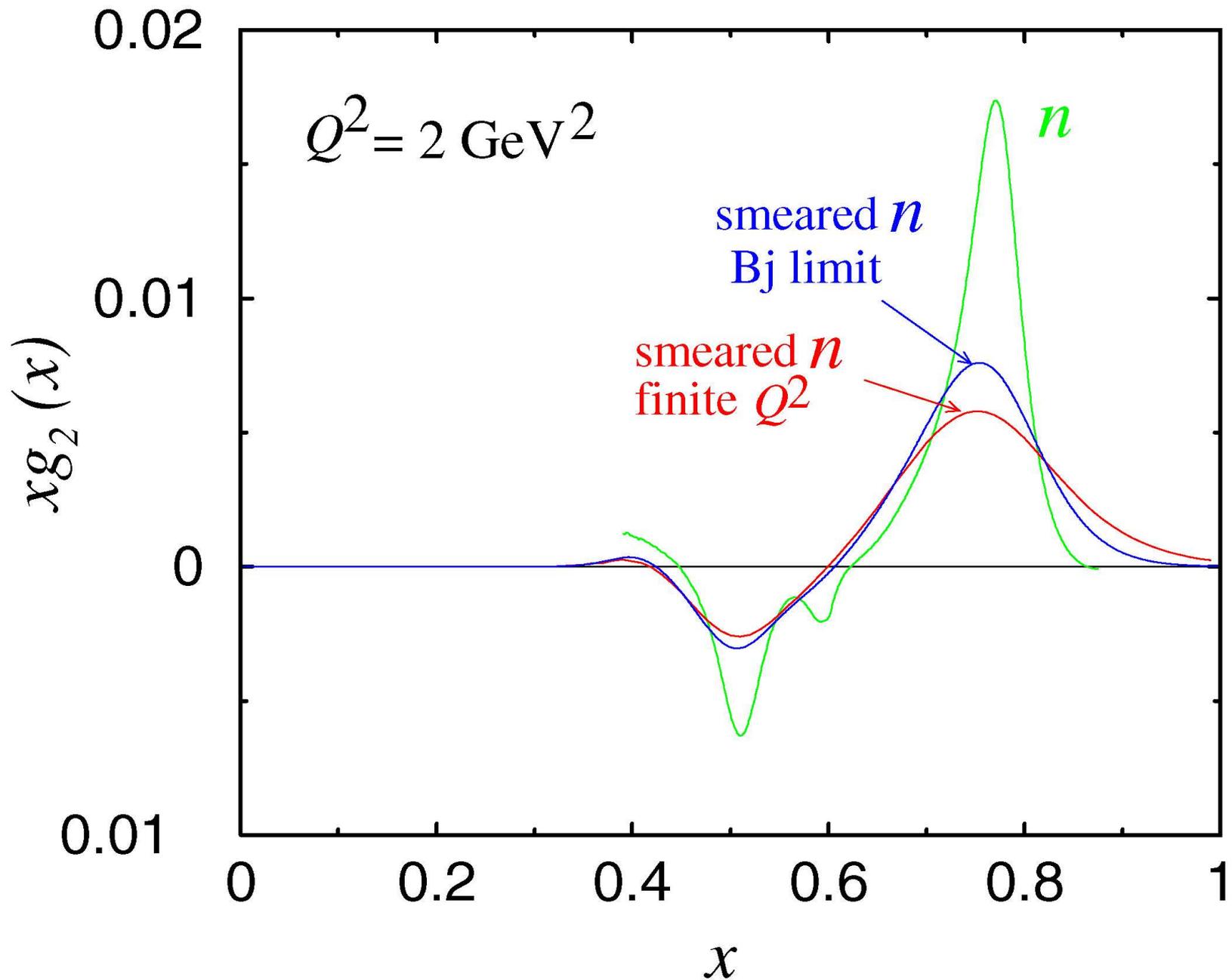
$$\begin{aligned} & x g_1^{3\text{He}}(x, Q^2) + (1 - \gamma^2) x g_2^{3\text{He}}(x, Q^2) \\ = & \sum_{N=p,n} \int d^3p dE (1 - \frac{\epsilon}{M}) \left\{ \left[ \left( 1 + \frac{\gamma p_z}{M} + \frac{p_z^2}{M^2} \right) f_1 + \left( -\frac{1}{3} + \hat{p}_z^2 + \frac{2\gamma p_z}{3M} + \frac{2p_z^2}{3M^2} \right) f_2 \right] z g_1^N(z, Q^2) \right. \\ & \left. + (1 - \gamma^2) \left( 1 + \frac{\epsilon}{M} \left[ f_1 + \left( \frac{p_z^2}{\vec{p}^2} - \frac{1}{3} \right) f_2 \right] \frac{z^2}{x} g_2^N(z, Q^2) \right) \right\} \end{aligned}$$

$$\begin{aligned} & x g_1^{3\text{He}}(x, Q^2) + x g_2^{3\text{He}}(x, Q^2) \\ = & \sum_{N=p,n} \int d^3p dE (1 - \frac{\epsilon}{M}) \left\{ \left[ \left( 1 + \frac{p_x^2}{M^2} \right) f_1 + \left( \vec{p}_x^2 - \frac{1}{3} + \frac{2p_x^2}{3M^2} \right) f_2 \right] z g_1^N(z, Q^2) \right. \\ & \left. + \left[ \left( 1 + \frac{p_x^2}{M^2} (1 - z/x) \right) f_1 + \left( \vec{p}_x^2 - \frac{1}{3} + \frac{2p_x^2}{3M^2} (1 - z/x) - \frac{\gamma p_z \hat{p}_x^2 z}{M x} \right) f_2 \right] z g_2^N(z, Q^2) \right\} \end{aligned}$$

with  $\gamma = \sqrt{1 + 4M^2 x^2 / Q^2}$  a kinematical factor parameterizing the finite  $Q^2$  correction,  $\epsilon \equiv \vec{p}^2 / 4M - E$ , and  $z = x / (1 + (\epsilon + \gamma p_z) / M)$ .

# Nuclear corrections (continued)





# How $g_2(x, Q^2)$ is usually obtained

$$g_2(x, Q^2) = \frac{\nu}{2E} \left[ \frac{\nu [1 + \epsilon \mathbf{R}(x, Q^2)] (1 + \gamma^2) \mathbf{F}_2(x, Q^2) \mathbf{A}_\perp(x, Q^2)}{(1 - \epsilon) 2x [1 + \mathbf{R}(x, Q^2)] E' \sin \theta_e} - \mathbf{g}_1(x, Q^2) \right]$$

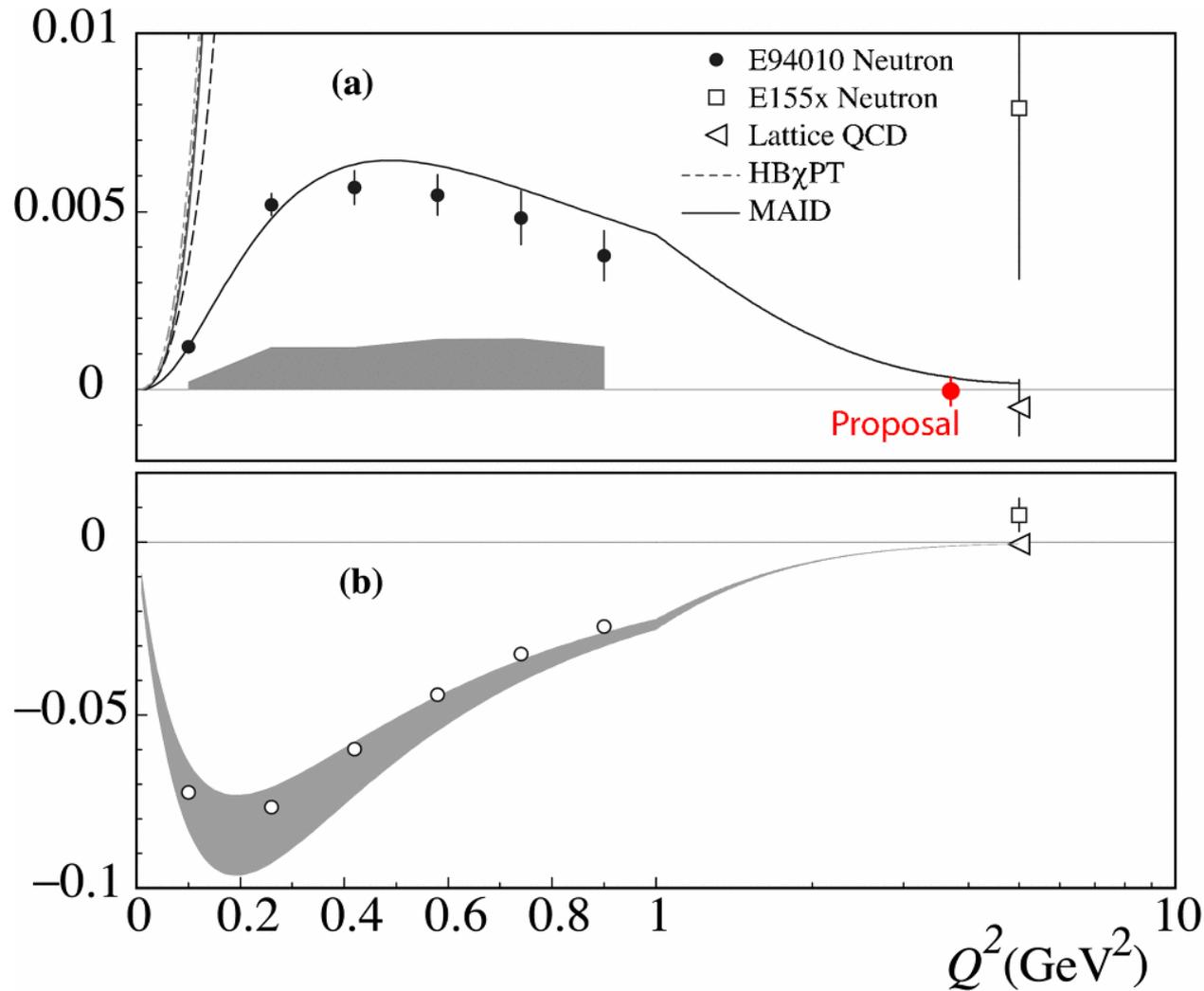
where  $\nu = E - E'$ ,  $\gamma^2 = Q^2/\nu^2$  and  $\epsilon^{-1} = 1 + 2 [1 + \gamma^{-2}] \tan^2 \theta/2$

$\mathbf{F}_2(x, Q^2)$  NMC fit

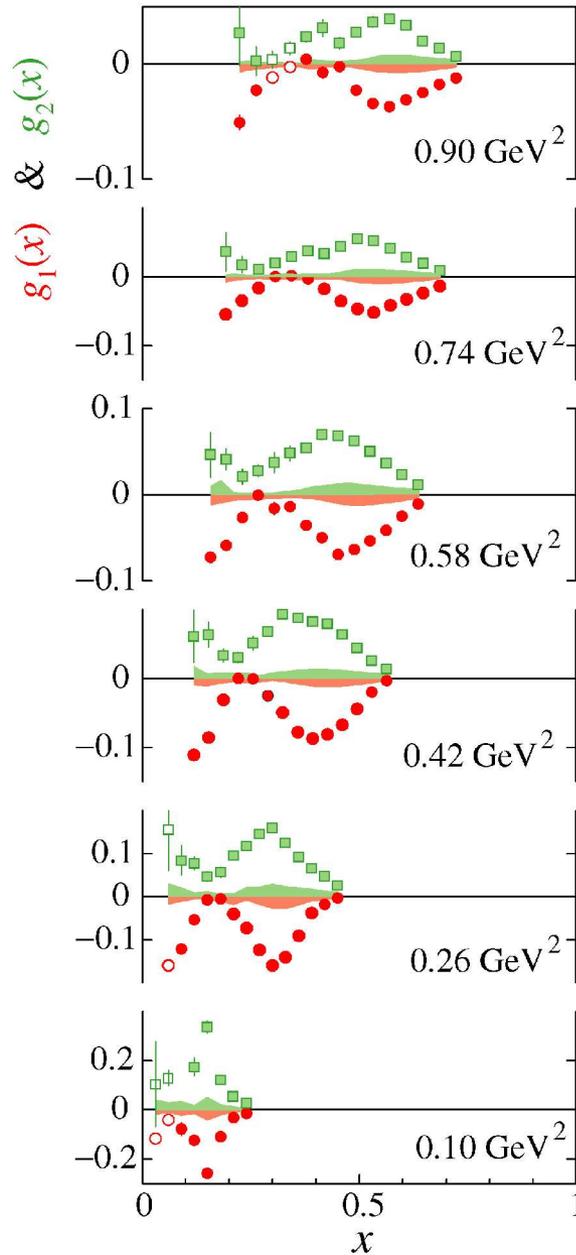
$\mathbf{g}_1(x, Q^2)$  Fit to the data and evolution to a constant  $Q^2$

$\mathbf{R}(x, Q^2)$  SLAC fit

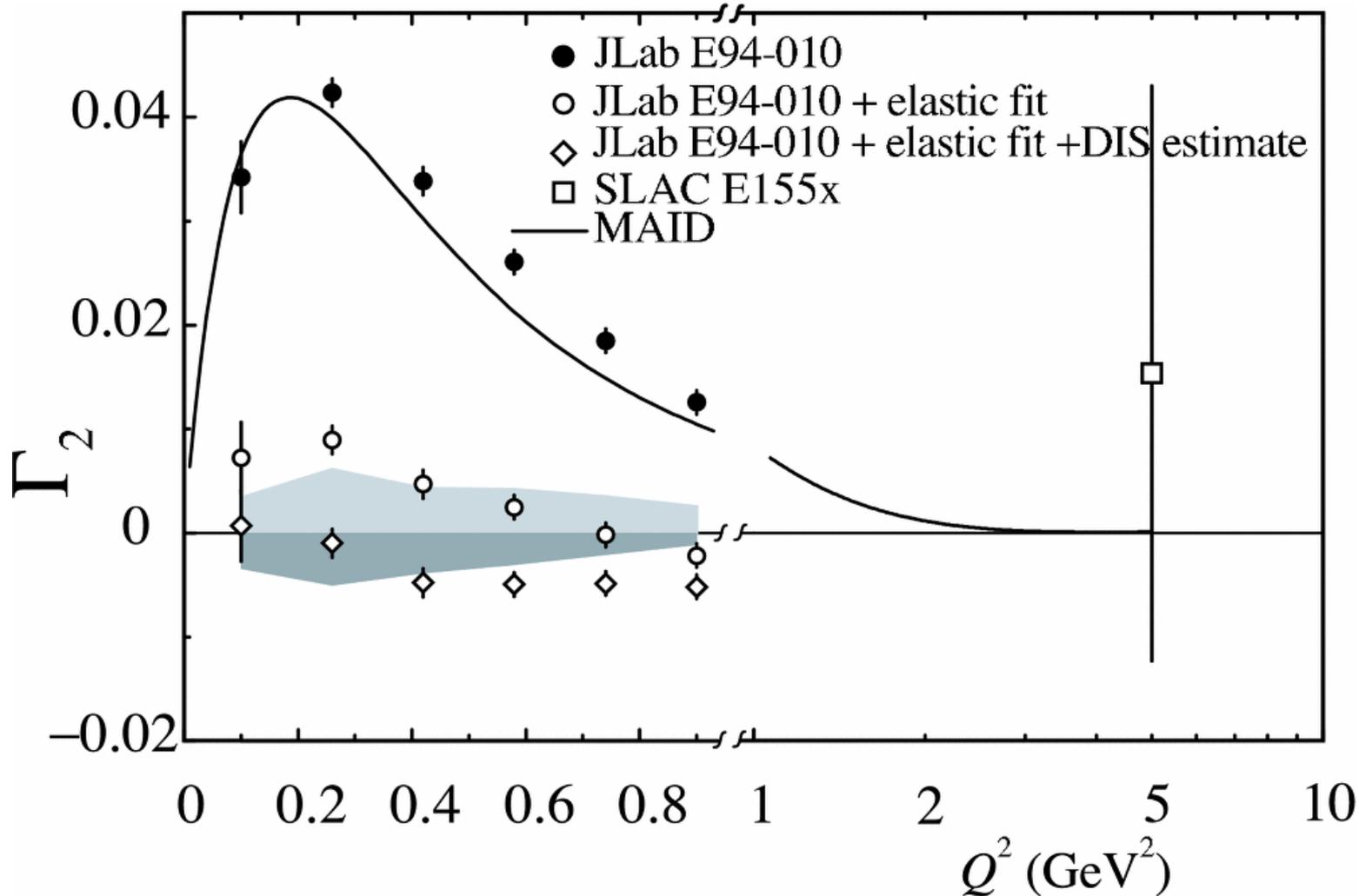
# World Data on $d_2^-$



JLab E94-010  $g_1(x)$  &  $g_2(x)$



# JLab and SLAC experimental results on $\Gamma_2$



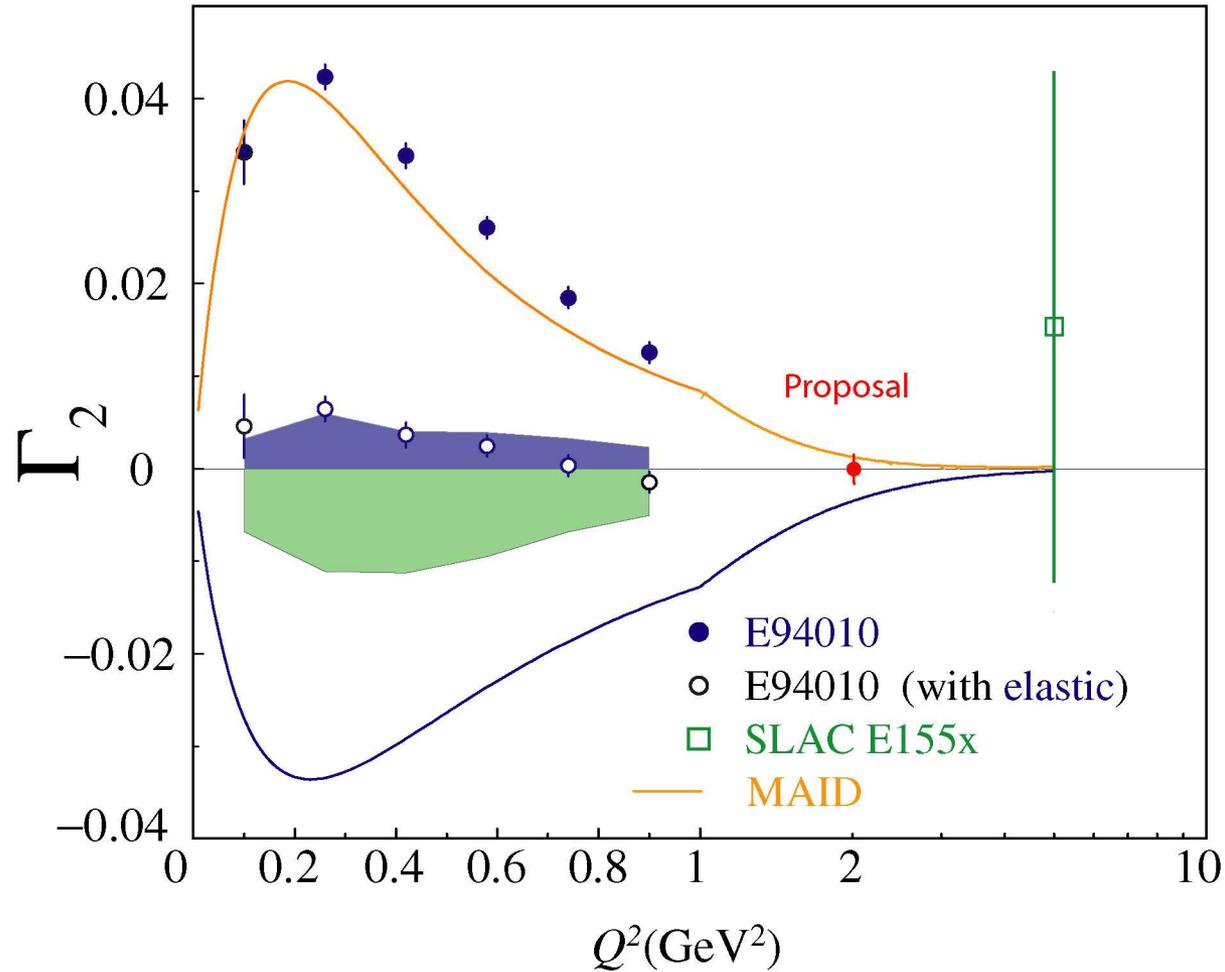
# Burkhardt-Cottingham Sum Rule (1965-1966)

$$\Gamma_2(Q^2) = \int_0^1 g_2(x, Q^2) dx = 0$$

- Dispersion relation for a spin-flip Compton amplitude
  - Causality
  - Analyticity
  - Absence of a  $J=0$  pole with non polynomial residue
- Doesn't follow from OPE and **is valid at all  $Q^2$**
- Many scenarios of  $g_2$ 's low  $x$  behavior which would invalidate the sum rule are discussed in the literature.

# Expected Error on $\Gamma_2$

UPDATE



# Kinematics of the proposed measurement

- Single beam energy 5.7 GeV
- BigBite fixed at single scattering angle (data divided into 10 bins during analysis)
- Avoid resonance region as much as possible.

