Measuring the Neutron g_2 and d_2 at 12 GeV

- Goal:
  - ➡ Clearly map out Q^2 evolution of neutron g_2 for $x > 0.5$
  - ➡ Determine the neutron d_2 at $\langle Q^2 \rangle = 3, 4, 5$ GeV^2
- An Experiment in Hall A:
  - ➡ A polarized electron beam of 6.6, 8.8 GeV and polarized ^{3}He target
  - ➡ Measure unpolarized cross section for $^3\vec{\mathrm{He}}(\vec{e}, e')$ reaction $\sigma_0^{^3\mathrm{He}}$ in conjunction with the transverse asymmetry $A_\perp^{^3\mathrm{He}}$ and the parallel asymmetry $A_\parallel^{^3\mathrm{He}}$ for $0.2 < x < 0.9$ with $2 < Q^2 < 9$ GeV^2.
- An Experiment in Hall C:
  - ➡ A polarized electron beam of 11.0 GeV and polarized ^{3}He target
  - ➡ Measure $\Delta\sigma_\parallel = \sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}$, $\Delta\sigma_\perp = \sigma^{\downarrow\Rightarrow} - \sigma^{\uparrow\Rightarrow}$ for $^3\vec{\mathrm{He}}(\vec{e}, e')$ reaction for $0.2 < x < 0.9$ with $2 < Q^2 < 6$ GeV^2.
    - Spokespeople:
      - ➡ Hall A: T. Averett, J.P. Chen, W. Korsch, B. Sawatzky
      - ➡ Hall C: T. Averett, W. Korsch, Z.E. Meziani, B. Sawatzky

Polarized deep inelastic cross sections

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E + E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right] = \Delta\sigma_{\parallel}$$

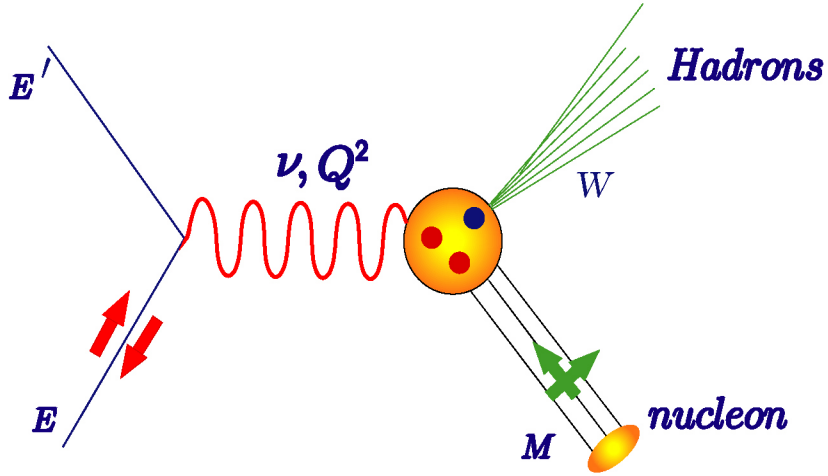
$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2Eg_2(x,Q^2)\right] = \Delta\sigma_{\perp}$$

Q^2 = 4-momentum transfer squared of the virtual photon.

ν = energy transfer.

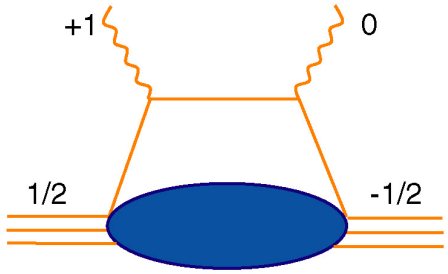
θ = scattering angle.

$x = \frac{Q^2}{2M\nu}$ fraction of nucleon momentum carried by the struck quark.



g_2 and Quark-Gluon Correlations

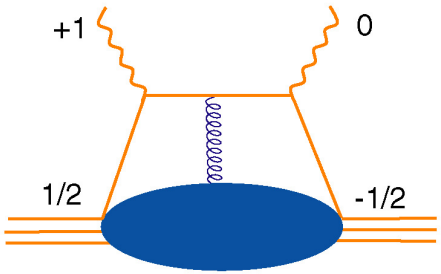
twist-2



Carry one unit of orbital angular momentum

QCD allows the helicity exchange to occur in two principle ways

twist-3



Couple to a gluon

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \bar{g}_2(x,Q^2)$$

- a twist-2 term (Wandzura & Wilczek, 1977):

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\frac{dy}{y}$$

- a twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston, 92):

$$\bar{g}_2(x,Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left(\frac{m_q}{M}h_T(y,Q^2) + \xi(y,Q^2)\right)\frac{dy}{y}$$

transversity

quark-gluon correlation

Moments of Structure Functions

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)\, dx = \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

leading twist

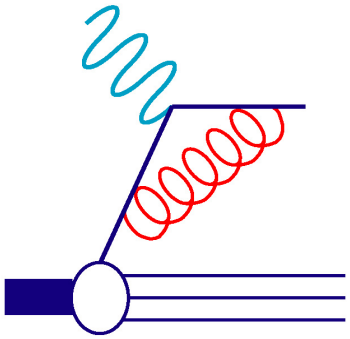
higher twist

$$\mu_2^{p,n}(Q^2) = (\pm \frac{1}{12} g_A + \frac{1}{36} a_8) + \frac{1}{9}\Delta\Sigma$$ + pQCD corrections

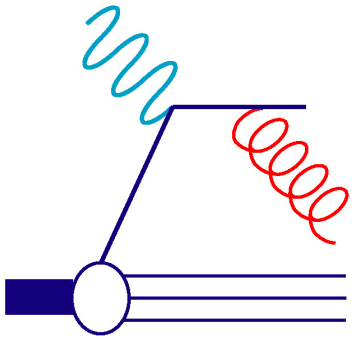
$g_A = 1.257$ and $a_8 = 0.579$ are the triplet and octet axial charge, respectively

$\Delta\Sigma$ = singlet axial charge

(Extracted from neutron and hyperon weak decay measurements)



$$\begin{aligned} g_A &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ \Delta\Sigma &= \Delta u + \Delta d + \Delta s \end{aligned}$$



pQCD radiative corrections

Moments of Structure Functions (continued)

$$\mu_4(Q^2) = \frac{M^2}{9}\left[a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)\right]$$

Twist - 2 (TMC) Twist - 3 Twist -4

where a_2, d_2 and f_2 are higher moments of g_1 and g_2

e.g. $$d_2(Q^2) = \int_0^1 x^2[2g_1(x,Q^2) + 3g_2(x,Q^2)]dx = 3\int_0^1 x^2\,\overline{g_2}(x,Q^2)dx$$

$$a_2(Q^2) = \int_0^1 x^2 g_1(x,Q^2)\,dx$$

- To extract f_2, d_2 needs to be determined first.
- Both d_2 and f_2 are required to determine the color polarizabilities

Color "polarizabilities"

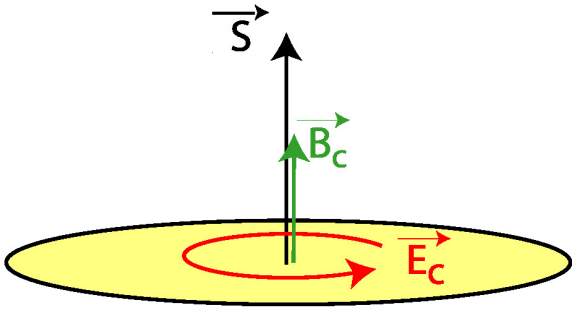
How does the gluon field respond when a nucleon is polarized ?

Define color magnetic and electric polarizabilities (in nucleon rest frame):

$$\chi_{B,E}\, 2M^2 \vec{S} \;=\; \langle PS|\vec{O}_{B,E}|PS\rangle$$

where $\vec{O}_B = \psi^\dagger g \vec{\boldsymbol{B}} \psi$

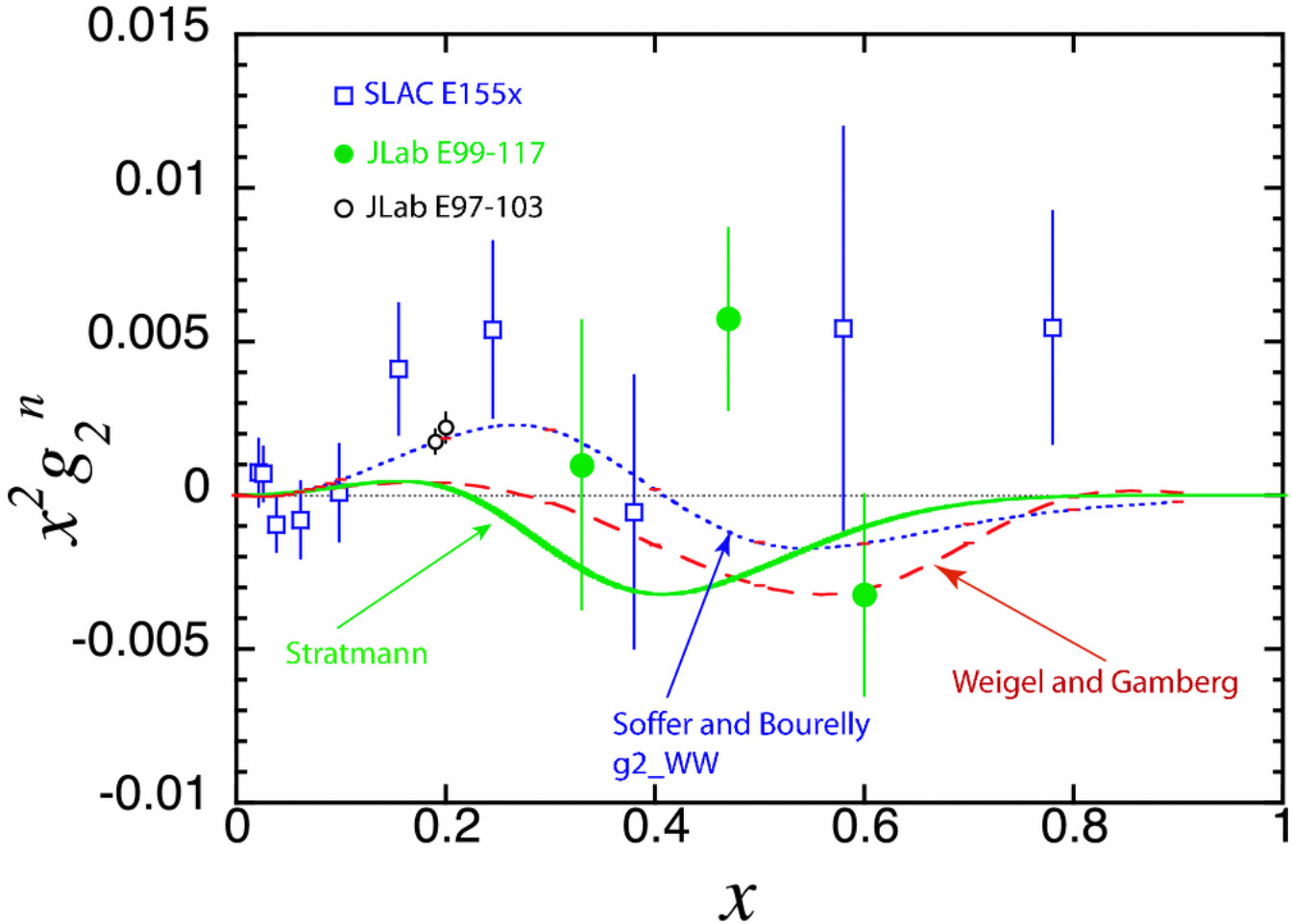
$\vec{O}_E = \psi^\dagger \vec{\alpha} \times g \vec{\boldsymbol{E}} \psi$



$$\chi_E^n = (4d_2^n + 2f_2^n)/3$$
$$\chi_B^n = (4d_2^n - f_2^n)/3$$

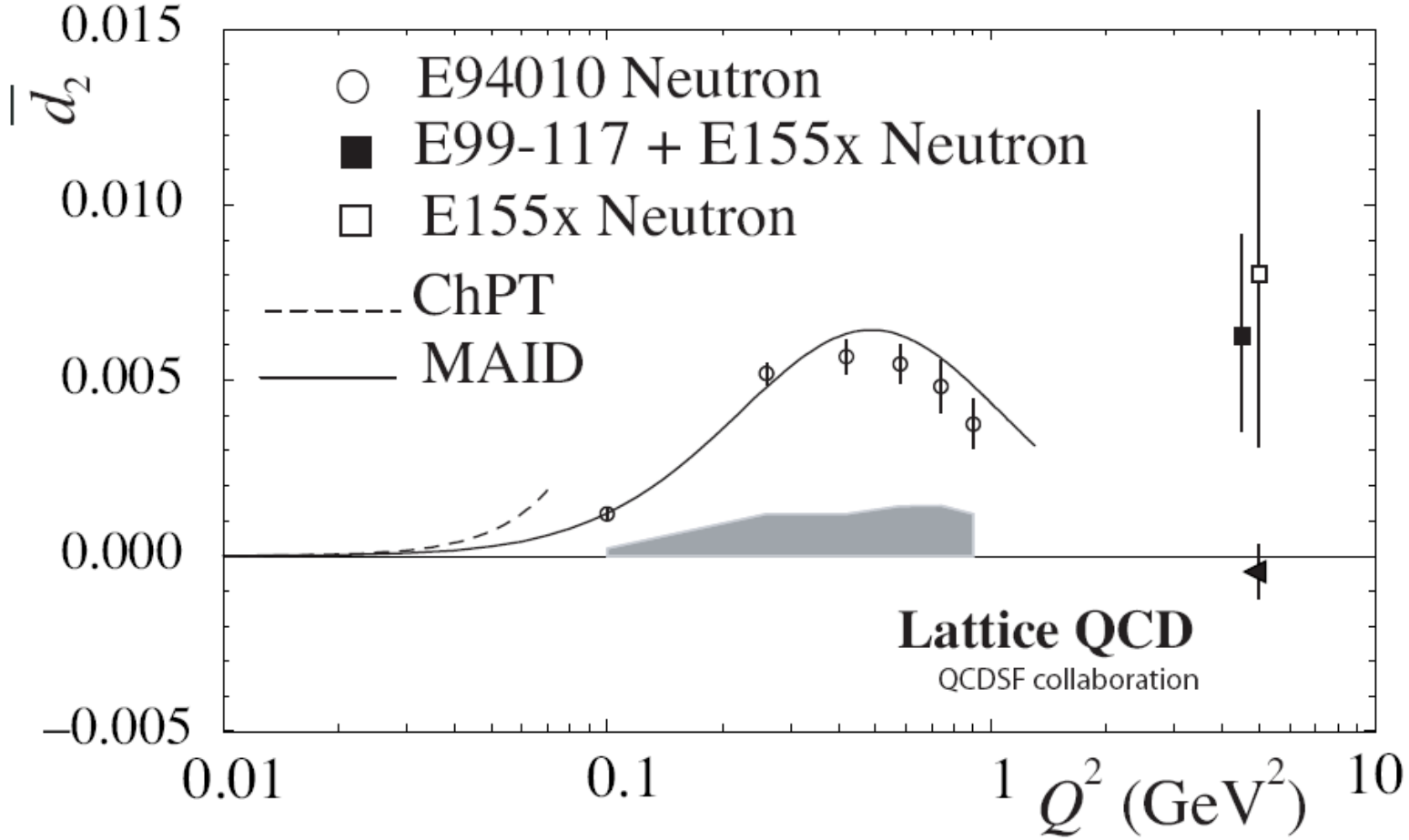
χ_E and χ_B represent the response of the color $\vec{B}$ & $\vec{E}$ fields to the nucleon polarization

World Data on on g_2^n



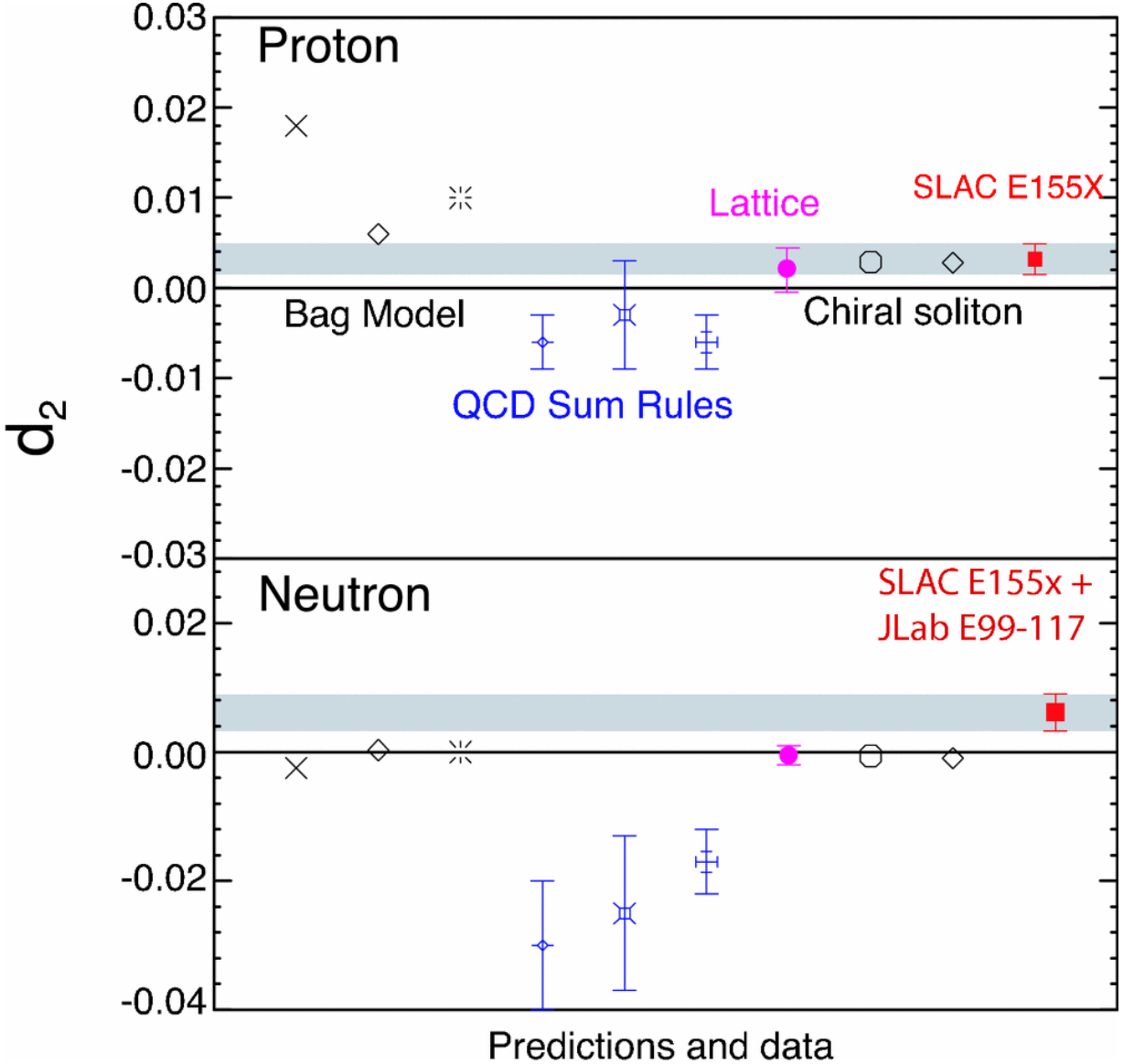
- However, Q^2 values for these data range from 0.1 – 15 GeV^2

World Data on $\bar{d}_2$



(nucleon elastic contribution suppressed)

Model evaluations of d_2



The proposal for Hall A and BigBite

- A 6.6 and 8.8 GeV polarized electron beam scattering off a polarized ^{3}He target
- Measure unpolarized cross section for $^3\vec{\mathrm{He}}(\vec{e},e')$ reaction $\sigma_0^{^3\mathrm{He}}$ in conjunction with the parallel asymmetry $A_\parallel^{^3\mathrm{He}}$ and the transverse asymmetry $A_\perp^{^3\mathrm{He}}$ for $0.2 < x < 0.9$ with $2 < Q^2 < 9$ GeV^2.
  - ➡ Asymmetries measured by BigBite at for two kinematics
    - ↳ E_{beam} = 6.6 GeV and $\theta = 40°$, E_{beam} = 8.8 GeV and $\theta = 30°$
  - ➡ Absolute cross sections measured simultaneously by L-HRS
- Determine d_2^n and g_2^n using the relations

$$\begin{aligned}\tilde{d}_2(x,Q^2) &= x^2[2g_1(x,Q^2)+3g_2(x,Q^2)] \\ &= \frac{MQ^2}{4\alpha^2}\frac{x^2y^2}{(1-y)(2-y)}\sigma_0\left[\left(3\frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}+\frac{4}{y}\tan\frac{\theta}{2}\right)A_\perp+\left(\frac{4}{y}-3\right)A_\parallel\right]\end{aligned}$$

$$g_2 = \frac{MQ^2}{4\alpha^2}\frac{y^2}{2(1-y)(2-y)}2\sigma_0\left[-A_\parallel+\frac{1+(1-y)\cos\theta}{(1-y)\sin\theta}A_\perp\right]$$

where,

$$A_\perp = \frac{\sigma^{\downarrow\Rightarrow}-\sigma^{\uparrow\Rightarrow}}{2\sigma_0} \qquad A_\perp^{^3He} = \frac{\Delta_\perp}{P_bP_t\cos\phi} \qquad A_\parallel^{^3He} = \frac{\Delta_\parallel}{P_bP_t}$$

$$A_\parallel = \frac{\sigma^{\downarrow\Uparrow}-\sigma^{\uparrow\Uparrow}}{2\sigma_0} \qquad \Delta_\perp = \frac{(N^{\uparrow\Rightarrow}-N^{\uparrow\Rightarrow})}{(N^{\uparrow\Rightarrow}+N^{\uparrow\Rightarrow})} \qquad \Delta_\parallel = \frac{(N^{\downarrow\Uparrow}-N^{\uparrow\Uparrow})}{(N^{\downarrow\Uparrow}+N^{\uparrow\Uparrow})}$$

I_{beam} = 10 μA
P_{beam} = 0.8
P_{targ} = 0.5

Kinematics for Hall A

	Spec.	E[GeV]	E'[GeV]	theta[deg]	Q2[GeV^2]	x	W[GeV]	total_e-[Hz]	e-(3He)[Hz]
BigBite	HRS	6.6	1.04	40	3.21	0.31	2.85	19.66	16.26
			1.2		3.7	0.37	2.71	15.17	12.57
			1.36		4.19	0.43	2.56	11.21	9.3
			1.52		4.68	0.49	2.4	7.79	6.48
			1.67		5.17	0.56	2.23	4.97	4.13
			1.83		5.66	0.63	2.05	2.79	2.32
			1.99		6.15	0.71	1.84	1.28	1.07
			2.15		6.64	0.79	1.62	0.41	0.35
			2.31		7.13	0.88	1.35	0.06	0.05
			2.47		7.61	0.98	1.02	0	0
BigBite	HRS	8.8	1.11	30	2.62	0.18	3.57	67.97	55.9
			1.41		3.33	0.24	3.38	53.67	44.26
			1.71		4.04	0.3	3.19	41.39	34.23
			2.01		4.74	0.37	2.98	30.52	25.31
			2.31		5.45	0.45	2.76	20.95	17.4
			2.61		6.16	0.53	2.52	12.85	10.7
			2.91		6.87	0.62	2.25	6.58	5.49
			3.21		7.58	0.72	1.95	2.44	2.04
			3.51		8.28	0.83	1.59	0.44	0.37
			3.81		8.99	0.96	1.12	0.01	0

- HRS used to measure σ_0 at 10 momentum settings for each beam energy.
  - ➡ will also reverse the field to monitor π^-/π^+ and e^-/e^+ asymmetries
- BigBite measures $A_{\parallel}^{^3\mathrm{He}}$ and $A_{\perp}^{^3\mathrm{He}}$ with single configuration at each beam energy.
  - ➡ Optimized to minimize error on extracted g_2
- **Hall A**: 200 hours for 6.6 GeV data set + 400 hours for 8.8 GeV data set

The proposal for Hall C and SHMS/HMS

- An Experiment in Hall C:
  - A polarized electron beam of 11.0 GeV and polarized ^{3}He target
  - Measure $\Delta\sigma_{\parallel} = \sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}$, $\Delta\sigma_{\perp} = \sigma^{\downarrow\Rightarrow} - \sigma^{\uparrow\Rightarrow}$ for $^3\vec{\mathrm{He}}(\vec{e}, e')$ reaction using both the SHMS and HMS running in parallel for 3 kinematic settings each
    - SHMS: (p_0= 8.0 GeV/c, θ = 11.0°), (p_0= 7.0 GeV/c, θ = 13.3°), (p_0= 6.3 GeV/c, θ = 15.5°)
    - HMS: (p_0= 4.2 GeV/c, θ = 13.5°), (p_0= 5.0 GeV/c, θ = 16.4°), (p_0= 3.4 GeV/c, θ = 20.0°)
- Determine d_2^n and g_2^n using the relations:

$$\tilde{d}_2 = x^2(2g_1+3g_2) = \frac{MQ^2\nu}{8\alpha_e^2}\frac{E}{E'}\frac{x^2(4-3y)}{(E+E')}\left[\Delta\sigma_{\parallel} + \left(\frac{4-y}{(1-y)(4-3y)\sin\theta_e} - \cot\theta_e\right)\Delta\sigma_{\perp}\right]$$

$$g_2 = \frac{MQ^2\nu^2}{4\alpha_e^2}\frac{1}{2E'(E+E')}\left[-\Delta\sigma_{\parallel} + \frac{E+E'\cos\theta_e}{E'\sin\theta_e}\Delta\sigma_{\perp}\right]$$

where $\Delta\sigma_{\parallel} = \sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}$, $\Delta\sigma_{\perp} = \sigma^{\downarrow\Rightarrow} - \sigma^{\uparrow\Rightarrow}$ and $y = \nu/E$.

I_{beam} = 10 μA
P_{beam} = 0.8
P_{targ} = 0.5

Kinematics in Hall C (E_{beam} = 11 GeV)

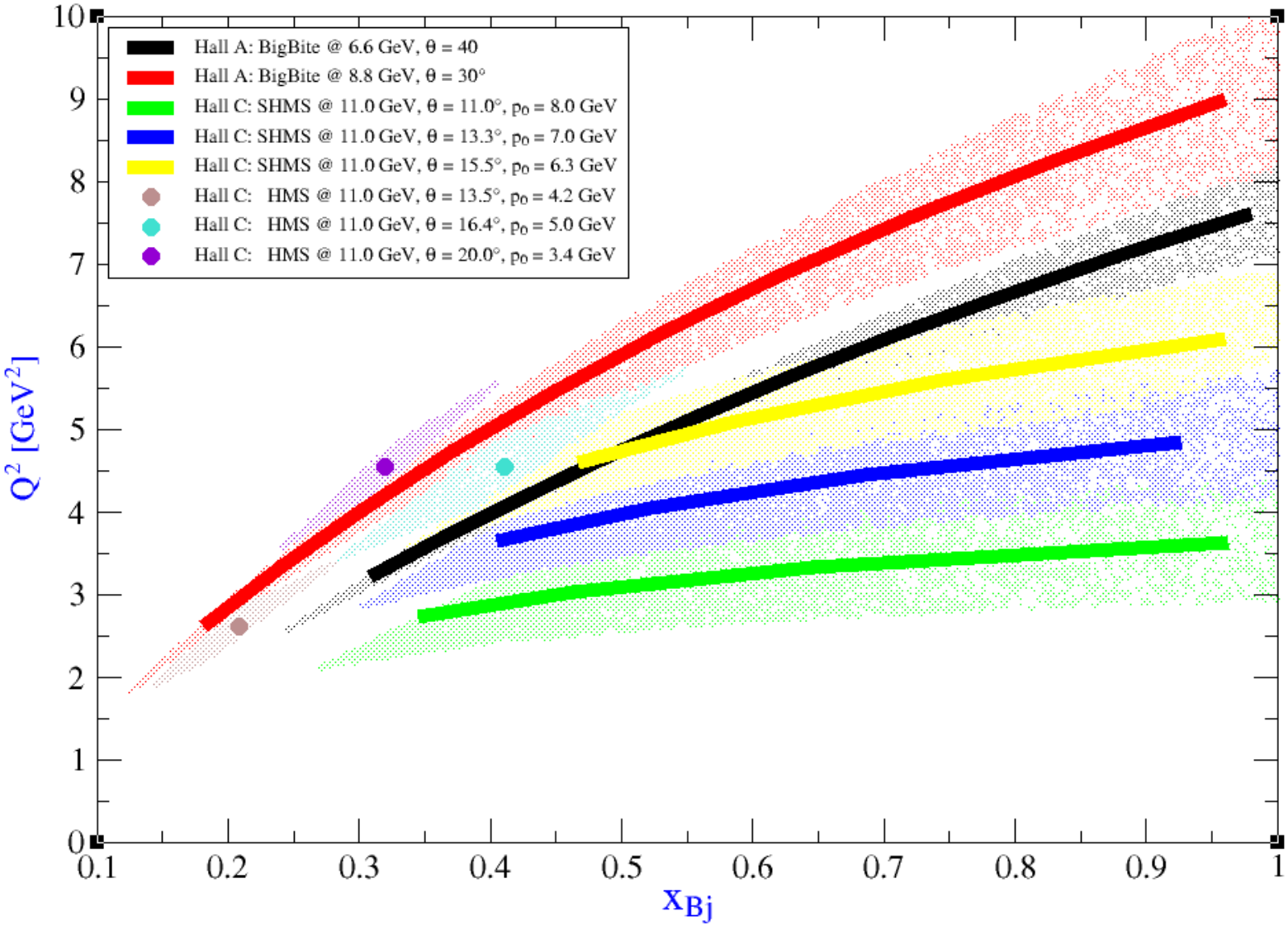
Spec.	p0 [GeV/c]	E[GeV]	theta[deg]	Q2[GeV^2]	x	W[GeV]	total_e-[Hz]	e-(3He)[Hz]
SHMS	8	11	11	2.74	0.34	2.47	257.6	213.3
				3.04	0.46	2.1	194.1	161.3
				3.34	0.65	1.65	77.72	64.83
				3.63	0.96	1.01	0.21	0.17
HMS	4.2	11	13.5	2.62	0.21	3.29	268.8	221.3
SHMS	7	11	13.3	3.65	0.4	2.5	79.98	66.37
				4.05	0.52	2.14	51.4	42.76
				4.45	0.69	1.71	17.75	14.82
				4.85	0.93	1.13	0.14	0.12
HMS	5	11	16.4	4.56	0.41	2.73	78.14	64.85
SHMS	6.3	11	15.5	4.6	0.47	2.48	28.63	23.79
				5.1	0.59	2.12	15.31	12.75
				5.6	0.74	1.68	3.89	3.26
				6.1	0.96	1.07	0.02	0.01
HMS	3.4	11	20	4.68	0.33	3.2	25.7	21.28

- SHMS collects data at Θ = 11°, 13.3° and 15.5° for 200 hrs each
  - ➡ data from each setting divided into 4 bins
- HMS collects data at Θ = 13.5°, 16.4° and 20.0° for 200 hrs each

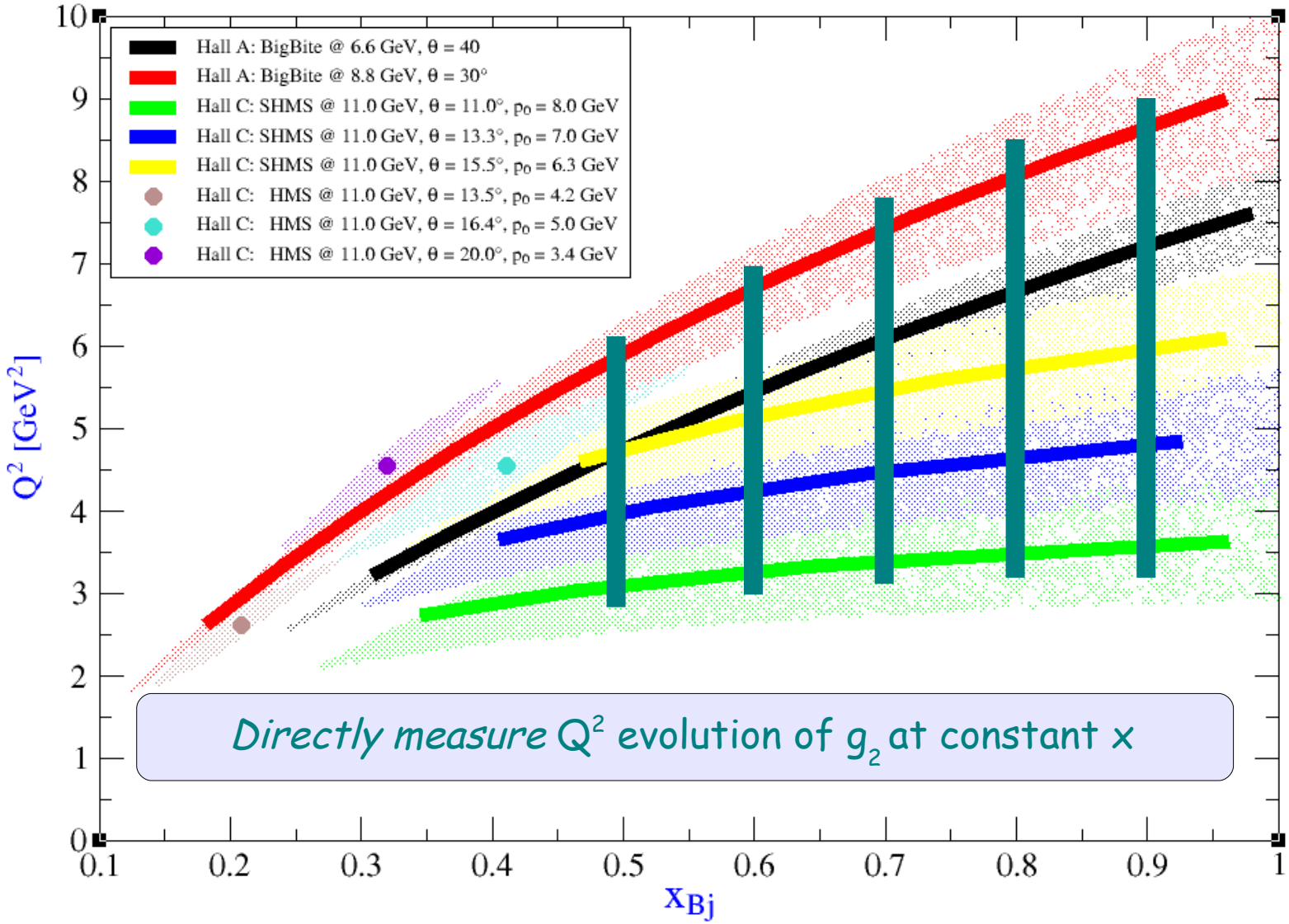
➡ SHMS and HMS collect data *in parallel*

↳ Total time: 600 hours

Combined Kinematics from both Halls

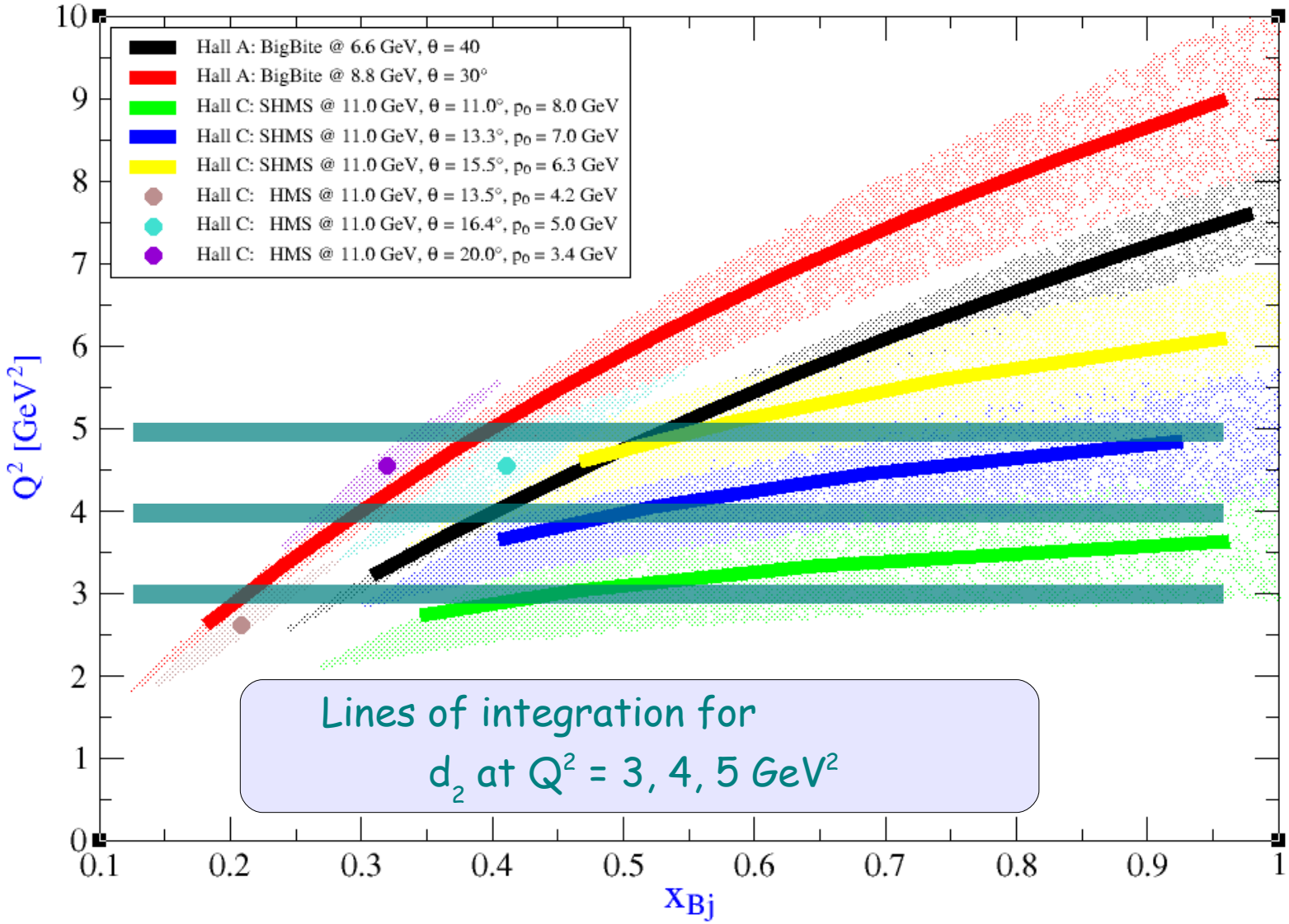


Q^2 evolution of d_2 (both Halls)



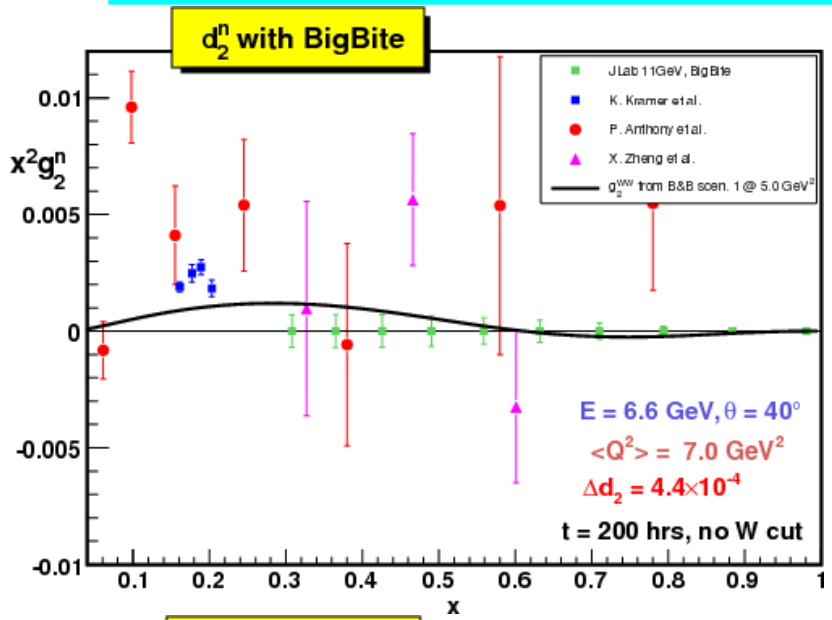
Directly measure Q^2 evolution of g_2 at constant x

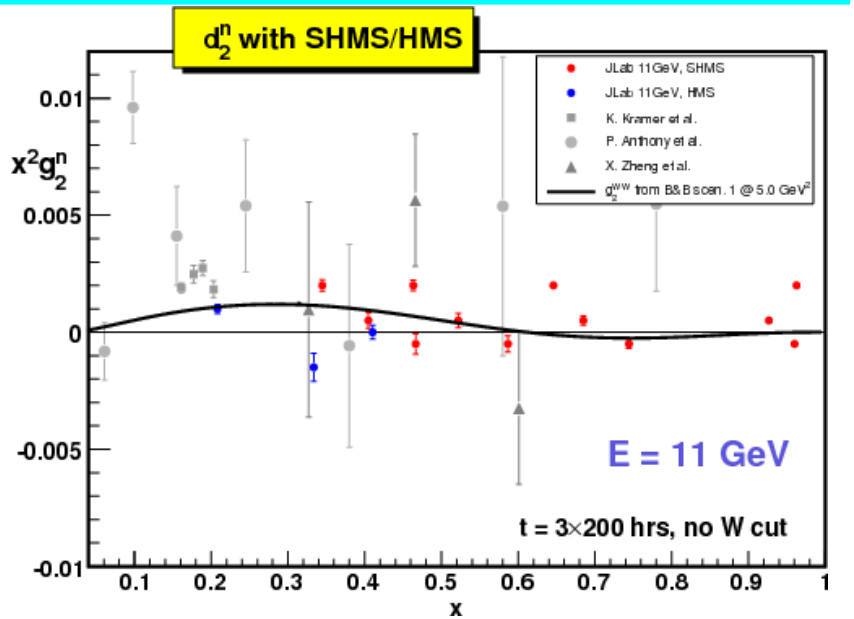
Q^2 evolution of d_2 (both Halls)

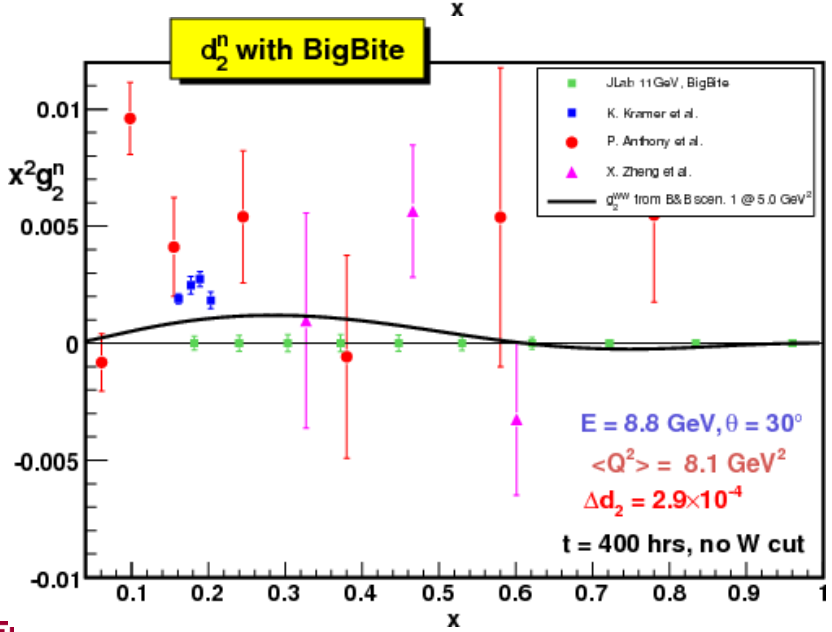


Lines of integration for
d_2 at Q^2 = 3, 4, 5 GeV^2

Projected $x^2g_2(x,Q^2)$ results

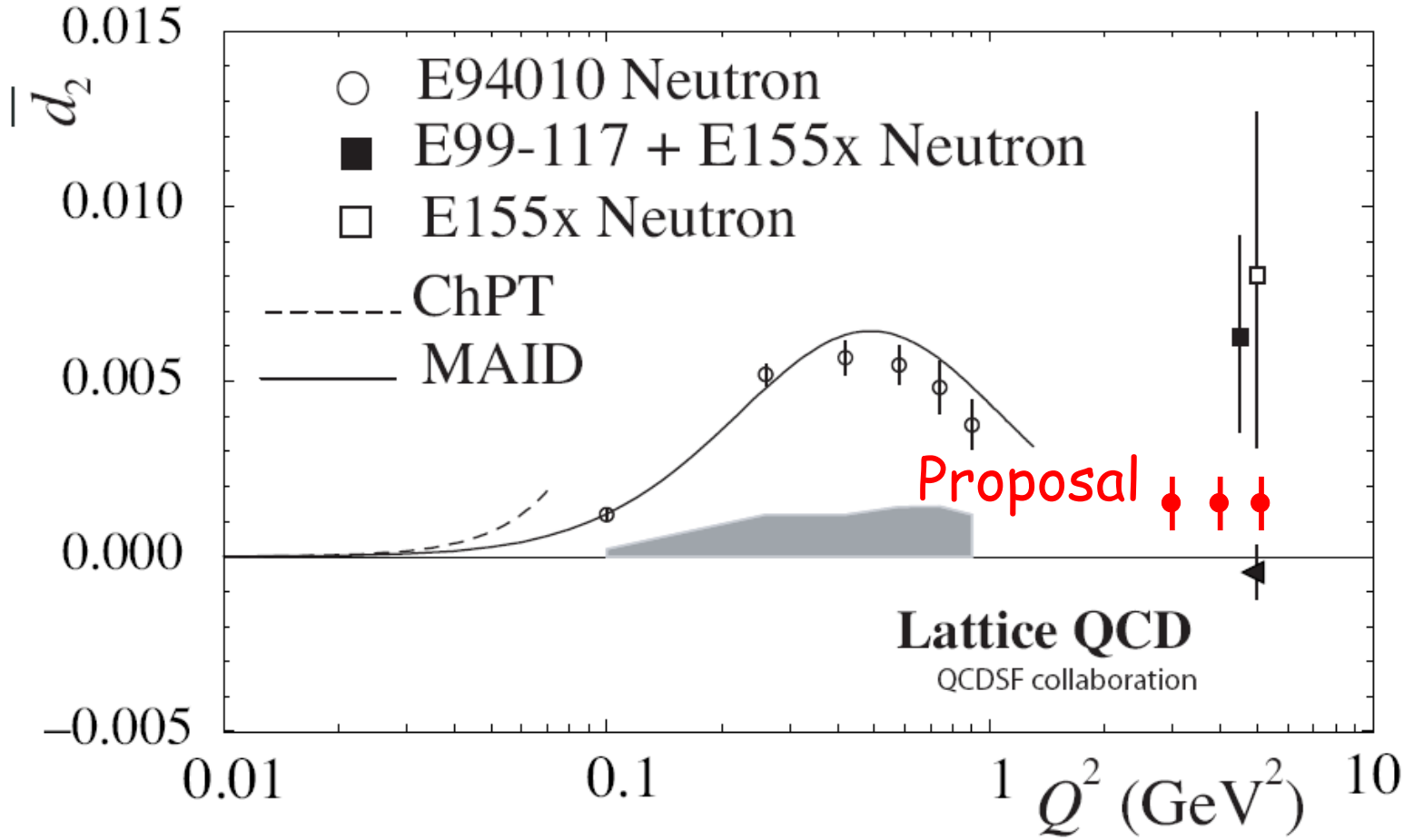






- g_2 for ^{3}He is extracted directly from L and T spin-dependent cross sections measurements within the same experiment.
- The nuclear corrections will be applied to the moments not to the structure functions.
- Strength of BigBite: large x coverage per bin (but large Q^2 variation)
- Strength of SHMS/HMS: nearly constant Q^2 (but less coverage for x < 0.3)

Expected Error on d_2



- The proposed measurements are at constant Q^2
- The dominant E155x point includes data evolved down from as far as 15 GeV^2!

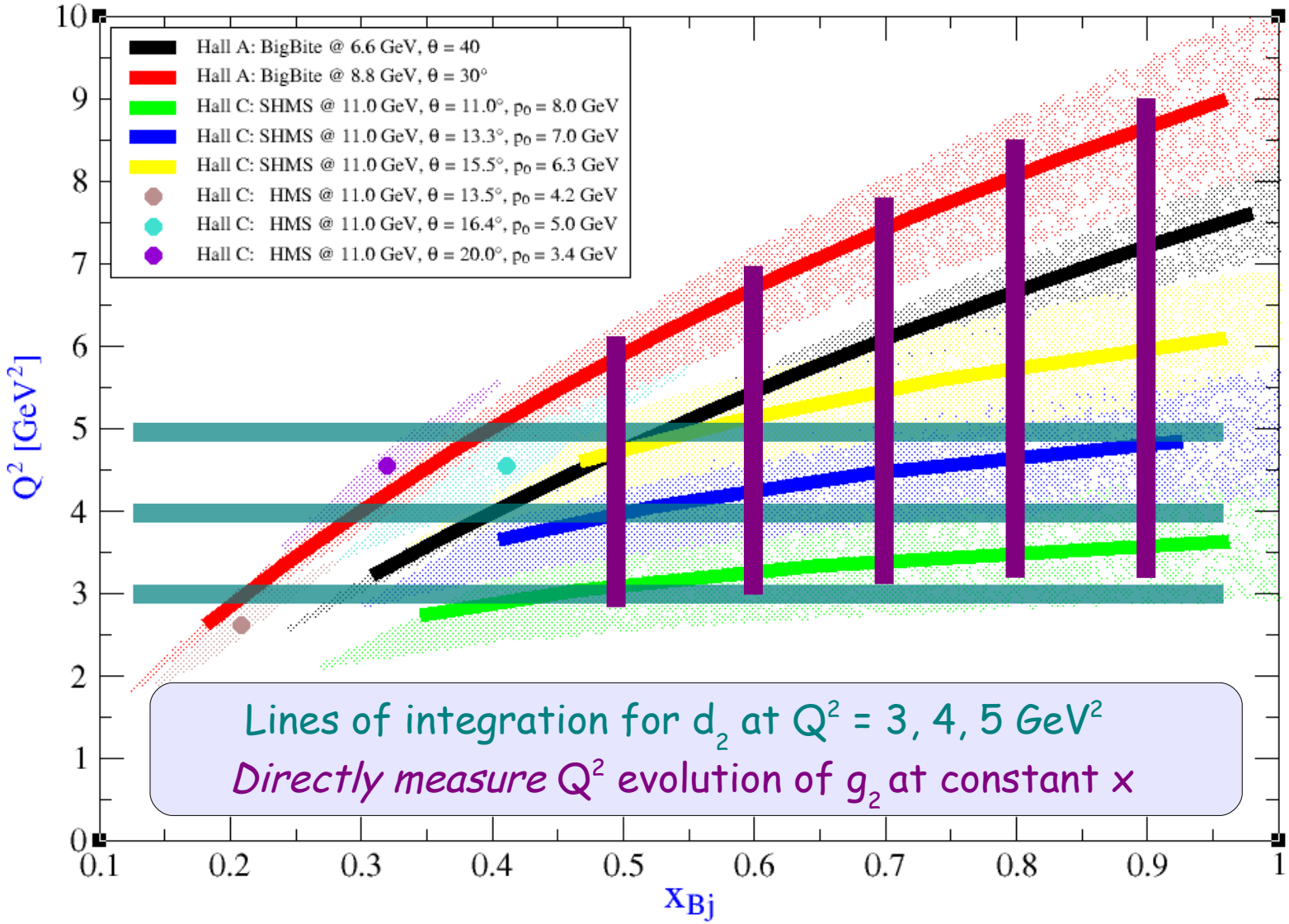
Summary

- We propose a pair of complementary measurements in Hall A and Hall C to
  - ➡ map out g_2 with unprecedented x, Q^2 coverage and precision
  - ➡ precisely measure the neutron d_2^n at Q^2 = 3, 4, and 5 GeV^2.
  - ➡ precisely measure Q^2 evolution of $g_2^n(x)$ for $x > 0.5$
- Provide a rigorous test for theory (lattice QCD).
  - ➡ we can achieve a statistical uncertainty of $\Delta d_2^n \sim 5 \times 10^{-4}$
    - measurements done at constant Q^2 (never been done before!)
- Dramatically improve our knowledge of $g_2^n(x)$
  - ➡ vastly improve the available data for $x > 0.2$, all with better precision and with an optimized distribution in x and Q^2
- Uses standard equipment in Hall A and Hall C

We request

- Hall A: 700 hours of polarized beam divided between 6.6 and 8.8 GeV.
  - ➡ ~ 600 hours of production, ~100 hours of overhead and calibration
- Hall C: 700 hours of polarized beam at 11 GeV.
  - ➡ ~ 600 hours of production, ~100 hours of overhead and calibration

d_2 and g_2 evolution (both Halls)



Lines of integration for d_2 at Q^2 = 3, 4, 5 GeV^2

Directly measure Q^2 evolution of g_2 at constant x

Extra Slides

PAC30 'Contribution' requirement

Spokespeople for Hall A measurement will commit to the following:

- Moeller upgrade (primary contribution)
- HRS electronics (commissioning, etc)
- Polarized ^{3}He target

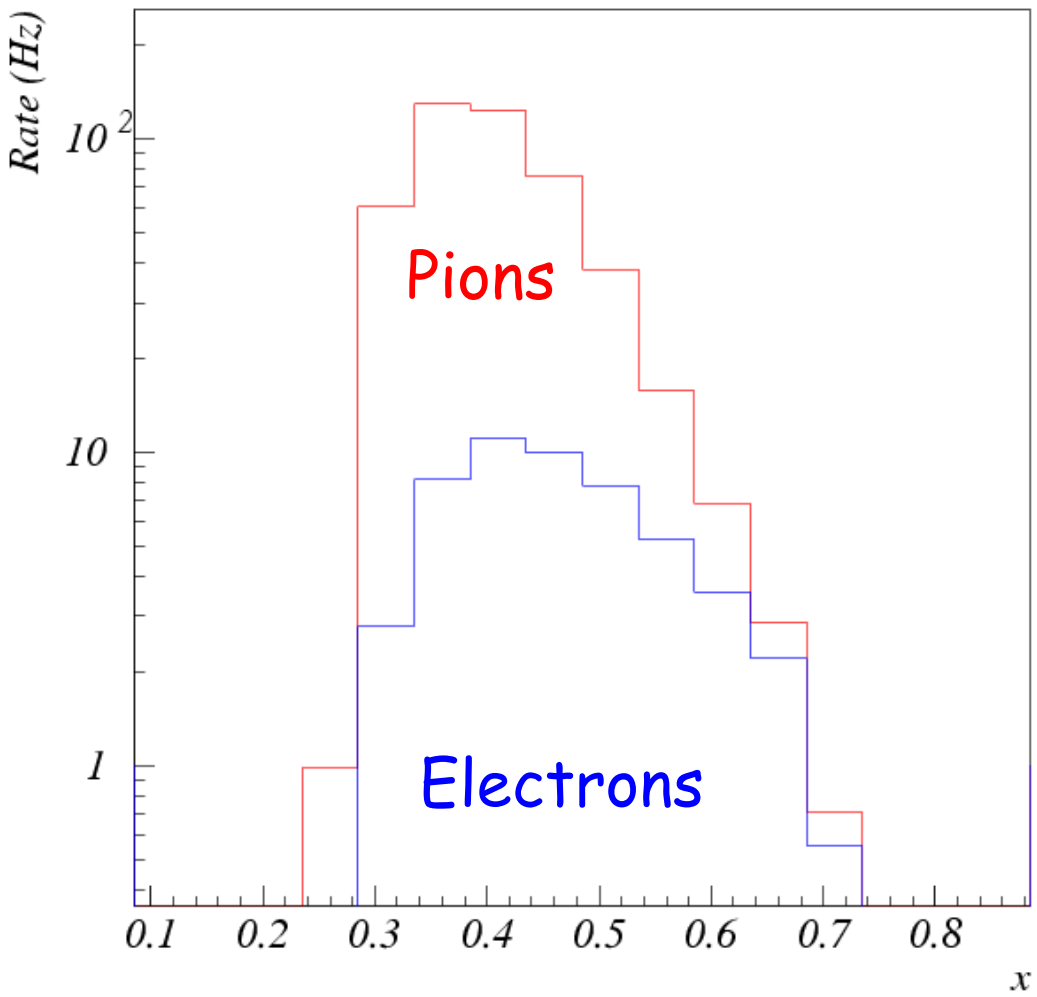
Spokespeople for Hall C measurement will commit to the following:

- Manpower: 1 full-time equiv. devoted to commissioning and/or construction of a base equipment item in Hall C (details pending)
- Polarized ^{3}He target

Systematic Error Contributions to g_2^n and d_2^n

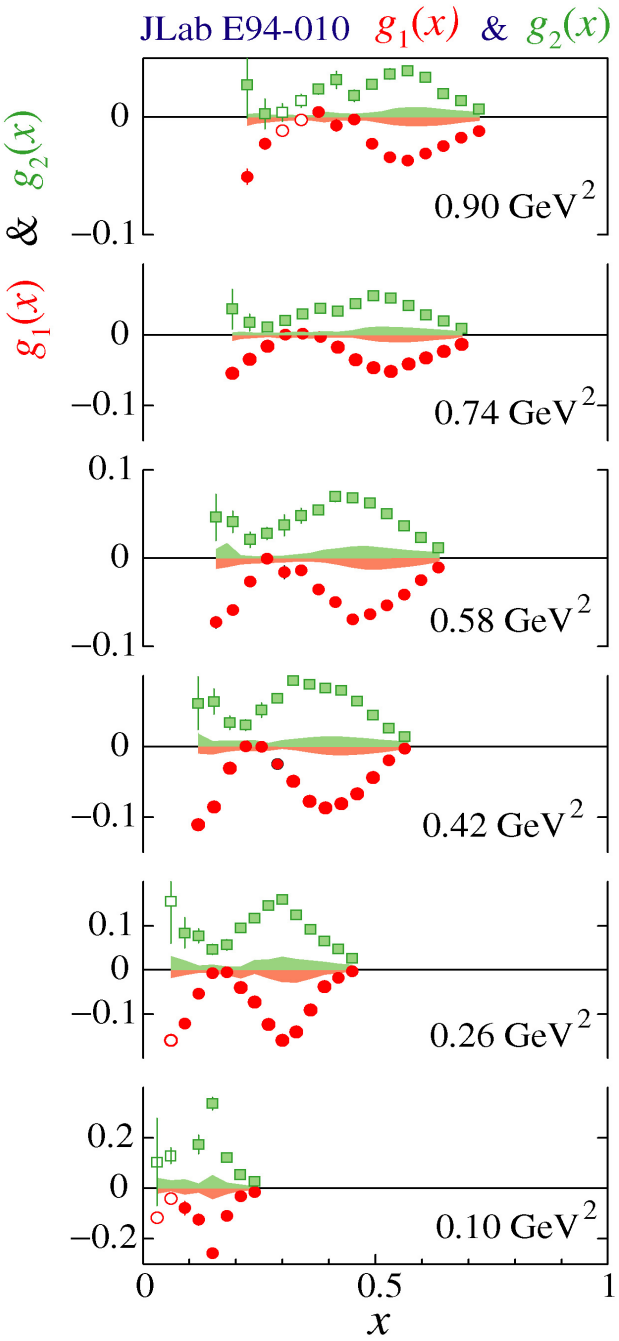
Target polarization		4 %
Beam polarization		3 %
Asymmetry (raw)		
	• Target spin direction (0.5°)	$\approx 1.5 \times 10^{-3}$
	• Beam charge asymmetry	200 ppm
Cross section (raw)		
	• PID efficiency	$\approx$ 1 %
	• Background Rejection efficiency	$\approx$ 1 %
	• Beam charge	$<$ 1 %
	• Beam position	$<$ 1 %
	• Acceptance cut	2-3 %
	• Target density	2-3 %
	• Nitrogen dilution	2-3 %
	• Dead time	<1 %
	• Finite Acceptance cut	<1%
Radiative corrections		$\leq$ 10 %
From ^{3}He to Neutron correction		5 %
Total effect		$\leq$ 10 %
Estimate of contributions	$\int_{0.003}^{0.23} \tilde{d}_2^n \, dx$	4.8×10^{-4}
from unmeasured regions	$\int_{0.70}^{0.999} \tilde{d}_2^n \, dx$	5.0×10^{-5}
Projected absolute statistical uncertainty		$\Delta d_2 \approx 5.4 \times 10^{-4}$
Projected absolute systematic uncertainty assuming $d_2 = 5 \times 10^{-3}$		$\Delta d_2 \approx 5 \times 10^{-4}$

e/π rates in BigBite at 8.8 GeV and 30 degrees



Q^2 evolution of $g_1(x)$ and $g_2(x)$ from E94-010

Q^2: 0.1 – 0.9 GeV^2

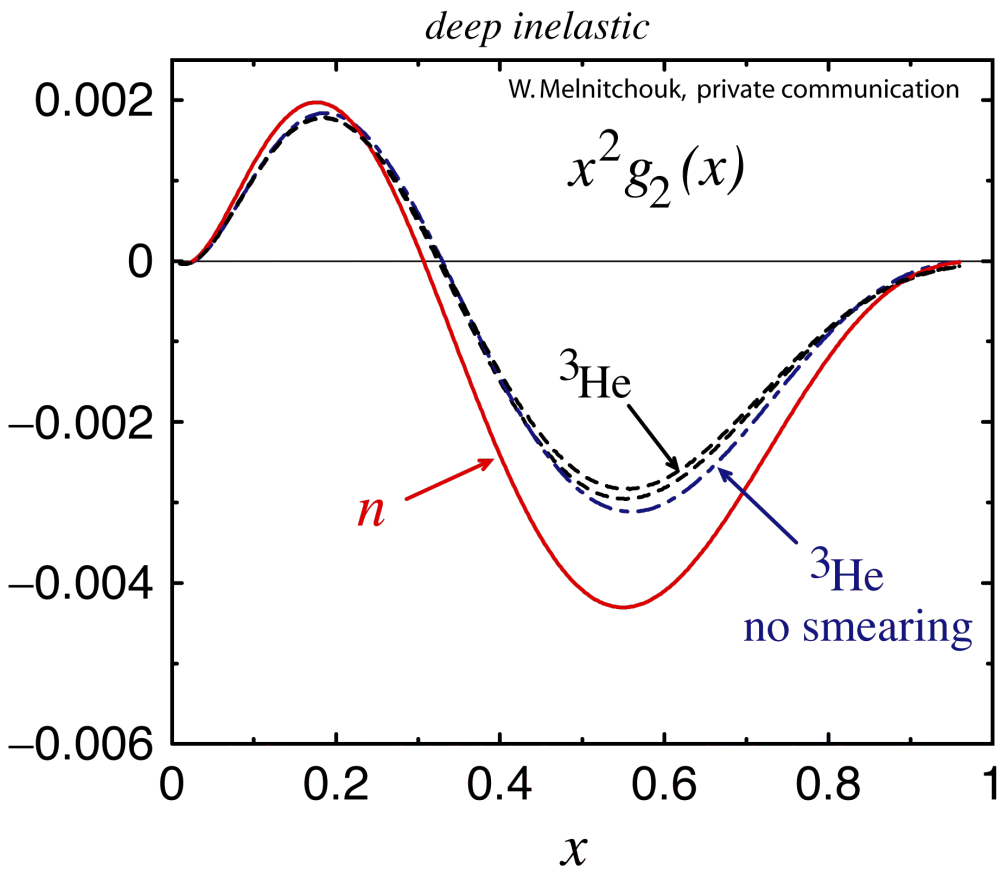


Nuclear corrections

- Convolution method using the impulse approximation and realistic ground state wave functions of ^{3}He (in Bjorken limit: $g_1^{^3He}$ related to g_1^N).
  - Variational Method,
    - C. Ciofi degli Atti & S. Scopetta, Phys. Lett. B 404 (1997) 223, for g_1, for g_2, S. Scopetta. private communication
  - Faddeev
    - F. Bissey et al. Phys. Rev. C 64 (2001) 024004

- Finite Q^2 effects (both g_1^N and g_2^N contribute to $g_1^{^3He}$ and to $g_2^{^3He}$)
  - S.A. Kulagin and W. Melnitchouk

From ^{3}He to Neutron



✓Correction large for g_2 but much smaller for d_2

✓About 5% difference between additive or convolution methods or between potential models

$$d_2^n = d_2^{^3He}/(1-\delta^c) \qquad \text{with} \qquad \delta^c \approx 0.35$$

$$\Delta\delta^c \approx 0.15\delta^c \approx 0.05 \qquad \Rightarrow \qquad \Delta d_2^n/d_2^n \approx 5\%$$

Nuclear corrections (continued)

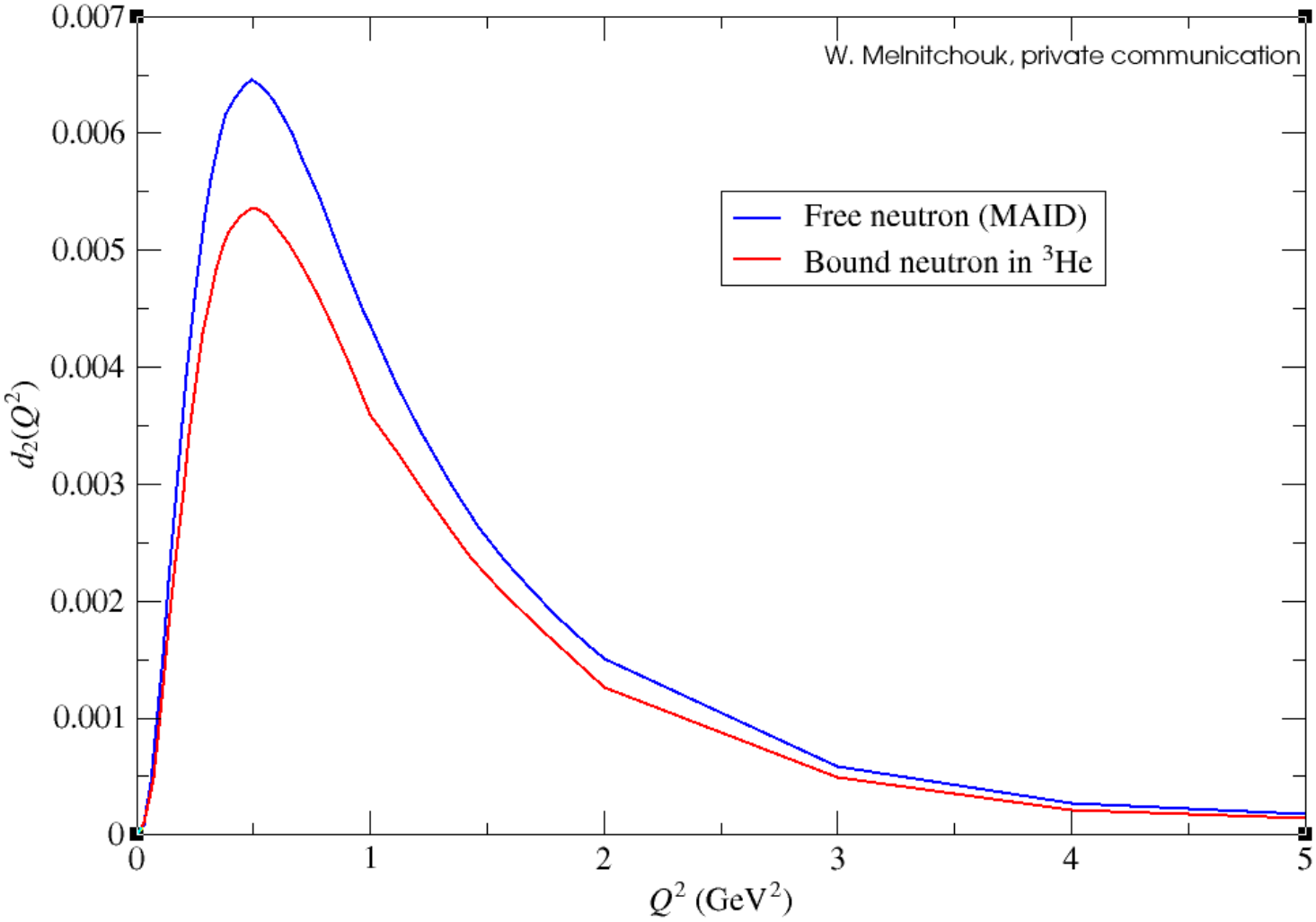
$$S(\vec{p},E) \;=\; \frac{1}{2}\left(f_0 + f_1\vec{\sigma}_N\cdot\vec{\sigma}_A + f_2\left[\vec{\sigma}_N\cdot\hat{p}\;\vec{\sigma}_A\cdot\hat{p}\;-\frac{1}{3}\vec{\sigma}_N\cdot\vec{\sigma}_A\right]\right)$$

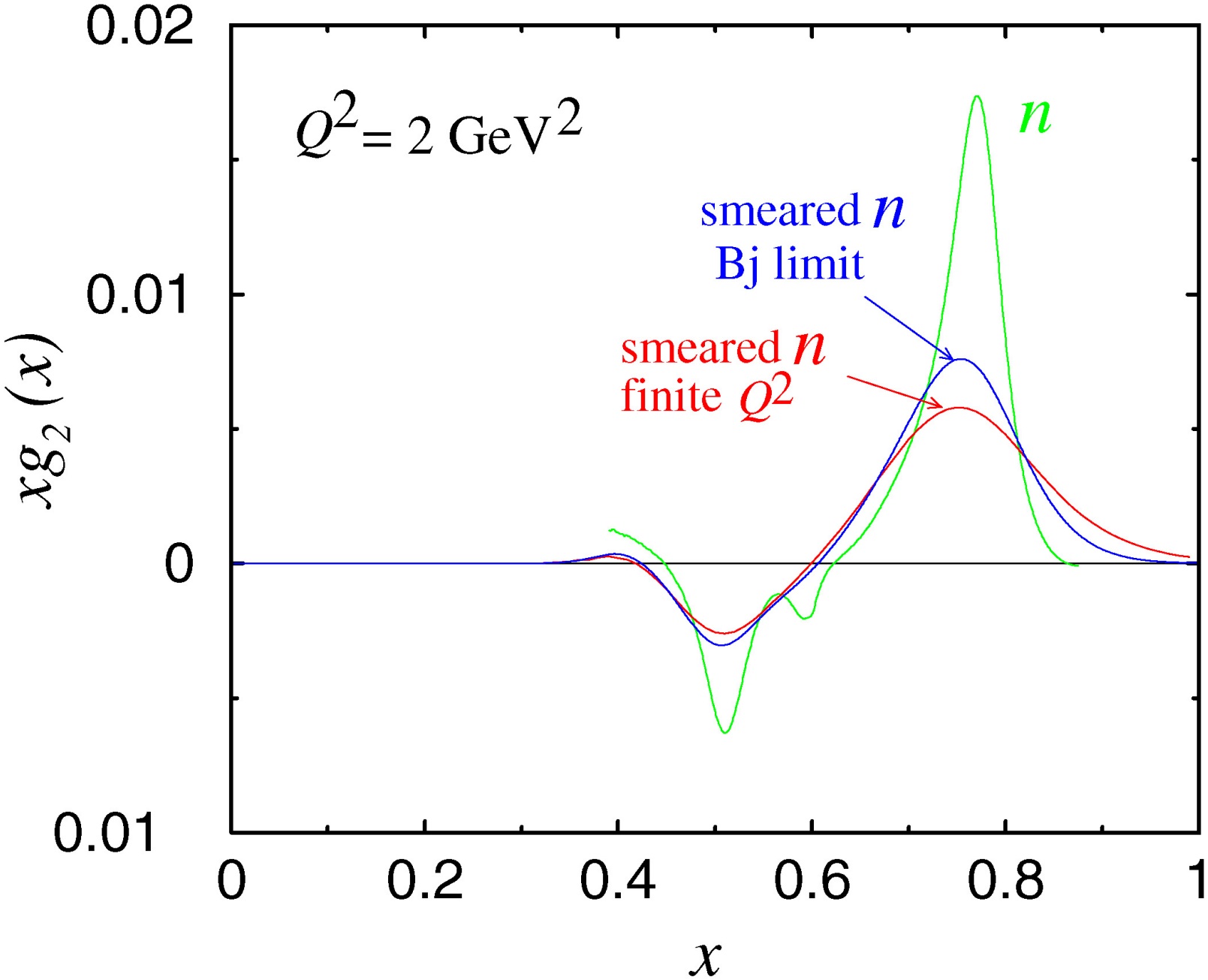
$$\begin{aligned}
&xg_1^{^3\mathrm{He}}(x,Q^2) + (1-\gamma^2)xg_2^{^3\mathrm{He}}(x,Q^2)\\
&= \sum_{N=p,n}\int d^3p\; dE\;(1-\frac{\epsilon}{M})\left\{\left[\left(1+\frac{\gamma p_z}{M}+\frac{p_z^2}{M^2}\right)f_1 + \left(-\frac{1}{3}+\hat{p}_z^2+\frac{2\gamma p_z}{3M}+\frac{2p_z^2}{3M^2}\right)f_2\right]\; zg_1^N(z,Q^2)\right.\\
&\qquad\left. + (1-\gamma^2)(1+\frac{\epsilon}{M}\left[f_1+\left(\frac{p_z^2}{\vec{p}^2}-\frac{1}{3}\right)f_2\right]\frac{z^2}{x}g_2^N(z,Q^2)\right\}
\end{aligned}$$

$$\begin{aligned}
&xg_1^{^3\mathrm{He}}(x,Q^2) + xg_2^{^3\mathrm{He}}(x,Q^2)\\
&= \sum_{N=p,n}\int d^3p\; dE\;(1-\frac{\epsilon}{M})\left\{\left[\left(1+\frac{p_x^2}{M^2}\right)f_1 + \left(\vec{p}_x^{\,2}-\frac{1}{3}+\frac{2p_x^2}{3M^2}\right)f_2\right]zg_1^N(z,Q^2)\right.\\
&\left. + \left[\left(1+\frac{p_x^2}{M^2}(1-z/x)\right)f_1 + \left(\vec{p}_x^{\,2}-\frac{1}{3}+\frac{2p_x^2}{3M^2}(1-z/x)-\frac{\gamma p_z\hat{p}_x^2}{M}\frac{z}{x}\right)f_2\right]zg_2^N(z,Q^2)\right\}
\end{aligned}$$

with $\gamma = \sqrt{1+4M^2x^2/Q^2}$ a kinematical factor parameterizing the finite Q^2 correction, $\epsilon \equiv \vec{p}^2/4M - E$, and $z = x/(1+(\epsilon+\gamma p_z)/M)$.

Nuclear corrections (continued)




$Q^2 = 2\ \mathrm{GeV}^2$
n
smeared n
Bj limit
smeared n
finite Q^2
$xg_2(x)$
x
0.02
0.01
0
0.01
0
0.2
0.4
0.6
0.8
1

How $g_2(x,Q^2)$ is usually obtained

$$g_2(x,Q^2) = \frac{\nu}{2E}\left[\frac{\nu\left[1+\epsilon \boldsymbol{R}(x,Q^2)\right](1+\gamma^2)\boldsymbol{F_2}(x,Q^2)\boldsymbol{A_\perp}(x,Q^2)}{(1-\epsilon)2x\left[1+\boldsymbol{R}(x,Q^2)\right]E'\sin\theta_e} - \boldsymbol{g_1}(x,Q^2)\right]$$

where $\nu = E - E'$, $\gamma^2 = Q^2/\nu^2$ and $\epsilon^{-1} = 1 + 2\left[1+\gamma^{-2}\right]\tan^2\theta/2$

$\boldsymbol{F_2}(x,Q^2)$ NMC fit

$\boldsymbol{g_1}(x,Q^2)$ Fit to the data and evolution to a constant Q^2

$\boldsymbol{R}(x,Q^2)$ SLAC fit