Precision Measurement of the Neutron Magnetic Form Factor up to $Q^2 = 8.0$ (GeV/c)² by the Ratio

Method

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Introduction

For spin $\frac{1}{2}$ target (one-photon-exchange approx):

$$\frac{d\sigma}{d\Omega} = \eta \frac{\sigma_{\text{\tiny Mott}}}{1+\tau} \left((G_E)^2 + \frac{\tau}{\epsilon} (G_M)^2 \right)$$

where: $\eta = \frac{1}{1+2\frac{E}{M_N}\sin^2(\theta/2)}$ $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta/2)$ $\tau = Q^2/4M_N^2$ Nucleon: $G_E(Q^2)$ and $G_M(Q^2)$ are Sachs Electric and Magnetic form factors.

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• Opportunity to make precision measurements related to the structure of hadrons



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where: $F_1 = \frac{1}{1+\tau} (G_E + \tau G_M)$

$$\Rightarrow \quad G_M^n \text{ important to understanding}$$
transverse charge distribution of neutron
$$\int_{-LS}^{0^n, \rho_T^n} \frac{[1/fm^2]}{\rho_0^n, \rho_T^n} \int_{-LS}^{0.5} \int_{$$



• Combining G^n_M with G^p_M allows direct extraction of flavor (neglecting strange quarks)

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• Sets sum rules constraining GPD's (at each Q^2) $F_1^q(Q^2) = \int_{-1}^{+1} dx H^q(x,\xi,Q^2) \qquad \qquad F_2^q(Q^2) = \int_{-1}^{+1} dx E^q(x,\xi,Q^2)$

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$$G_M^{u/d,p} = \int_{-1}^{+1} dx (H^{u/d}(x,\xi,Q^2) + E^{u/d}(x,\xi,Q^2))$$

Previous Data ($Q^2 \ge 1$ (GeV/c)²)



Previous Data ($Q^2 \ge 1$ (GeV/c)²) and CLAS e5



Previous Data ($Q^2 \ge 1$ (GeV/c)²) and CLAS e5 and projected error bars



Ratio Method

Measure quasi-elastic scattering from the deuteron *tagged* by coincident nucleon: d(e,e'p) and d(e,e'n)

$$R'' = \frac{\frac{d\sigma}{d\Omega}|_{d(e,e'n)}}{\frac{d\sigma}{d\Omega}|_{d(e,e'p)}}$$

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<1% nuclear corrections (common factors cancel in ratio)

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 \approx 1% correction (Galster parameterization) for electric form factor

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<1% nuclear corrections (common factors cancel in ratio) \approx 1% correction (Galster parameterization) for electric form factor $G_M^n \propto \sqrt{R}$...given proton elastic cross section Subsequent improvements... nuclear corrections, G_E^n , $\frac{d\sigma}{d\Omega}|_{p(e,e')}$

...can be applied retrospectively to measured value of $R^{\prime\prime}.$

$$\frac{\sigma_{G_M^n}}{G_M^n} = \frac{1}{2} \frac{\sigma_R}{R}$$



Ratio is insensitive to:

- target thickness
- target density
- beam current
- beam structure
- live time
- (electron) trigger efficiency
- electron track reconstruction efficiency
- electron acceptance ...





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- electron acceptance ...

Important to understand:

- neutron efficiency / proton efficiency
- neutron acceptance / proton acceptance

calibration reactions

Kinematics

\mathbf{Q}^2	E _{beam}	$ heta_e$	$ heta_N$	E′	P_N
(GeV/c) 2	(GeV)			(GeV)	(GeV/c)
3.5	4	37.5°	28.7 °	2.1	2.65
4.5	4	49.5°	21.7 °	1.6	3.2
5.25	5	40.4 °	22.7 °	2.3	3.6
6	5	48.1 °	18.7 [°]	1.8	4.0
7	6	42.0 °	18.7 [°]	2.3	4.6
8	6	52.0 °	14.9 [°]	1.7	5.1

Apparatus



Experience from GEn experiment with BigBite/BigHAND combination

Adding "BigBen" deflector magnet

 $\mathcal{L}=10^{37}/cm^2/s$ (100 $imes\mathcal{L}$ of CLAS12)

BigBite spectrometer

Electron arm (and π^+ for H(γ, π^+)n calibration)



Reconfigured for higher momentum running.

pprox 50 msr acceptance

0.5-0.6% momentum resolution (ppprox 2 GeV/c)

<1 mr angular resolution

Gas Cerenkov \Rightarrow reduced singles rates \Rightarrow Single-arm trigger

"BigHAND" Hall A Nucleon Detector

(neutron and proton arm)

244 scintillator bars in 7 layer with $\frac{1}{2}$ " iron converters Two veto layers with 2" lead and 1" iron shields

$$\label{eq:light} \begin{split} \mathbf{L}_{\mathsf{flight}} &= 17 \; \mathsf{m} \qquad \sigma_{\theta} < .25^{\circ} \\ \mathsf{Time Resolution} &\approx 0.35 \; \mathsf{ns} \end{split}$$

n vs. p PID would be complicated by hadronic interactions in shielding



 ×









Choose $\Delta P_x = 100$ MeV/c 95% probability position will be shifted by less than $\Delta = \frac{\Delta P_x}{|\vec{\sigma}|}$ Lflight

Deflect proton by ≈ 200 MeV/c for clean PID. $\Rightarrow \int Bdl \approx .7$ Tm Remaining 5% corrected based on veto-based PID, opposite-side distribution



 \Leftarrow Efficiency (ϵ) for

neutron/proton detection with

20 MeV (electron equivalent)

threshold

BigHAND efficiency and stability



 \Leftarrow Efficiency (ϵ) for neutron/proton detection with 20 MeV (electron equivalent)

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BigHAND calibration reactionsp(e,e'p) $p(\gamma,\pi^+n)$ $\geqslant \approx$ elastic kinematics (massless e, γ, π)

Neutron calibration $p(\gamma, \pi^+ n)$ uses bremsstrahlung end-point method

Require p_{π^+} at least 1.5% <u>above maximum</u> for three-body background reaction: $p(\gamma, \pi)N\pi$.

\mathbf{Q}^2	E _{beam}	$ heta_e$	E_{π}^{max}	E_{π}^{max}	E_{π}^{limit}	E_{γ}^{min}	$\int \Gamma dk$
			(γ,π)	$(\gamma, 2\pi)$	(γ,π)		
(GeV/c) 2	(GeV)		(GeV)	(GeV)	(GeV)	(GeV)	
3.5	4	37.5°	2.12	2.043	2.074	3.83	0.0029
4.5	4	49.5°	1.603	1.540	1.563	3.78	0.0039
6	5	48.1 °	1.805	1.747	1.773	4.80	0.0028
8	6	52. °	1.73	1.688	1.713	5.79	0.0025

Fiducial cut on $\vec{q} \Rightarrow$ Acceptance losses <5%

(... and tend to cancel in ratio)



Fermi-motion spreads events beyond region calibrated by single BigBite position

Two calibration settings (at 4 of the 6 kinematic points)



 $Q^2=3.5\,{\rm (GeV/c)}^2$

Q^2	a) Fraction (%) in	b) Fraction (%) in
$\left(\text{GeV/c}\right)^2$	Single Cal. Zone	Double Cal. Zone
3.5	93.3	100.
4.5	71.1	95.3
6.0	68.4	94.4
8.0	71.7	88.6

BigHAND Kinematics 4





Simulation

Quasi-elastic

- On shell spectator (\Rightarrow Struck nucleon off shell)
- Boost to struck nucleon rest frame
- Isotropic $\cos(heta)$, ϕ distribution
- Dipole (&Galster) ightarrow cross-section for weight
- Boost back to lab
- Fold in resolution, (weighted) increment of spectra

Inelastic

- GENEV physics Monte Carlo (Genoa/CLAS)
- On-shell initial nucleons ($ec{p}_F$ and $-ec{p}_F$)
- Boost to struck nucleon rest frame
- Generate GENEV event (with boosted beam energy)
- Boost back to lab
- Fold in resolution, increment (un-weighted) spectra

Inelastic normalized empirically to quasi-elastic



SLAC data (Stuart/Lung at E=5.5 GeV) and (Rock at $\theta = 10^{\circ}$)

Simulation Results ($Q^2 = 3.5$ (GeV/c)²)

acceptance-integrated cross section (tb) 3.5 0.5 -0.5 0 1 1.5 2 2.5 3 4 $W^2 \, (\text{GeV}/\text{c}^2)^2$ neutron coincidence acceptance-integrated cross section (tb) 8 6 4 2 0 -0.5 0.5 1.5 3.5 0 1 2 2.5 3 $W^2 (GeV/c^2)^2$



Simulation Results ($Q^2 = 8.0$ (GeV/c)²)

cross section (tb) 05 09 acceptance-integrated 20 0 3.5 1.5 2.5 -0.5 0 0.5 1 2 3 4 $W^2 \, (\text{GeV}/\text{c}^2)^2$ neutron coincidence -0.5 0.5 1.5 2 2.5 3 3.5 -1 0 1 $W^2 \, (\text{GeV}/\text{c}^2)^2$



Simulation θ_{pq} cuts ($Q^2 = 8.0$ (GeV/c)²)





Rates (inputs)

 $\mathcal{L}=10^{37}/ ext{A}$ /cm 2 /s Livetime=80% BigBite tracking eff=75%

Q^2 (GeV/c) 2	3.5	4.5	5.25	6.0	7.0	8.0
E (GeV)	4.	4.	5.	5.	6.	6.
$ heta_e$	37.5 °	49.5 °	40.4 °	48.1 °	42.0 °	52 .°
p efficiency (%)	78.4	86.0	90.1	93.8	96.5	97.6
n efficiency (%)	73.0	80.9	84.7	86.6	89.8	90.6
Quasi-elastic						
p-coinc. $\int rac{d\sigma}{d\Omega} d\Omega$ (fb)	172	293	228	124	85.5	27
n-coinc. $\int rac{d\sigma}{d\Omega} d\Omega$ (fb)	74	131	102	60	40.6	12.4
W^2 cut (%)	98	92	89	84	80	77
Proton elastic (calibration)						
Full $\Delta\Omega$ (mSr)	39.5	53.6	_	53.4	_	53.2
$rac{d\sigma}{d\Omega}{}_{p(e,e')}$ (pb/sr)	71.3	10.9	—	3.00	—	0.57
$p(\gamma,\pi^+)n$ (calibration)						
$\int \Gamma dk$	0.0030	0.0039		0.0028		0.0025
$ heta_{\gamma\pi}^*$	93°	110 [°]	_	114 °	_	123 °
$rac{d\sigma}{d\Omega}_{p(\gamma,\pi^+n)}$ (pb/sr)	2380	1730	_	626	_	313

Rates

Predicted coincidence rates (counts per hour)

$Q^2~({ m GeV/c})^2$	3.5	4.5	5.25	6.0	7.0	8.0
d(e, e'p)	1400	2500	1700	1050	710	220
d(e, e'n)	570	1050	830	470	315	93
p(e, e'p)	47000	11000	_	3200	_	640
$p(\gamma, \pi^+ n)$	1100	1580	_	440		200

Systematic Error Estimates

Estimated contributions (in percent) to systematic errors on R.

Q^2 (GeV/c) 2	3.5	4.5	5.25	6.0	7.0	8.0
Nuclear correction,						
G_{E}^{n} , proton cross-section	-	-	-	-	-	-
Accidentals	-	-	-	-	-	-
Target windows	.2	.2	.2	.2	.2	.2
Acceptance losses	.5	.5	.5	.5	.5	.25
Inelastic contamination	.1	.4	.3	1.	.36	.1
Nucleon mis-identification	.6	.6	.6	.6	.6	.6
BigHAND calibration	0	.13	2.8	.16	1.5	.32
Total (quadrature sum)	.81	.91	2.9	1.3	1.7	.76

\Rightarrow Statistical error goals:

 \leq 2% statistical errors on R at Q^2 =3.5, 4.5, 6.0 (GeV/c) 2

$$\leq$$
 3% statistical errors on R at $Q^2=$ 5.25, 7.0, 8.0 (GeV/c) 2

Beam Time Request

Beam Time Request (beam hours)

Q^2 (GeV/c) 2	3.5	4.5	5.25	6.0	7.0	8.0	
E (GeV)	4.	4.	5.	5.	6.	6.	
$ heta_e$	37.5 ⁰	49.5 ⁰	40.4 ^O	48.1 ⁰	42.0 ^O	52. ⁰	
d(e,e')							
Normal $\mathcal L$	36	24	24	48	36	80	
Dummy target	3	2	2	4	3	8	
Half ${\cal L}$	12	6					
Dummy half ${\cal L}$	2	1					
H(e, e')							
Normal ${\cal L}$	24	6		6		6	
Half ${\cal L}$	3	3		6		6	
Quarter ${\cal L}$	3	6					
BigBen off	6	6		6		6	
Dummy target	4	1		1		1	
$H(\gamma, \pi^+)$							
Radiator	24	24		12		20	
Dummy target	3	3		2		3	
No radiator	6	6		3		5	
Total	126	88	26	88	39	136	\Rightarrow 502
Commissioning							72
2 Energy changes							16
13 angle changes							52
8 polarity changes							32
Beam request							674
							pprox28 days

28 Days

Previous Data ($Q^2 \ge 1$ (GeV/c)²) and CLAS e5 and projected error bars



Backup Slides



 $Q^2=8$ SLAC (E=18.5 GeV, heta=10) and prediction for (E=6 Gev, heta=25)



 $Q^2=3.5$ prediction for d (E=4 Gev, heta=37.5) and GEn on p and 3 He (E=3.3 GeV, heta=51.6)



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