Extracting Azimuthal Asymmetries in Transversity Data

& Tech Note: http://www.jlab.org/~jinhuang/Transversity/MLE.pdf

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In This Talk

What are Azimuthal Asymmetries

- TMD & SIDIS
- Definition
- Related Experiments

Extracting Azimuthal Asymmetries

- Binning and Fitting
- A Maximum Likelihood Method
- Systematic Uncertainties
  - Spin Flips
  - Angular Acceptance

Summary

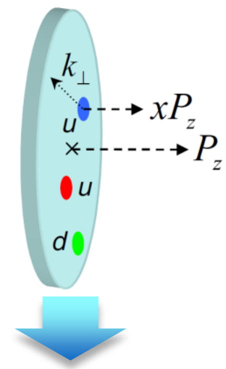
What are Azimuthal Asymmetries in SIDIS

- TMD & SIDIS
- Definition
- Related Experiments

Transverse Momentum Dependent (TMD) Partonic Distributions

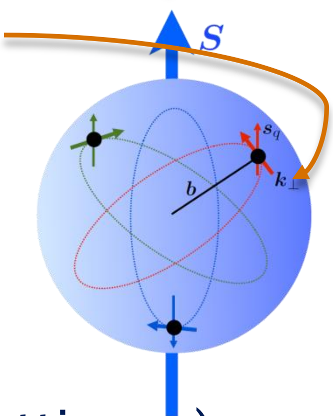
- TMDs link
  - Intrinsic motion of partons
  - Parton spin
  - Spin of the nucleon
- Probes orbital motion of quarks through quark transverse momentum distribution
- A new phase of study, fast developing field
  - Great advance in theories (models, factorization, Lattice …)
  - Not measured until recent years
    - Semi-Inclusive DIS (SIDIS): HERMES, COMPASS, JLab, …
    - p-p(p_bar) process (Drell-Yan, hadron prod, jets) : FNAL, BNL, …

in DIS
Nucleon -> Plate



$\mathbf{L} = \mathbf{r} \times \mathbf{p} \neq^? 0$

→Transverse motion



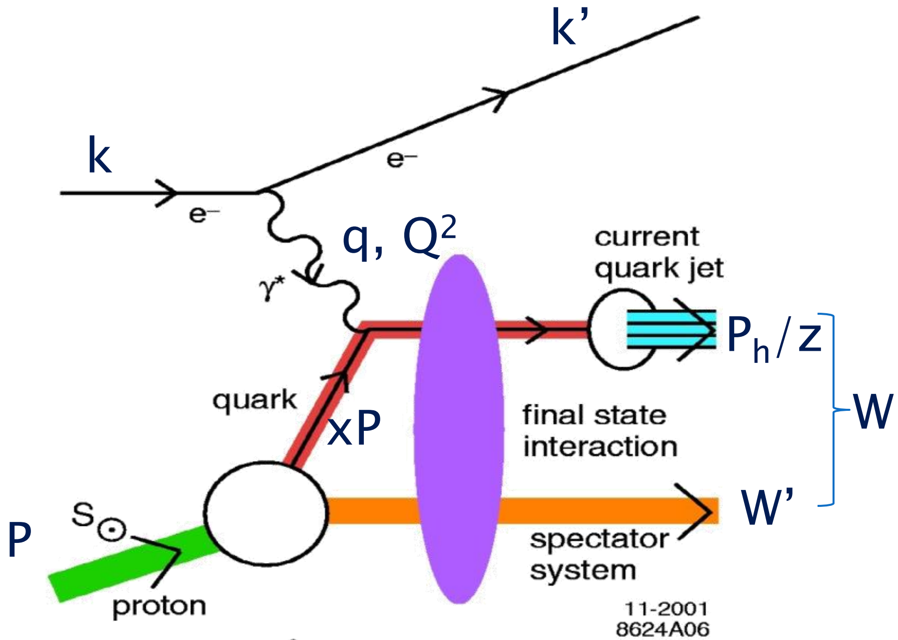
Leading-Twist TMDs

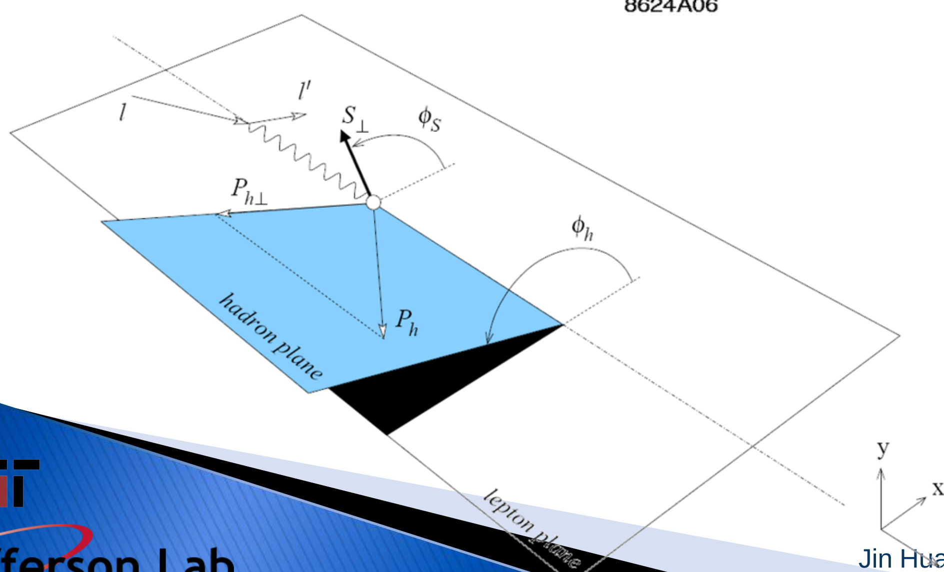
→ Nucleon Spin
→ Quark Spin

		Quark polarization		
		Un-Polarized	Longitudinally Polarized	Transversely Polarized
Nucleon Polarization	U	f_1 =		$h_1^{\perp}$ = − Boer-Mulder
	L		g_1 = − Helicity	$h_{1L}^{\perp}$ = − Worm Gear
	T	$f_{1T}^{\perp}$ = − Sivers	g_{1T} = − Worm Gear	h_{1T} = − Transversity $h_{1T}^{\perp}$ = − Pretzelosity

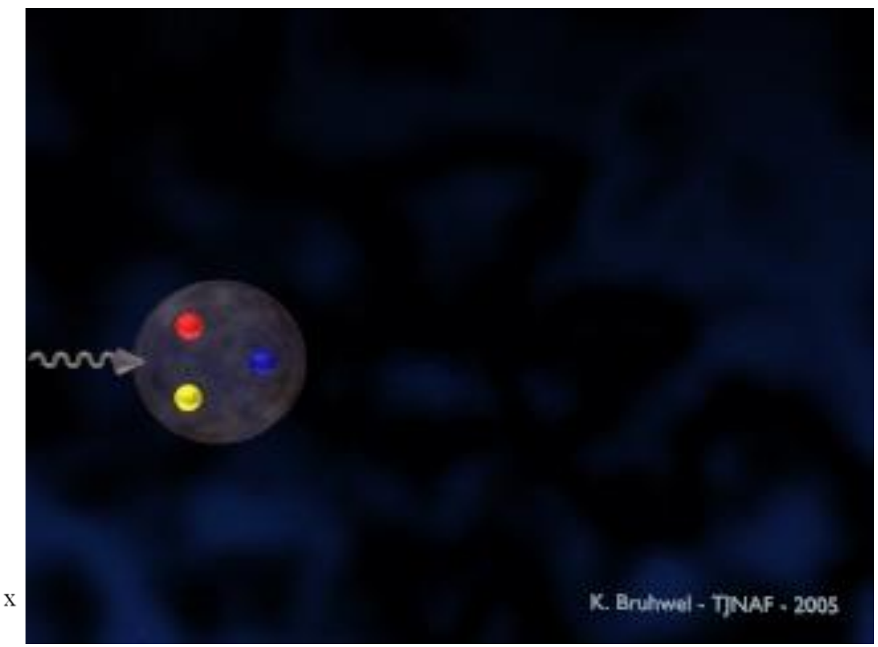
: Contribute to inclusive DIS (quark mass ignored)

One Tool to Study TMDs : SIDIS





- Access to new TMDs not accessible in inclusive DIS ($m_{quark}=0$)
- Variables:
  x, q, Q2, z, W, W'



TMDs in SIDIS Cross Section

$$\frac{d\sigma}{dxdyd\phi_S dzd\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\cdot$$

$$\{F_{UU,T} + \ldots$$
$$+\varepsilon\cos(2\phi_h)\cdot F_{UU}^{\cos(2\phi_h)} + \ldots$$

Unpolarized

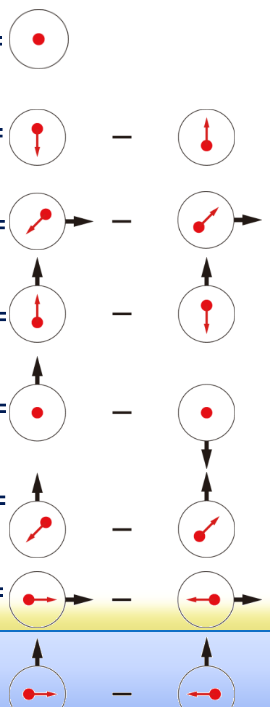
$$+S_L[\varepsilon\sin(2\phi_h)\cdot F_{UL}^{\sin(2\phi_h)} + \ldots]$$
$$+S_T[\varepsilon\sin(\phi_h+\phi_S)\cdot F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+\sin(\phi_h-\phi_S)\cdot(F_{UL}^{\sin(\phi_h-\phi_S)} + \ldots)$$
$$+\varepsilon\sin(3\phi_h-\phi_S)\cdot F_{UT}^{\sin(3\phi_h-\phi_S)} + \ldots]$$

Polarized Target

$$+S_L\lambda_e[\sqrt{1-\varepsilon^2}\cdot F_{LL} + \ldots]$$
$$+S_T\lambda_e[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\cdot F_{LT}^{\cos(\phi_h-\phi_S)} + \ldots]\}$$

Polarized Beam and Target

- $f_1 =$
- Boer-Mulder $h_1^\perp =$
- Worm Gear $h_{1L}^\perp =$
- Transversity $h_{1T} =$
- Sivers $f_{1T}^\perp =$
- Pretzelosity $h_{1T}^\perp =$
- Helicity $g_1 =$
- Worm Gear $g_{1T} =$



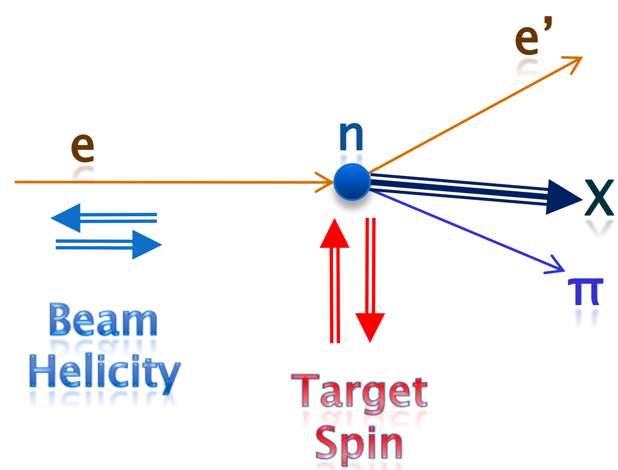
S_L, S_T: Target Polarization; λ_e: Beam Polarization

Extract structure function ratio with azimuthal asymmetries

- Example: $F_{LT}^{\cos(\phi_h-\phi_S)} \propto g_{1T}$ is extractable from Double Beam–Target Spin Asymmetry (DSA) in with transversely polarized target: A_{LT}
- $$\frac{d\sigma}{dxdyd\phi_S dzd\phi_h dP_{h\perp}^2} \propto F_{UU,T} + S_T\lambda_e[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\cdot F_{LT}^{\cos(\phi_h-\phi_S)} + \ldots]$$
- Define Asymmetry: $d\sigma \propto 1 + S_T\lambda_e[\cos(\phi_h-\phi_S)\cdot A_{LT}^{\cos(\phi_h-\phi_S)} + \ldots]$
- Then

$$A_{\mathrm{LT}}^{\cos(\phi_h-\phi_s)} \equiv 2\frac{\int d\phi_s^h\left(d\vec{\sigma} - d\overleftarrow{\sigma}\right)\cos(\phi_h-\phi_s)}{\int d\phi_s^h\left(d\vec{\sigma} + d\overleftarrow{\sigma}\right)}$$

$$= \sqrt{1-\varepsilon^2}\,\frac{F_{LT}^{\cos(\phi_h-\phi_s)}}{F_{UU,T}} \propto g_{1T}^q \otimes D_{1q}^h$$



Related Experiments

- Hall A 6GeV Neutron Transversity
  - Transversely polarized ^{3}He target, Polarized Beam
  - Single Target Spin Asymmetry , A_{UT}
    - Probing TMDs : Sivers, Collins & Pretzelosity
  - Double Spin Asymmetry, A_{LT}
    - Probing worm-gear TMD: g1T
- Hall A 12GeV SIDIS Programs
  - Neutron Transversity with SoLID/Super-Bigbite
  - New proposal: SIDIS asymmetries with long. pol. ^{3}He
  - High precision and large acceptance
- Jlab Hall B, COMPASS, HERMES
- Azimuthal Asymmetries are Popular Observable for Many Other Experiments

Extracting Azimuthal Asymmetries in Transversity

- A Maximum Likelihood Method
- A Binning and Fitting Method

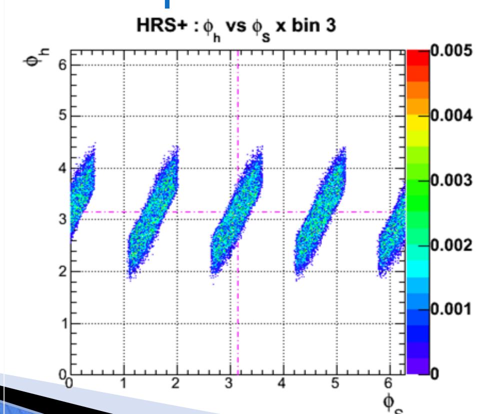
During experiment Transversity

- Input: Physics events
  - Each event provide as set of azimuthal angles
- Full Azimuthal Yield

$$y(\phi_h,\phi_S) = \rho\cdot\sigma\cdot a_{T/V\pm}(\phi_h,\phi_S)\left(1+P_T\sum_i \epsilon_i SSA_i(\phi_h,\phi_S) + |P_{Beam}|\cdot h\cdot\left(P_T\sum_j \epsilon_j DSA_j(\phi_h,\phi_S)+P_L\cdot\left(\epsilon_{LL}+\epsilon_{LL}^{\cos\phi_h}\cdot\cos(\phi_h)\right)\right)\right) \quad (1)$$

$\propto$Luminosity

Azimuthal Acceptance (Partial)



Target Polarization

Modulation Function
Ex. A=$\cos(\phi_h-\phi_S)$

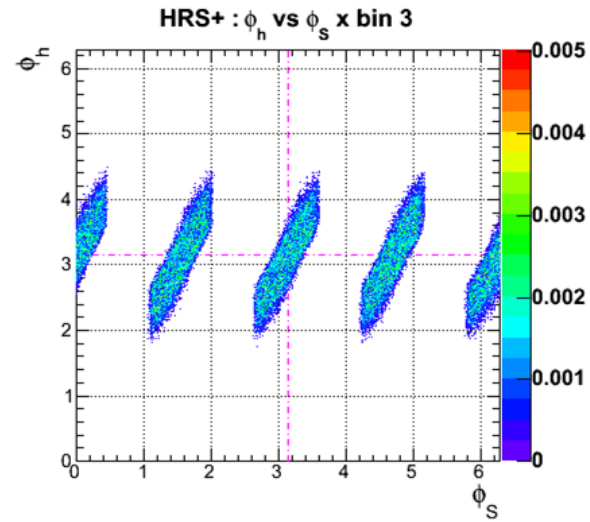
Azimuthal Asymmetry
$\propto$Structure Function
& TMDs

Binning and Fitting Method

1. Make fine bins on Azimuthal Distributions
2. Form spin asymmetry within each bin
   - Techniques to suppress yield drift:
     Local pair method: See X. Qian's thesis
3. Fit with modulation functions over all bins and extract:
   - Angular Modulation
   - Uncertainty
   - Correlation

- Difficult for binning if stat. is low

Azimuthal Distribution



Why MLE

- Maximum likelihood Estimation (MLE) is a popular statistical method providing estimates for the model's parameters
- MLE is
  - No binning, more stable at low stat.
  - asymptotically unbiased
    - its bias -> zero as the sample size increases
  - asymptotically efficient
    - Low mean squared error with the MLE
  - asymptotically normal
    - Gaussian interpretation for the results

Example of low stat stability:
Yield Estimation with multiple data sections

- MLE yield estimation expression is simple:

$$\hat{y}_{MLE} = \sum_i N_i / \sum_i \widetilde{C}_i$$

  - effective charge (life time, target density corrected)

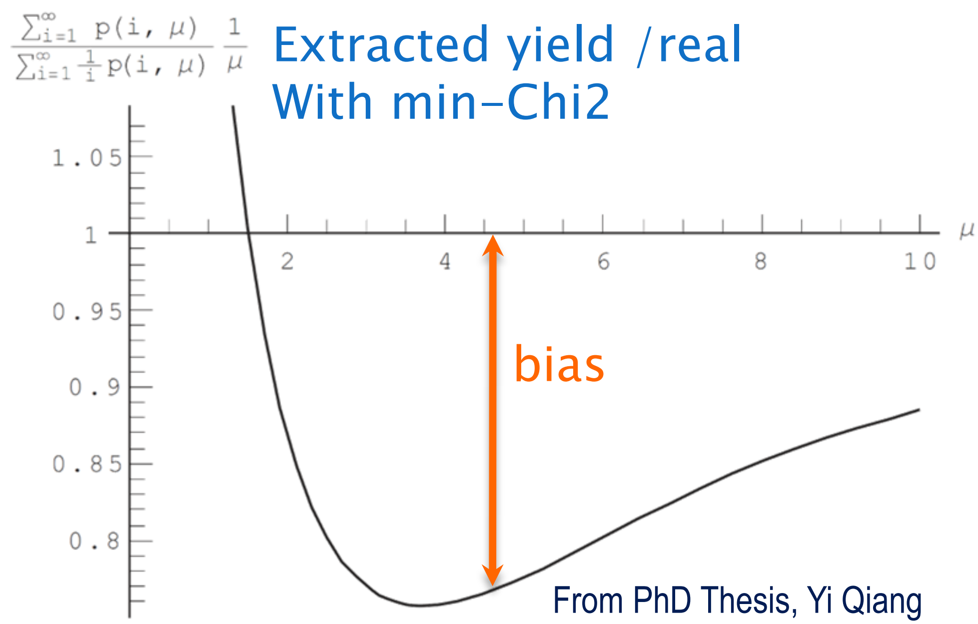
$$\widetilde{C}_{i\pm} \equiv \widetilde{L}_{i\pm} \times LT_{i\pm} / \left(Average\ \ Target\ \ Density\right)$$

- Comparing with min-Chi2 fit
  - min-Chi2 results is a weighted sum:

$$\begin{aligned} \hat{y}_{ws} &= \sum_i \frac{N_i}{\widetilde{C}_i} w_i \\ &= \sum_i \widetilde{C}_i / \sum_j \frac{\widetilde{C}_j^2}{N_i} \end{aligned}$$

Bias of min-Chi2 fit at low stat.

- min-Chi2 show bias when statistics of each bin is low (<10)
- Similar situation for angular binned fitting



The ratio of weighted average to true value μ as a function of μ.

Azimuthal Asymmetry w/ MLE

- Define matrixes

$$\mathbf{F} \equiv \begin{pmatrix} \sum [P^2 A_1 A_1] - N A_{CP,1}^2 & \sum [P^2 A_1 A_2] - N A_{CP,1} A_{CP,2} & \cdots \\ \sum [P^2 A_2 A_1] - N A_{CP,2} A_{CP,1} & \sum [P^2 A_2 A_2] - N A_{CP,2}^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$B \equiv \begin{pmatrix} \sum [P A_1] - N A_{CP,1} \\ \sum [P A_2] - N A_{CP,2} \\ \vdots \end{pmatrix}$$

$$\epsilon \equiv \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \end{pmatrix}$$

Sum over all events: A Function of polarization and azimuthal angle

Corrections related to luminosity asym. And acceptance to be discussed

- The estimator for azimuthal asymmetry:

$$\hat{\epsilon} = \mathbf{F}^{-1} B + O\left(\frac{N_+ - N_-}{N} \sum_{mn} \epsilon_m \epsilon_n\right)$$

- Covariance Matrix: uncertainty & correlations

$$\mathbf{V}(\hat{\epsilon}) = \mathbf{F}^{-1} + O\left(\frac{\sum_{ijk} A_{CPi} A_{CPj} A_{CPk} \sum_i \epsilon_i}{N}\right)$$

Ignore High Order Terms

Comments on correlations

- Example: two-modulation case, no corrections

$$\mathbf{F} = \begin{pmatrix} \sum[P^2 A_1^2] & \sum[P^2 A_2 A_1] \\ \sum[P^2 A_1 A_2] & \sum[P^2 A_2^2] \end{pmatrix}$$

- Covariant Matrix:

Uncertainty

Correlation Coefficient

$$\mathbf{V}(\boldsymbol{\varepsilon}) = \mathbf{F}^{-1} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

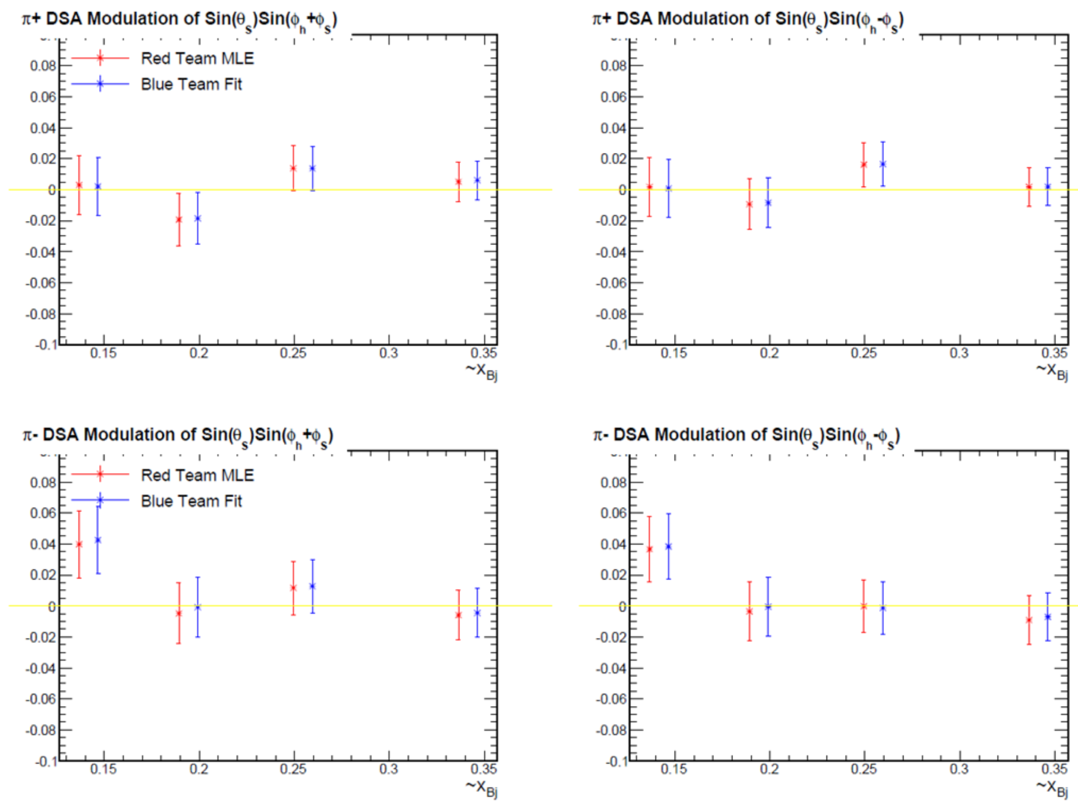
- If are cross-talk between modulations due to limited azimuthal acceptance: $\sum[P^2 A_1 A_2] \neq 0$
- Then
  - uncertainty for each asymmetry is larger
  - Non-zero correlation between extracted asymmetries

Cross check of between methods

- Results are consistent

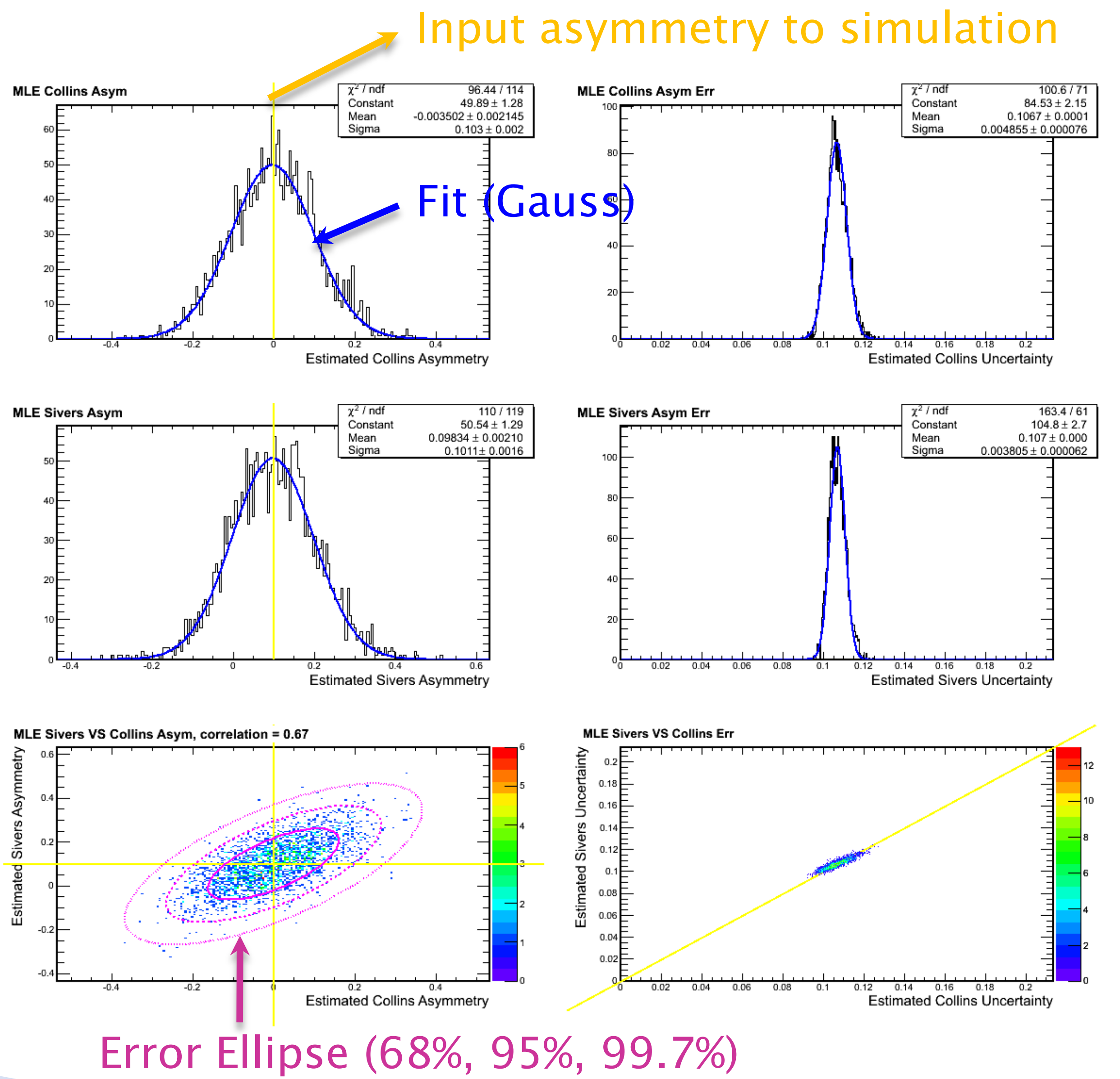
Test: 2 term extraction of an arbitrary azimuthal asymmetries

- **Zero expected under single photon exchange**
- **Raw results shown**



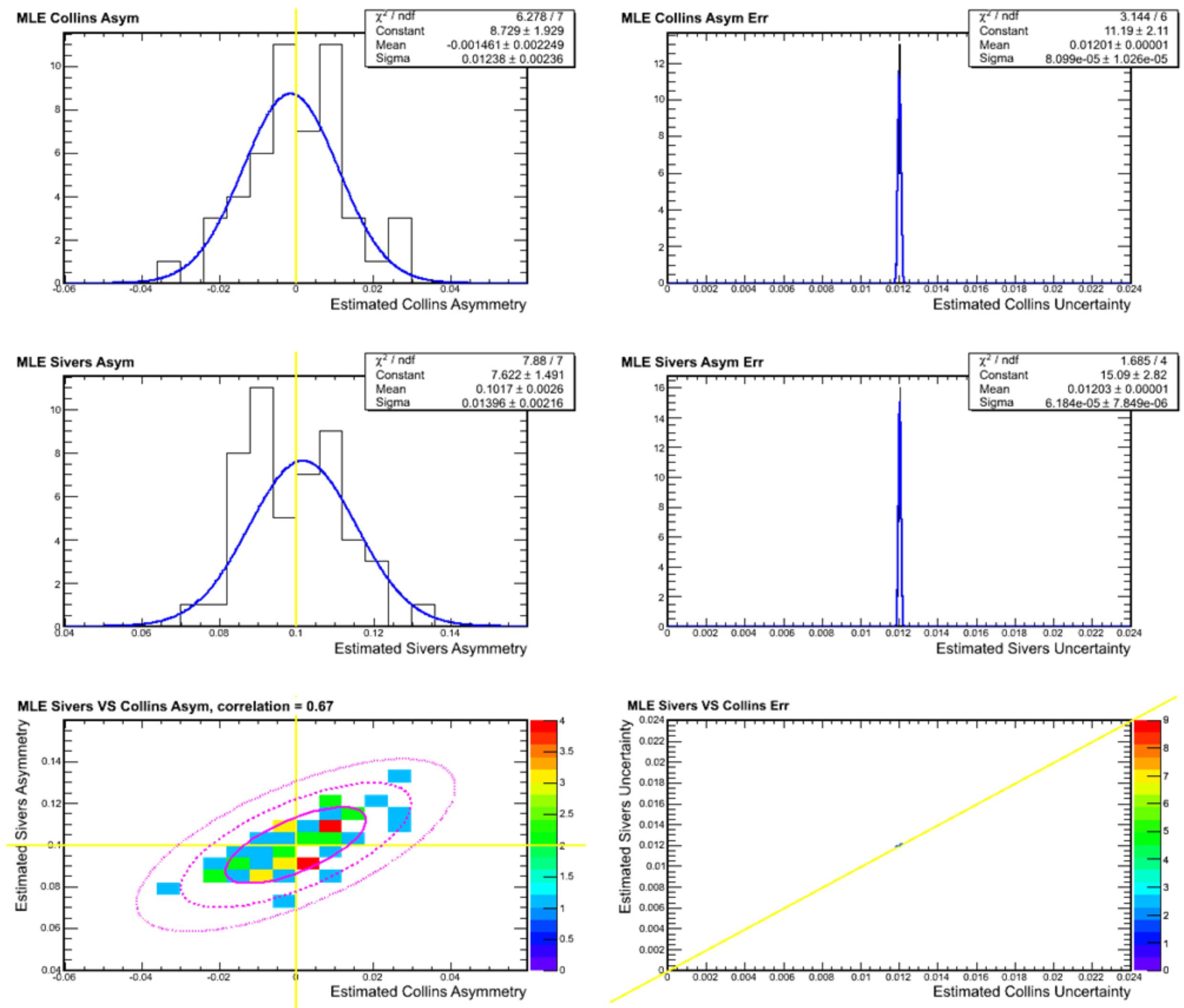
Check w/ Simulations

- 2500 separate simulations with SIMC
- For each data, extract asym and err
- Histogram results
- Verified both calculations



Check with higher statistics per data sets

- More stat. in each data set
- ~= Transversity pion data



Systematic Uncertainties

- Normalization correction A_{CP}, suppressed with
  - Spin/helicity flip
    - suppression $\propto$ Luminosity Asym.
  - Symmetric Acceptance
    - Suppression $\propto$ integral of acceptance $\times$ angular function
- Contamination of other modulation terms
  - Terms not included in the fitting
  - Contamination ratio can be estimated with both methods

Summary

- Methods for extracting SIDIS azimuthal asymmetries are discussed, with consideration of
  - Spin/helicity flip
  - Incomplete azimuthal acceptance
  - Normalization corrections:
    Luminosity, DAQ Life time, etc.
  - Stable at low stat.
- Transversity results shown tomorrow! (K. Allada)
- Methods can be used for
  - 12GeV SIDIS experiments
  - These are generic method
    Studies of other process with needs of extracting azimuthal asymmetries