Jefferson Lab
E01-012 Analysis Report 07
April 2005

Estimate of the total radiation lengths during E01-012

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Abstract

The radiation lengths needed for the radiative corrections, for the nitrogen dilution and elastic analyses are calculated in this note.

1 Polarized ^{3}He cells

1.1 Wall and window thicknesses

1.1.1 Duke

The wall and window thickness measurements were done by Jaideep Singh [1]. The average value for wall thicknesses measured by Vince Sulkosky [2] were very similar: 1.42 mm for the left side* and 1.38 for the right side.

LEFT		RIGHT	
position	thickness (mm)	position	thickness (mm)
A	1.52	E	1.42
B	1.38	F	1.47
I	1.42	J	1.39
C	1.37	G	1.42
D	1.38	H	1.22
avg.	**1.41**	**avg.**	**1.38**

Table 1: Duke wall thicknesses.

measurement method	upstream (A) (μm)	downstream (B) (μm)
opt.	108	137
mech.	105	132

Table 2: Duke window thicknesses.

*If window A is upstream, then walls (A,B,I,C,D) faced the left HRS.

1.1.2 Exodus

The wall and window thickness measurements were done by Vince Sulkosky [2]. A negative value for the distance from the center of the cell corresponds to an upstream position on the cell.

LEFT		RIGHT	
position (cm)	thickness (mm)	position (cm)	thickness (mm)
-15.5	1.167	-16.2	1.167
-9.0	1.264	-8.8	1.146
-3.0	1.211	-2.5	1.120
+4.3	1.155	+4.5	1.126
+10.5	1.122	+10.2	1.111
+16.0	1.137	+15.0	1.209
avg.	**1.176**	**avg.**	**1.147**

Table 3: Exodus wall thicknesses.

measurement method	upstream (μm)	downstream (μm)
opt.1	99.5	126.8
opt.2	100.6	126.9
opt. avg.	**100.05**	**126.85**

Table 4: Exodus window thicknesses.

1.2 Radiation length calculation

Cell	$n_o^{^3He}$ (amg)	$n_o^{N_2}$ (amg)	L_{tc} (cm)	D_{tc} (cm)
Duke	9.18	0.0846	39.4	1.92
Exodus	9.62	0.0865	39.6	1.88

Table 5: Cell characteristics: n_o is the filling density, L_{tc} and D_{tc} are the target chamber length and diameter respectively.

The ratio $N_2/^3He<1\%$ so the radiation length from N_2 is negligeable. In running condition, Duke's target chamber density is about 11.6 amg and Exodus's at 12.0 amg [4]. Thus their radiation lengths are different. With $X_0 = 67.42$ g/cm^2 and $d = n \cdot AMG^{\dagger} \cdot (M_{^3He}/\mathcal{N}_A)$, we get:

$$R_0 = 43423 \text{ cm for Duke}$$

$$R_0 = 41975 \text{ cm for Exodus}$$

For ^{4}He cooling jet:

$$X_0 = 94.32 \text{ g/cm}^2 \text{ and } d = 0.1783 \text{ g/l} \rightarrow R_0 = 528107 \text{ cm}$$

For air at room temperature and 1 atm:

$$X_0 = 36.66 \text{ g/cm}^2 \text{ and } d = 1.205 \text{ g/l} \rightarrow R_0 = 30423 \text{ cm}$$

In the following, the wall and window thicknesses used will be the ones measured by the interferometry method.

†AMG $= 2.6868 \times 10^{19}$ atoms per cm^3

1.2.1 Radiation lengths before scattering

Material	R_0 (cm)		thickness (cm)		# of X_0	
	Duke	Exodus	Duke	Exodus	Duke	Exodus
Be	35.28		0.01778		5.04×10^{-4}	
^{4}He	528107		28.28		5.35×10^{-5}	
glass	7.04		0.0108	0.0101	1.53×10^{-3}	1.43×10^{-3}
^{3}He	43423	41975	19.7	19.8	4.54×10^{-4}	4.72×10^{-4}
Total					$\mathbf{2.55\times10^{-3}}$	$\mathbf{2.46\times10^{-3}}$

Table 6: Radiation lengths of the material encountered before scattering.

1.2.2 Radiation lengths after scattering

Material	X_0 (cm)		thickness (cm)		# of X_0	
	Duke	Exodus	Duke	Exodus	Duke	Exodus
^{3}He	43423	41975	$0.958/\sin\theta$	$0.940/\sin\theta$	$2.21\times10^{-5}/\sin\theta$	$2.24\times10^{-5}/\sin\theta$
glass (L)	7.04		$0.141/\sin\theta$	$0.118/\sin\theta$	$0.0200/\sin\theta$	$0.0168/\sin\theta$
glass (R)	7.04		$0.138/\sin\theta$	$0.115/\sin\theta$	$0.0196/\sin\theta$	$0.0163/\sin\theta$
^{4}He	528107		45.7		8.65×10^{-5}	
Al	8.9		0.04064		4.57×10^{-3}	
Air	30423		65.1		2.14×10^{-3}	
Kapton	28.6		0.01778		6.22×10^{-4}	

Table 7: Radiation lengths of the material encountered after scattering.

1.2.3 Summary

The radiation lengths to be used for the radiative corrections are summarized in table 8.

kinematics				total # of X_0	
E (GeV)	kin	arm	θ	before	after
1.046	all	l	15.993	2.55×10^{-3}	0.0802
1.046	all	r	16.010	"	0.0786
4.018	all	l	24.998	"	0.0549
4.018	all	r	24.998	"	0.0539
5.009	all	l	24.998	"	0.0549
5.009	$6.\{4,5\}$	l	31.979	"	0.0453
5.009	$6.\{1,2,4,5\}$	r	32.001	"	0.0444
3.028	all	l	24.994	2.46×10^{-3}	0.0471
3.028	all	r	25.018	"	0.0461
5.009	6.3	l	31.977	"	0.0391
5.009	6.3	r	31.997	"	0.0383

Table 8: Total radiation lengths before and after scattering.

1.2.4 Uncertainties

The total uncertainty on the wall and window measurements is about 0.6% and the errors arising with using the average value for the wall and window thicknesses are 3% and 1% respectively.

Before the experiment, I measured the aluminium windows with a caliper. The average is 17.06 mils for the right side and 17.76 mils for the left arm. They are supposed to be 15 mils thick but they are recovered by a layer of black paint in order to avoid laser reflexion inside. Morever they had tiny bumps therefore the caliper couldn't be positionned perfectly flat. Since the paint has certainly a lower radiation length than the aluminium thus taking as thickness (16.0±1.5) mils should be safe.

The maximum relative errors on the radiation length before and after scattering are 3.5% and 3.6% respectively (see tables 8-9).

material	X_0 (cm)	σ	dX_0
Be	35.28	0.5 mils	3.60×10^{-5}
^{4}He	528107	2.5 cm	4.73×10^{-6}
glass (window)	7.04	1.5 μm	2.13×10^{-5}
^{3}He	41975	1.0 cm	2.38×10^{-5}
Error total			$\mathbf{8.58\times10^{-5}}$
^{3}He	41975	0.24 cm	5.72×10^{-6}
glass (wall)	7.04	0.05 mm	7.10×10^{-4}
^{4}He	528107	2.5 cm	4.73×10^{-6}
Al	8.9	1.5 mils	4.28×10^{-4}
air	30423	5.0 cm	1.64×10^{-4}
Kapton	28.6	0.5 mils	4.44×10^{-5}
Error total			$\mathbf{1.36\times10^{-3}}$

Table 9: Error summary.

2 Reference cells

2.1 Wall and window thicknesses

2.1.1 Reference cell for Duke

The measurements were done by Jaideep. This cell was already used during the experiment E97-103.

LEFT		RIGHT	
position	thickness (mm)	position (cm)	thickness (mm)
A	1.620	E	1.411
B	1.635	F	1.404
I	1.408	J	1.276
C	1.392	G	1.342
D	1.603	H	1.434
avg.	**1.532**	**avg.**	**1.373**

Table 10: Duke reference cell wall thicknesses.

window	Thickness (μm)
upstream (Short)	118
downstream (Long)	126

Table 11: Duke reference cell window thicknesses.

2.1.2 Reference cell for Exodus

Measurements and a detailed analysis were done by Nathan Justis [3] and then reanalyzed by Vince. A negative value for the distance from the center of the cell corresponds to an upstream position on the cell.

LEFT				RIGHT			
position (cm)	thickness (mm)	σ_{stat} (mm)	σ_{syst} (mm)	position (cm)	thickness (mm)	σ_{stat} (mm)	σ_{syst} (mm)
-15.7	1.533	0.0045	0.0072	-13.5	1.660	0.0051	0.0092
-14.4	1.557	0.0046	0.0064	-12.5	1.678	0.0048	0.0067
-11.7	1.511	0.0050	0.0063	-8.0	1.584	0.0097	0.0063
-8.6	1.482	0.0036	0.0058	-7.0	1.608	0.0053	0.0063
-7.5	1.488	0.0042	0.0058	-4.4	1.648	0.0050	0.0085
-5.5	1.445	0.0051	0.0060	-2.2	1.646	0.0059	0.0085
-3.0	1.507	0.0056	0.0060	0.0	1.648	0.0040	0.0081
+0.5	1.504	0.0044	0.0072	+4.0	1.558	0.0041	0.0080
+3.7	1.484	0.0033	0.0081	+8.5	1.564	0.0044	0.0085
+8.0	1.500	0.0045	0.0071	+9.8	1.555	0.0041	0.0054
+11.8	1.528	0.0044	0.0058	+14.3	1.562	0.0047	0.0085
+15.7	1.542	0.0042	0.0067	+15.7	1.612	0.0031	0.0085
avg.	**1.507**	**0.0045**	**0.0065**	**avg.**	**1.610**	**0.0050**	**0.0077**

Table 12: Exodus reference cell wall thicknesses.

Window	Thickness (μm)	σ_{stat} (μm)	σ_{syst} (μm)
Upstream	125.7	0.10	0.50
Downstream	125.6	0.07	0.71

Table 13: Exodus reference cell window thicknesses.

2.2 Radiation length calculation

Duke reference cell has a target length of 39.2 cm but other dimensions are not known. No measurement of Exodus reference cell dimensions was done.

The reference cells were filled with N_2 or ^{3}He at different pressures. In this note, only the radiation length of the gas inside the reference cells is not taken into account but it will be added in the nitrogen dilution [5] and pressure curve [6] analyses.

Material	X_0 (cm)	thickness (cm)		# of X_0	
		Ref. Duke	Ref. Exodus	Ref. Duke	Ref. Exodus
Be	35.28	0.01778		5.04×10^{-4}	
^{4}He	528107	28.28		5.35×10^{-5}	
glass	7.04	0.0118	0.0126	1.68×10^{-3}	1.79×10^{-3}
Total				$\mathbf{2.23\times10^{-3}}$	$\mathbf{2.35\times10^{-3}}$

Table 14: Radiation lengths of the material encountered before scattering.

Material	X_0 (cm)	thickness (cm)		# of X_0	
		Ref. Duke	Ref. Exodus	Ref. Duke	Ref. Exodus
glass (L)	7.04	$0.153/\sin\theta$	$0.151/\sin\theta$	$0.0217/\sin\theta$	$0.0214/\sin\theta$
glass (R)	7.04	$0.137/\sin\theta$	$0.161/\sin\theta$	$0.0195/\sin\theta$	$0.0229/\sin\theta$
^{4}He	528107	45.7		8.65×10^{-5}	
Al	8.9	0.04064		4.57×10^{-3}	
Air	30423	65.1		2.14×10^{-3}	
Kapton	28.6	0.01778		6.22×10^{-4}	

Table 15: Radiation lengths of the material encountered after scattering.

The radiation lengths to be used for the radiative corrections are summarized in table 16.

kinematics				total # of X_0	
E (GeV)	kin	arm	θ	before	after
1.046	all	l	15.993	2.23×10^{-3}	0.0863
1.046	all	r	16.010	"	0.0780
4.018	all	l	24.998	"	0.0588
4.018	all	r	24.998	"	0.0535
5.009	all	l	24.998	"	0.0588
5.009	$6.\{4,5\}$	l	31.979	"	0.0485
5.009	$6.\{1,2,4,5\}$	r	32.001	"	0.0441
3.028	all	l	24.994	2.35×10^{-3}	0.0582
3.028	all	r	25.018	"	0.0615
5.009	6.3	l	31.977	"	0.0479
5.009	6.3	r	31.997	"	0.0506

Table 16: Total radiation lengths before and after scattering for empty reference cells.

References

[1] J. Singh, *Cell Data Table*, http://galileo.phys.virginia.edu/research/groups/spinphysics/celldata.htm

[2] V. Sulkosky, *Cell Wall Thickness Measurements*, http://www.jlab.org/%7Evasulk/cell thickness.html

[3] N. Justis, *Measuring glass thickness of a reference cell used in a polarized ^{3}He experiment*, Brigham Young University, August 2004

[4] P. Solvignon, *Target density for E0-012*, E01-012 Analysis Report 05, March 2005.

[5] P. Solvignon, *Nitrogen dilution analysis for E0-012*, in progress.

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