Scattering Off a Moving Pair in Nuclei

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INTRODUCTION

TJNAF proposal E01-106 uses the (e,epN) reaction on ^{12}C in hall A as a tool to measure short range nucleon-nucleon correlations (NN SRC). In the context of this proposal we refer to NN SRC has a pre-existing pair of nucleons which have back-to-back high momenta balancing each other. The two existing magnetic spectrometers will be used to measure the (e,ep) part of the reaction. The measurement requires a third spectrometer (BigBite) and an array of scintillation counters, to simultaneously measure neutrons and protons in coincidence with the outgoing high momentum electron and proton.

For the purpose of estimating the counting rate we assumed that, at the high momenta, we are focusing on, all counts are due completely to 2N SRC. We also assumed that under this condition the "quasi Deutron" picture of FS is applicable and the measured cross section can be expressed, neglecting the CM motion of the pair, as:

$$\frac{d\sigma}{dE_e d\Omega_e d\Omega_p} = a_2 \times Z \times \frac{d\sigma}{dE_e d\Omega_e d\Omega_p}_d \tag{1}$$

where $\frac{d\sigma}{dE_e d\Omega_e d\Omega_p}_d$ is the deuteron differential cross section. a_2 is a scaling ratio obtained from high-Q^2 inclusive (e,e') scattering at $x > 1$:

$$a_2(^{12}C) = 4.5 \pm 0.5. \tag{2}$$

The purpose of this work is to take into account the CM motion of the np-pair in the nucleus using MCEEP. This motion is expected to broaden the angular and momentum (TOF) correlation of the proton and the recoil nucleon. We will study its effect on the expected counting rate and the signal to background ratio.

Changes to MCEEP v3.5

The following refers to the MCEEP d(e,e'p)n option with the bound state (B) option entered by the user at startup.

DESCRIPTION OF MCEEP

MCEEP is capable of simulating the d(e,e'p)n scattering process in the laboratory. The beam described by the user determines the incident electron momentum, while the target nucleus is assumed to be at rest. An event is characterized by five randomly sampled quantities. They are: the in-plane and out-of-plane electron scattering angles, the momentum of the scattered electron and the in-plane and out-of-plane ejectile proton angles. The missing mass specified by the user input file is then used together with these five randomly sampled quantities to calculate the momentum of the ejectile proton. The momentum of the recoil nucleus (neutron) is defined by the other particles. An event is passed if the outgoing electron and proton fall within the spectrometers' acceptance and the kinematical calculations are sound. In this case the event cross section is calculated and used as a weighting factor for the kinematical quantities which are histogramed at this point.

DESCRIPTION OF CHANGES

We wanted to incorporate a target nucleus (d) which moves with a gaussian distributed center of mass momentum. To achieve this, an energy-momentum 4-vector is generated at the beginning of each event to describe the moving nucleus.

MCEEP assumes that the target nucleus is at rest when it calculates kinematical quantities and cross sections so these parts of the program were changed using the following considerations:

a) Kinematics, mainly the ejectile proton momentum and the recoil nucleus momentum, are calculated according to new expressions derived from the equations of conservation of energy and conservation of momentum taking into account the momentum of the center of mass of the target nucleus.

These new expressions revert back into the original MCEEP expressions when the target nucleus is at rest:

(i) invariant hadronic mass:

was:

$$W^2 = [(M_T, \vec{0}) + (\omega, \vec{q})]^2. \tag{3}$$

and now is:

$$W^2 = [(E_{CM}, \vec{P}_{CM}) + (\omega, \vec{q})]^2, \tag{4}$$

$$E_{CM} = \sqrt{M_T^2 + |\vec{P}_{CM}|^2}. \tag{5}$$

(ii) ejectile momentum (from solution of quadratic equation):

was:

$$a|\vec{p}|^2 + b|\vec{p}| + c = 0, \tag{6}$$

where:

$$a = 4(\alpha - |\vec{q}|^2 \cos^2 \theta_{pq}), \tag{7}$$

$$b = 4|\vec{q}|\beta \cos \theta_{pq}, \tag{8}$$

$$c = 4\alpha m_p^2 - \beta^2, \tag{9}$$

with:

$$\alpha = (\omega + M_T)^2, \tag{10}$$

$$\beta = |\vec{q}|^2 - \alpha + (\epsilon_m + M_T - m_p)^2 - m_p^2, \tag{11}$$

and now is:

$$a'|\vec{p}|^2 + b'|\vec{p}| + c' = 0, \tag{12}$$

where:

$$a' = 4(\alpha' - |\vec{q} + \vec{P}_{CM}|^2 \cos^2 \theta_{p(q+P_{CM})}), \tag{13}$$

$$b' = 4|\vec{q} + \vec{P}_{CM}|\beta' \cos\theta_{p(q+P_{CM})}, \tag{14}$$

$$c' = 4\alpha' m_p^2 - \beta'^2, \tag{15}$$

with:

$$\alpha' = (\omega + E_{CM})^2, \tag{16}$$

$$\beta' = |\vec{q} + \vec{P}_{CM}|^2 - \alpha' + (\epsilon_m + M_T - m_p)^2 - m_p^2. \tag{17}$$

(iii) recoil momentum:

was:

$$\vec{p}_r = \vec{q} - \vec{p}, \tag{18}$$

and now is:

$$\vec{p}_r = \vec{q} + \vec{P}_{CM} - \vec{p}. \tag{19}$$

b) The differential cross section is now calculated in the pair CM frame. The 4-momentum of the incident electron, 4-momentum transfer, ejectile proton and recoil nucleus are Lorentz transformed into the center of mass frame. These transformed quantities are used in the calculation of the cross section. The result is then expressed in the lab frame using Jacobians.

A Lorentz transformation of an arbitrary 4-momentum $(E, \vec{P})$ into the rest frame of a target particle whose laboratory 4-momentum is $(E_{CM}, \vec{P}_{CM})$ is given by:

$$E' = \frac{E E_{CM} - \vec{P} \cdot \vec{P}_{CM}}{M_T} \tag{20}$$

and:

$$\vec{P}' = \vec{P} - \left(\frac{E + E'}{E_{CM} + M_T}\right) \vec{P}_{CM}. \tag{21}$$

The differential cross section in the lab frame is related to the differential cross section in the CM frame by:

$$\frac{d^5\sigma}{dE_e d\Omega_e d\Omega_p} = \frac{d^5\sigma'}{dE_e' d\Omega_e' d\Omega_p'} \times \frac{\partial(E_e'\Omega_e'\Omega_p')}{\partial(E_e\Omega_e\Omega_p)} \tag{22}$$

where:

$$\frac{\partial(E_e'\Omega_e'\Omega_p')}{\partial(E_e\Omega_e\Omega_p)} = \frac{\partial(E_e'\Omega_e')}{\partial(E_e\Omega_e)} \times \frac{\partial\Omega_p'}{\partial\Omega_p} \tag{23}$$

with:

$$\frac{\partial(E_e'\Omega_e')}{\partial(E_e\Omega_e)} = \frac{|\vec{p}_e|}{|\vec{p_e'}|}, \tag{24}$$

$$\frac{\partial\Omega_p'}{\partial\Omega_p} = \frac{|\vec{p}_p|^2}{|\vec{p_p'}|^2} \left(\gamma sin\theta_p sin\theta_p' + cos\theta_p cos\theta_p'\right). \tag{25}$$

See Appendix 1 for derivation of (24) and (25).

TECHNICAL DESCRIPTION OF CHANGES

(I) common area VAR.CMN:

added the following variables:

VARIANCE: target nucleus momentum distribution variance
The following common arrays are used for storing the various 4-momenta:
DEUTB: target nucleus 4-momentum in laboratory frame
ELECB,ELECF: incident electron 4-momenta
QB,QF: 4-momentum transfer
PFB,PFF: ejectile proton 4-momenta
PRB,PRF: recoil nucleus 4-momenta

(II) Following is a description of the changes to the main mceep routine and to the relevant subroutines in the order they are called by MCEEP:

(a) subroutine PHYS_CHOICE: Option 351 added. It is essentially the same as Option 350 (Relativistic PWBA for d(e,e'p)n) with the addition that the user is prompted to enter the deuterium momentum distribution variance which is read into variable VARIANCE.

(b) main mceep routine:

i) Immediately after target mass MNUC is determined, the target nucleus 4-momentum array DEUTB is initialized since at this point subroutine RANGES is called which in turn makes a call to subroutine KINEM (changes to subroutine KINEM are described in (III)).

ii) At the beginning of the main event loop a random momentum vector is generated as follows: A call to the CERN library function V100 NORRAN returns a randomly normal gaussian distributed value. This is then multiplied by VARIANCE and the result is used as the x component of the momentum which is then put into the array DEUTB. This process is then repeated for the y and z momentum components.

(c) subroutine KINEM:

The following is an explanation of the names of the (laboratory) kinematic variables:
ECM: target nucleus energy
PCM: target nucleus momentum vector
OMEGA: energy transfer
Q: momentum transfer vector
PF: proton final momentum vector
PR: recoil nucleus momentum vector

i) The invariant hadronic mass is calculated:

$$W = SQRT((ECM + OMEGA) * *2 - (PCM + Q) * *2). \qquad (26)$$

The condition:

$$W \geq (MISS\_M + MNUC) \tag{27}$$

is the same as before.

ii) A call to the subroutine PFIN is made (changes to subroutine PFIN are described in (IV)).

iii) The recoil momentum is calculated:

$$PR = Q + PCM - PF \tag{28}$$

iv) A forward Lorentz transformation is performed on each back frame 4-momentum array $(E, \vec{P})$ and the result is stored in the forward frame 4-momentum array $(E', \vec{P}')$.

v) The recoil factor RECFAC is calculated in the forward frame. This is achieved by giving subroutine FREC only forward Lorentz transformed quantities.

(d) subroutine PFIN: Ejectile momentum is calculated like before with the following changes:

$$Q \rightarrow Q + PCM \tag{29}$$

$$CONST2 = (OMEGA + ECM) * *2. \tag{30}$$

(e) subroutine PHYSICS: Option 351 added. It is essentially the same as Option 350 (Relativistic PWBA for d(e,e'p)n) except that the cross section SIGMA_EEP is calculated in the forward frame. This is achieved by giving subroutine WQPWBA only forward Lorentz transformed quantities. The differential cross section is then multiplied by a Jacobian to get the lab frame result:

$$SIGMA\_EEP = JACOBIAN * SIGMA\_EEP \tag{31}$$

THE SIGNAL/BACKGROUND RATIO

In a dedicated beam development time during April 2001 we performed a triple coincidence d(e,epn) measurement. See detail report in []. The Kinematics and the detector setups for this measurement is called K20. We would like to use the simulation and the beam development measurements to estimate the expected signal to background ratio in the proposed experiment. The Kinematics and the detector setups for the 300 MeV/c missing momentum measurement is called K30.

The main factors that determine the signal to background ratio are:

- singles rates
- The ratio of (e,ep) - double coincidence events, to the (e,epn) - triple coincidence events.
- The width of the neutron TOF peak.

In the Carbon case he CM motion of the pair smear the exact equality between the Pn vector and Pmiss vector in the deutron case. This cause smaller ratio of triple /double coincidence events ratio and a wider neutron TOF peak. We use the simulation to estimate these effect. We assumed that the CM momentum has a Gaussian distribution with $\sigma_x = \sigma_y = \sigma_z = 100$ MeV/c.

For kinematics K20 using a neutron angle cuts correspond to detector of size $\pm 3°$ (V) and $\pm 10°$ (H) located at the center of the acceptance, we found that 7.5 % of the (e,ep) have also neutron in the detector. For kinematics K30 using the same neutron angle cuts we found that about 1 % of the (e,ep) have also neutron in the detector.

We also found in the simulation (see Fig. 9) that the relevant neutron momentum distribution is between 200 and 400 MeV/c, which for a counter at 5 meter from the target is translated to a TOF window of about 40 ns. This is about the same time window as we used in the "carbon-like" analysis of the test run data (35 ns).

(comment: If we ignore the CM motion for K30, about 5 % of the (e,ep) have also neutron in the detector. So the CM motion reduced the detected ratio by a factor of 5. This is in agreement with the K0=0.2 factor we used in the proposal (eq. 12) to estimate the counting rates.)

This mean that the signal to background ratio will be no more that a factor off 7.5 less due to this difference between the test run and the proposed measurement.

The singles rates measured in the triple coinc deuterium test (15cm, LD2, 4.2m) were 1.5 times higher than expected in the running conditions of the proposal. A factor 5 reduction due to extra shielding and the sweaping magnet (Big Bite) will compensate for the expected CM motion. Therefore, one expects the measured signal/background in the proposed experiment to be as good as we got for the "carbon-like" analysis of the test run.

Triple coincidence rates

We can use the MCCEP results to obtain the expected yield. The K30 MCEEP simulation is for 100 micro Amper on 15 cm, 0.17 gr/cm^2, LD2 target. The proposal is for 1mm (0.2 gr/cm^2), C target and 1 micro Amper. If we assume $a_2 \times Z = 4.5 \times 6$ we gets that the expected triple coincidence rate of the proposal is a third of the calculated K30 rate. If we assume a 3 times larger solid angle for the neutron array (calculations:$\pm 3°$ (V) and $\pm 10°$ (H); array planed: $\pm 17.2°$ (V) and $\pm 5°$ (H)) and attenuation*efficiency of 0.5*0.9*0.3 (0.5-attenuation in the lead, 0.9 attenuation in the first veto layer, 0.3 detection efficiency of 3 layers of 10 cm wide scintillators) the proposal rates should be 13.5% of the K30 calculated once.

K30 simulations yield 100 events per hour. Therefore, the expected triple coincidence rate is 13.5 events/hour. For the 100 hours devoted for the "300 MeV/c" missing momentum measurement that leads to an expected total of about 1350 np events.

How to integrate to obtain the full correlated neutron yield

As can be seen in Figs 7 and 8 the CM motion of the pair cause the angular range of the third particle to be larger than the acceptance of Big

Bite and the planed neutron array (with a match acceptance to Big Bite). To obtain the total triple over coincidence ratios one will have to extrapolate the mesurments to a wider angular range. This can be done using the vertical angular distribution. As can be seen in Figs 10-15 The vertical angular distribution is very sensitive to the pair CM momentum in the vertical (σ_y) direction and almost not sensitive to the other two orthogonal components of the pair CM momentum (σ_x, σ_z). Each figure show the vertical distribution for both $\sigma_x = \sigma_z = 0$ and $\sigma_x = \sigma_z = 100MeV/c$. We will used the very large vertical angular range ($\pm17.2^\circ$) to obtain σ_y and assuming ($\sigma_x = \sigma_y = \sigma_z$) we will perform the integration/extrapolation over the full range. The horizontal measured angular distribution (maybe in two positions of the BigBite / neutron array) will allow to check the assumptions made.

Appendix 1

Differential cross section Jacobian

The Lorentz transformations between two inertial frames K and K' where the relative motion is along the z axis are given by:

$$
\begin{array}{ll}
p'_z = \gamma(p_z - \beta E) & p_z = \gamma(p'_z + \beta E') \\
p'_y = p_y & p_y = p'_y \\
p'_x = p_x & p_x = p'_x \\
E' = \gamma(E - \beta p_z) & E = \gamma(E' + \beta p'_z)
\end{array}
$$

and in spherical coordinates:

$$
\begin{array}{l}
p_x = psin\theta cos\phi \\
p_y = psin\theta sin\phi \\
p_z = pcos\theta
\end{array}
$$

become:

$$
\begin{array}{ll}
p'cos\theta' = \gamma(pcos\theta - \beta E) & pcos\theta = \gamma(p' + \beta E') \\
p'sin\theta' = psin\theta & psin\theta = p'sin\theta' \\
\phi' = \phi & \phi = \phi' \\
E' = \gamma(E - \beta pcos\theta) & E = \gamma(E' + \beta p'cos\theta').
\end{array}
$$

(i) Calculation of the solid angle Jacobian $\frac{\partial\Omega'}{\partial\Omega}$:

$$\frac{\partial\Omega'}{\partial\Omega} = \frac{sin\theta' d\theta' d\phi'}{sin\theta d\theta d\phi} = \frac{pd\theta'}{p'd\theta} \tag{32}$$

Implicitly differentiating the equation $E = \gamma(E' + \beta p'cos\theta')$ gives:

$$0 = \frac{dE'}{d\theta} + \beta\frac{dp'}{d\theta}cos\theta' - \beta p'sin\theta'\frac{d\theta'}{d\theta} \tag{33}$$

By inserting:

$$\frac{dE'}{d\theta} = \gamma\beta psin\theta \tag{34}$$

$$\frac{dp'}{d\theta} = \frac{pcos\theta}{sin\theta'} - \frac{p'cos\theta'}{sin\theta'}\frac{d\theta'}{d\theta} \tag{35}$$

into (33) we get:

$$0 = \gamma\beta p sin\theta + \beta \left(\frac{pcos\theta}{sin\theta'} - \frac{p' cos\theta'}{sin\theta'} \frac{d\theta'}{d\theta} \right) cos\theta' - \beta p' sin\theta' \frac{d\theta'}{d\theta} \tag{36}$$

and by collecting terms:

$$\frac{d\theta'}{d\theta} = \frac{p}{p'} \left(\gamma sin\theta sin\theta' + cos\theta cos\theta' \right). \tag{37}$$

Therefore (32) gives:

$$\frac{\partial \Omega'}{\partial \Omega} = \frac{p^2}{p'^2} \left(\gamma sin\theta sin\theta' + cos\theta cos\theta' \right). \tag{38}$$

(ii) Calculation of $\frac{\partial(E'\Omega')}{\partial(E\Omega)}$ [1]:

$$\frac{\partial(E'\Omega')}{\partial(E\Omega)} = \frac{\partial(E'\Omega')}{\partial(p'\Omega')} \frac{\partial(p'\Omega')}{\partial(p\Omega)} \frac{\partial(p\Omega)}{\partial(E\Omega)} = \frac{p'}{E'} \frac{\partial(p'\Omega')}{\partial(p\Omega)} \frac{E}{p} \tag{39}$$

and:

$$\frac{\partial(p'\Omega')}{\partial(p\Omega)} = \frac{p^2 E'}{p'^2 E} \tag{40}$$

Therefore (39) gives:

$$\frac{\partial(E'\Omega')}{\partial(E\Omega)} = \frac{p}{p'} \tag{41}$$

Bibliography

[1]*Relativistic Kinematics, R. Hagedorn CERN 1963 pp. 42-43.*

Appendix 2

MCEEP results

The MCEEP simulations were done for: 15cm CD2 traget, 100 micro Amper beam, 1 hour run. The cuts we applied are:

- abs(PREC.TH)< 3 deg. (out of plane neutron angle).
- 80 deg. < PREC.PH < 100 deg. (in plane neutron angle).
- PMAG.F >200 MeV/c (n/p momenta in the pair CM system).

The number of events are:

Kinematics	tests	no CM motion	$\sigma_x = \sigma_y = \sigma_z = 100$ MeV/c.
K20	no tests	675,000	
K20	$1+2$	78,000	
K20	$1+2+3$	50,000	
K30	no tests	7580	8470
K30	$1+2$	441	124
K30	$1+2+3$	323	96

Fraction of (e,ep) events with a neutron in a detector of a size ±3 deg. horizontal and ±10 deg. vertical located at 90 deg. to the beam.

Kinematics	no CM motion	$\sigma_x = \sigma_y = \sigma_z = 100$ MeV/c.
K20	7.4 %	
K30	5.2 %	1.1 %

The MCEEP simulations for the sized of the planed n-array which has the Big Bite angular acceptance:

- abs(PREC.TH)$<$ 17.2 deg. (out of plane neutron angle).
- 85 deg. $<$ PREC.PH $<$ 95 deg. (in plane neutron angle).

The number of events are:

Kinematics	tests	no CM motion	$\sigma_x = \sigma_y = \sigma_z = 100$ MeV/c.
K30	no tests		8448
K30	$1+2$		661
K30	$1+2+3$		524

Fraction of (e,ep) events with a neutron in a detector of a size $\pm$17.2 deg. horizontal and $\pm$5 deg. vertical located at 90 deg. to the beam.

Kinematics	no CM motion	$\sigma_x = \sigma_y = \sigma_z = 100$ MeV/c.
K30		3.1 %

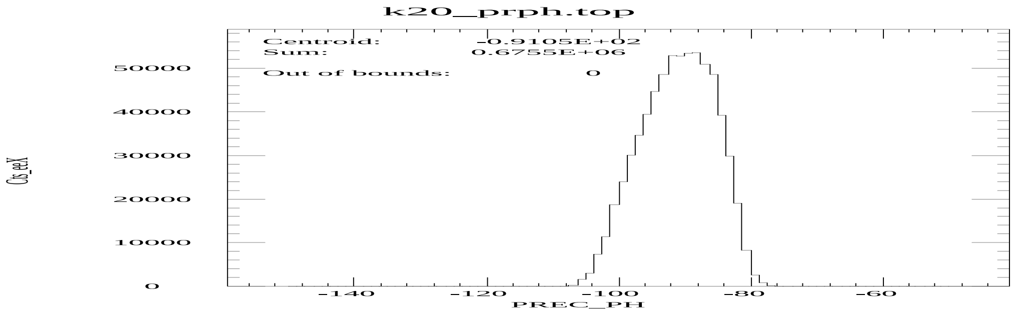

Figure 1: K20 , no CM motion. Angular distribution in the scattering plane.

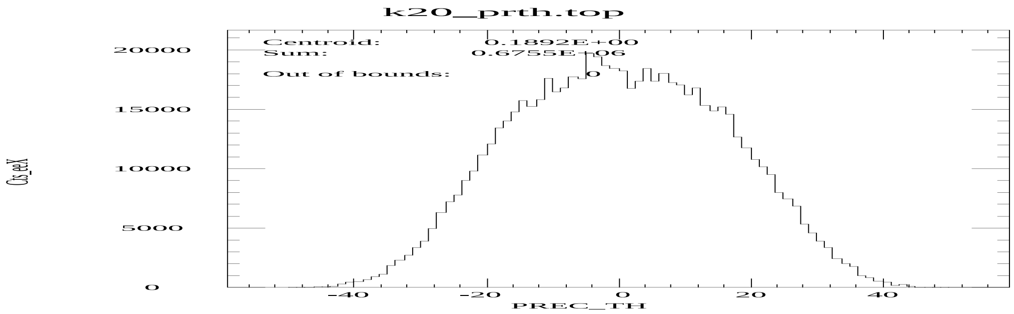

Figure 2: K20 , no CM motion. Angular distribution out of the scattering plane.

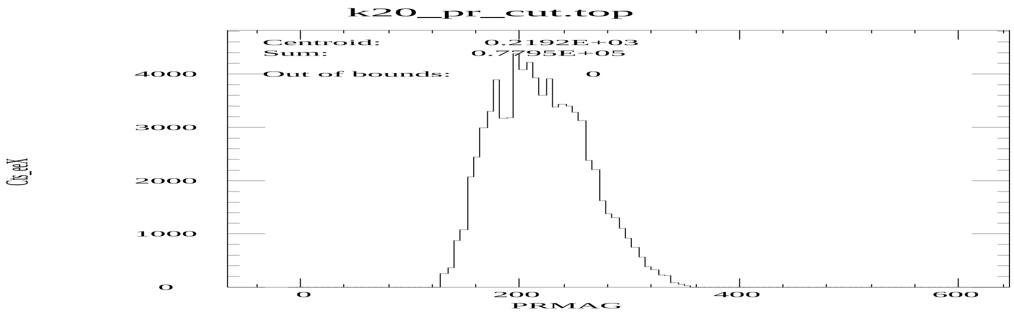

Figure 3: K20 , no CM motion. Neutron momentum distribution with angular cuts.

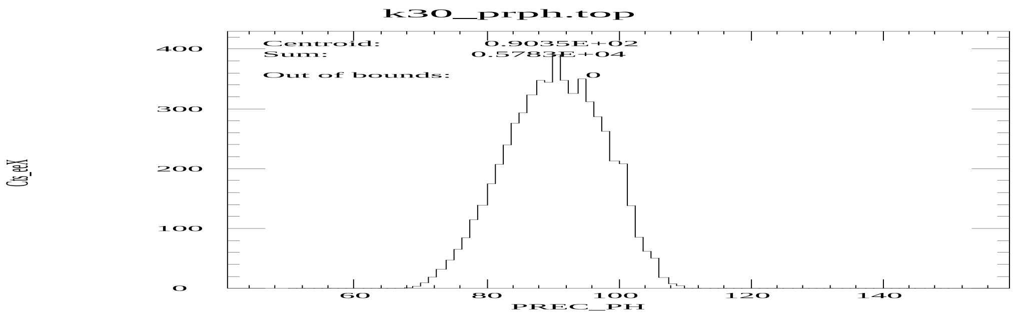

Figure 4: K30 , no CM motion. Angular distribution in the scattering plane.

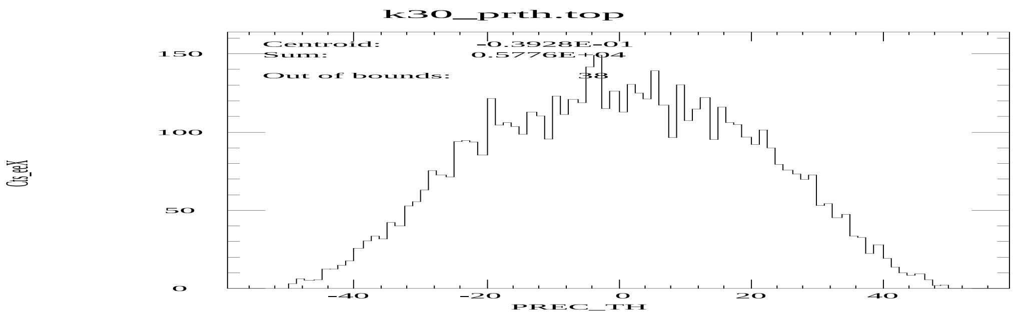

Figure 5: K30 , no CM motion. Angular distribution out of the scattering plane.

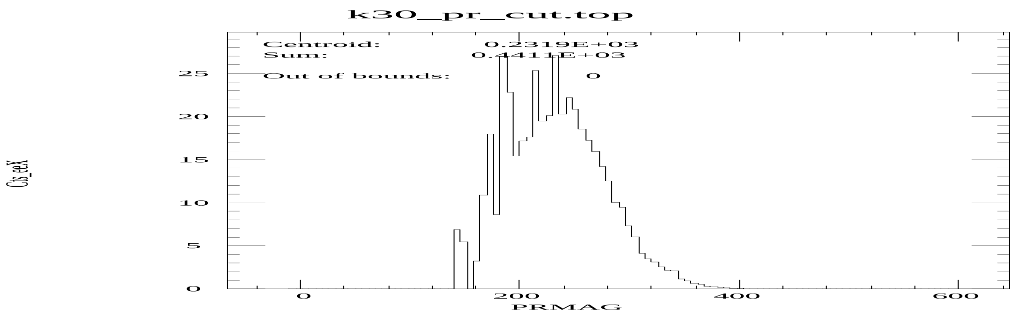

Figure 6: K30 , no CM motion. Neutron momentum distribution with angular cuts.

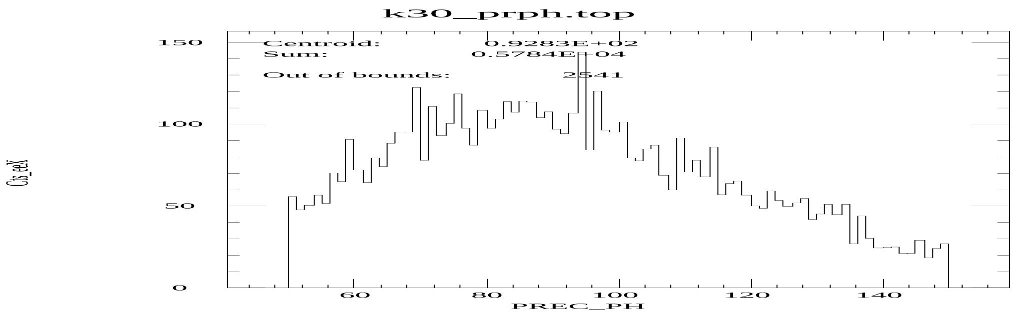

Figure 7: K30 , CM motion. Angular distribution in the scattering plane.

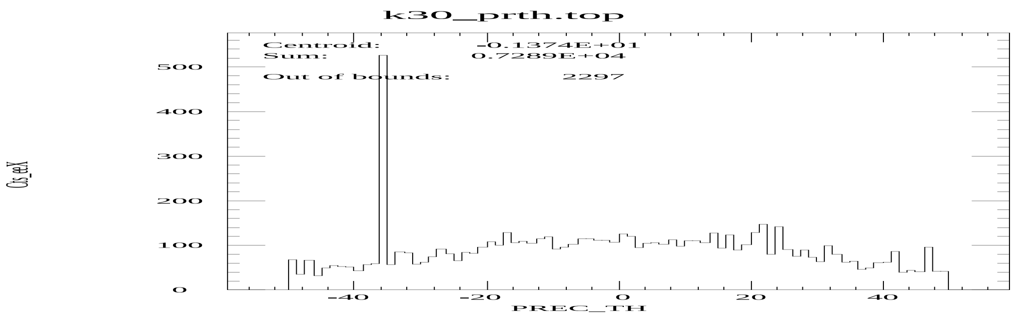

Figure 8: K30 , CM motion. Angular distribution out of the scattering plane.

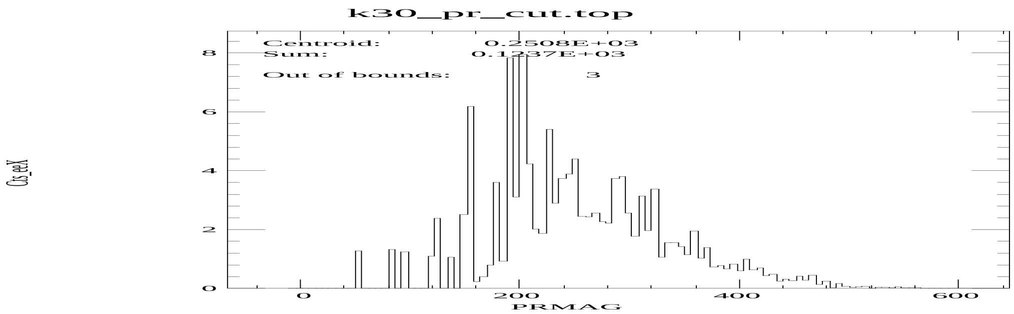

Figure 9: K30 , CM motion. Neutron momentum distribution with angular cuts.

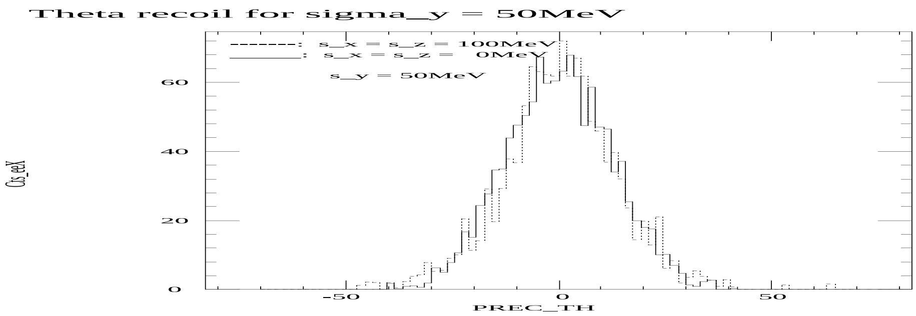

Figure 10: Cuts: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-10^o < \theta_p^{in} < 10^o$

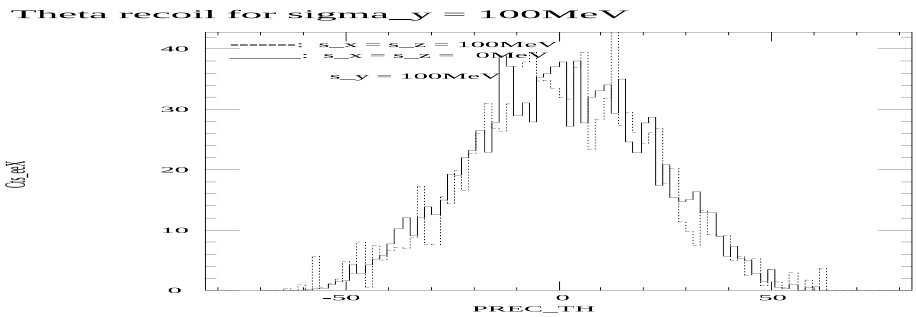

Figure 11: Cuts: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-10^o < \theta_p^{in} < 10^o$

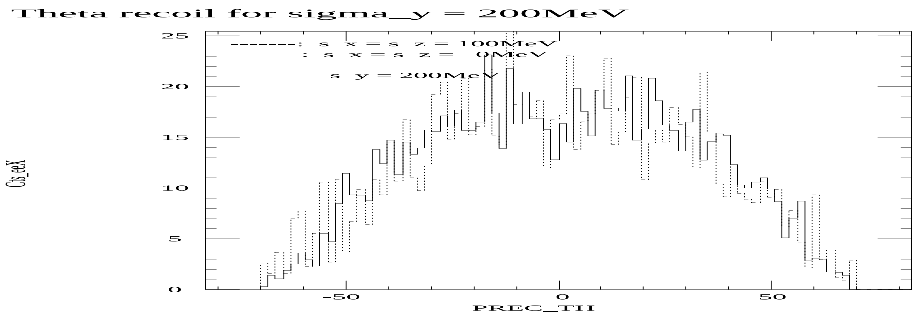

Figure 12: Cuts: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-10^o < \theta_p^{in} < 10^o$

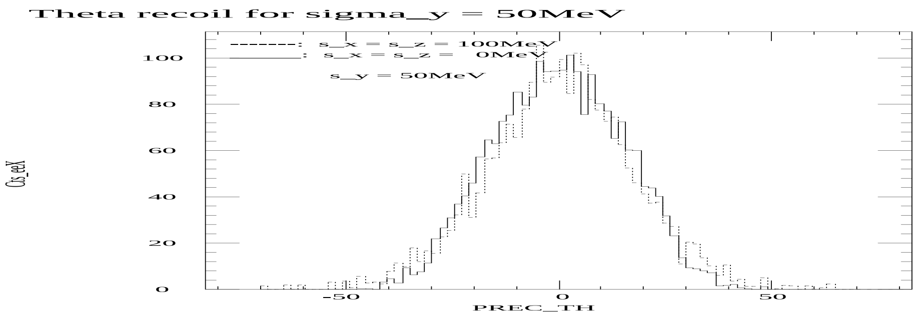

Figure 13: Cuts:: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-20^o < \theta_p^{in} < 20^o$

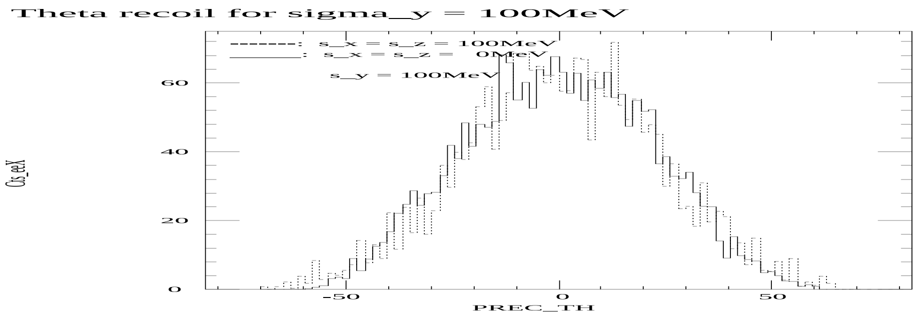

Figure 14: Cuts: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-20^o < \theta_p^{in} < 20^o$

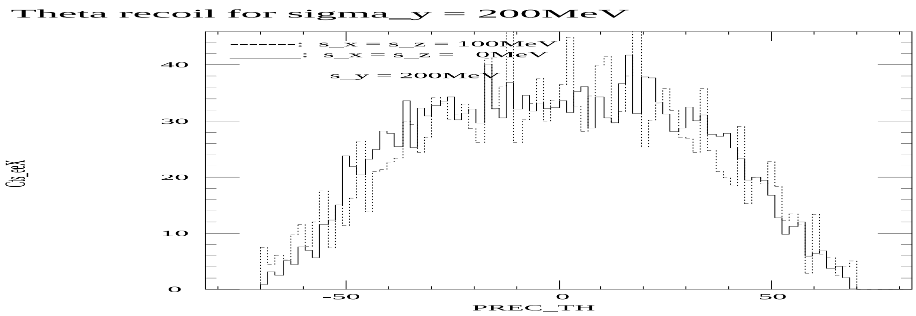

Figure 15: Cuts: $80^o < \phi_r < 100^o$; $0.2GeV < p_r < 1GeV$; $-20^o < \theta_p^{in} < 20^o$