Comments on Nucleon Form Factors

Carl Carlson

Nuclear and Particle Theory Group

College of William and Mary

and

Theory Group

Thomas Jefferson National Accelerator Facility

GEN collaboration meeting, Newport News, 23 October 2003

Topics

- Motivation
- Polarization asymmetries in $e^-e^+ \to p\bar{p}$
- Radiative Corrections to elastic eN scattering

Collaborators (on parts that are original)

Andrei Afanasev

Stan Brodsky

John Hiller

Dae Sung Hwang

. . .

Introduction

We know this figure.

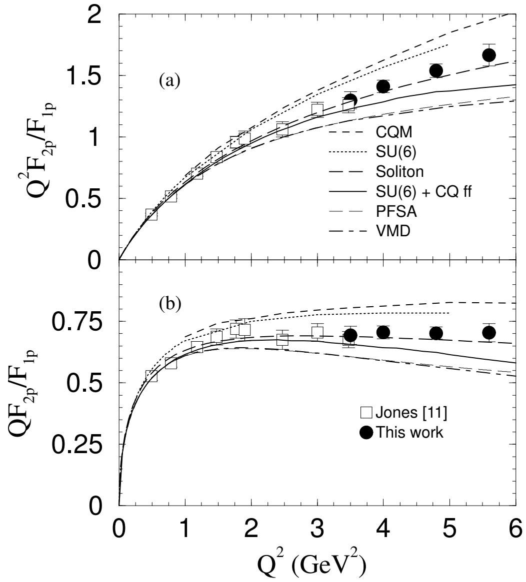

FIG. 1: Form factor.

Topic I: SSA and Timelike Form Factors

- Exclusive form factors in the timelike region are fundamental.
- Currently, there is more and better data in the spacelike region, and separation of the form factors has been done only there. In particular, have JLab F_2 measurements in Figure. Number of fits to data now available (see below).
- (Discuss discrepancy between Rosenbluth and polarization transfer measurements under next topic.)
- Measurements in the timelike region could relatively soon be available (Cornell, Frascati, Beijing, . . .). Not only the cross section, but also polarization observables.
- Difference between the timelike and spacelike region is that in the timelike region the form factors can have a relative phase. Then possible for a single outgoing baryon to be polarized even with no polarization in the initial state.

Timelike Measures

Differential cross section

Analog of the Rosenbluth method for measuring the magnitudes of helicity form factors.

For $e^- e^+ \to B\bar{B}$, when B is a spin-1/2 baryon, in CM,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{4q^2}\; D \;, \tag{1}$$

where $\beta = \sqrt{1 - 4m_B^2/q^2}$ and

$$D = |G_M|^2 \left(1 + \cos^2\theta\right) + \frac{1}{\tau}\, |G_E|^2 \sin^2\theta \;; \tag{2}$$

Note:

$$\begin{aligned} G_M &= F_1 + F_2 \;, \\ G_E &= F_1 + \tau F_2 \;, \end{aligned} \tag{3}$$

with $\tau \equiv q^2/4m_B^2 > 1$.

Polarization observables (Dubnickova, Dubnicka, and Rekalo)

Pin down the relative phases of the timelike form factors.

Polarization $\mathcal{P}_y$ is a single spin asymmetry,

$$\mathcal{P}_y = \frac{\sin 2\theta \, \mathrm{Im} G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau - 1)\sin 2\theta \, \mathrm{Im} F_2^* F_1}{D\sqrt{\tau}} \ . \tag{4}$$

The other two proton polarizations require initial state polarization. Electron polarization is P_e,

$$\mathcal{P}_x = -P_e \frac{2\sin\theta \, \mathrm{Re} G_E^* G_M}{D\sqrt{\tau}} \ , \tag{5}$$

and

$$\mathcal{P}_z = P_e \frac{2\cos\theta |G_M|^2}{D} \ . \tag{6}$$

Note 1: The sign of $\mathcal{P}_z$ can be determined from physical principles.

Note 2: Polarization $\mathcal{P}_y$ is a manifestation of the T-odd observable $\vec{k} \times \vec{p} \cdot \vec{S}_p$, and $\vec{S}_p$ is the proton polarization. Zero in the spacelike case.

Note 3: Can also argue (the zeros in) the factor $\sin 2\theta$ in $\mathcal{P}_y$ from physical principles.

Analytic continuation

• Any model which fits the spacelike form factor data with an analytic function can be continued to the timelike region.

• Have $Q^2 = -q^2$. The question is, what is -1?

• Get answer by examining denominators in loop calculations in perturbation theory: $Q^2 \to q^2 e^{-i\pi}$, or

$$\ln Q^2 = \ln(-q^2) \to \ln q^2 - i\pi \ . \tag{7}$$

Polarization in the timelike region

Select some fits to the spacelike data, concentrating on the ratio F_2/F_1.

Odd-Q fits.

The JLab experimenters themselves note that a good fit for Q^2 in the 2 to 5.6 GeV^2 is

$$\frac{F_2}{F_1} = \frac{1.25 \text{ GeV}}{Q} \; . \tag{8}$$

Compare to Ralston, Miller, and others.

Modify this form to

$$\frac{F_2}{F_1} = \left(\frac{1}{\kappa_p^2} + \frac{Q^2}{(1.25 \text{ GeV})^2} \right)^{-1/2} \; , \tag{9}$$

just to avoid trouble at $q^2 = 0$. ($\kappa_p = 1.79$.)

Fits involving logarithms.

Pick Belitsky, Ji, and Yuan as an example. Have traditional power law behavior, with significant log corrections. One of their fits is, with $\Lambda = 300$ MeV,

$$\frac{F_2}{F_1} = 0.17 \text{ GeV}^2 \, \frac{\ln^2(Q^2/\Lambda^2)}{Q^2} \ . \tag{10}$$

Improved fit which matches the above asymptotically and is good at low Q^2,

$$\frac{F_2}{F_1} = \kappa_p \frac{[1 + (Q^2/0.791 \, \text{GeV}^2)^2 \ln^{7.1}(1 + Q^2/4m_\pi^2)]}{[1 + (Q^2/0.380 \, \text{GeV}^2)^3 \ln^{5.1}(1 + Q^2/4m_\pi^2)]} \ . \tag{11}$$

Two-component fits.

• The 1973 Iachello, Jackson, and Lande model. See also Gari and Krumpelmann and Lomon.

• Two-component, core and meson cloud, structure for the nucleon with parameters fit to the then existing data.

$$\begin{aligned} F_1 &= \frac{1}{2} g \Bigg[(1-\beta_\omega-\beta_\phi) - \beta_\omega \frac{m_\omega^2}{q^2-m_\omega^2} - \beta_\phi \frac{m_\phi^2}{q^2-m_\phi^2} \\ &+ (1-\beta_\rho) - \beta_\rho \frac{m_\rho^2 + 8\Gamma_\rho m_\pi/\pi}{q^2-m_\rho^2+(q^2-4m_\pi^2)\Gamma_\rho \alpha(q^2)/m_\pi} \Bigg] , \end{aligned}$$

$$\begin{aligned} F_2 &= \frac{1}{2} g \Bigg[(0.120+\alpha_\phi) \frac{m_\omega^2}{q^2-m_\omega^2} - \alpha_\phi \frac{m_\phi^2}{q^2-m_\phi^2} \\ &- 3.706 \frac{m_\rho^2 + 8\Gamma_\rho m_\pi/\pi}{q^2-m_\rho^2+(q^2-4m_\pi^2)\Gamma_\rho \alpha(q^2)/m_\pi} \Bigg] , \end{aligned} \tag{12}$$

where

$$\begin{aligned} \alpha(q^2) &= \left(\frac{q^2-4m_\pi^2}{q^2} \right)^{1/2} \\ &\times \left\{ \frac{2}{\pi} \ln \left(\frac{\sqrt{q^2-4m_\pi^2}+\sqrt{q^2}}{2m_\pi} \right) - i \right\} . \end{aligned} \tag{13}$$

Factor $g = g(q^2)$ cancels in polarizations.

Parameters: $\beta_\rho = 0.672$, $\beta_\omega = 1.102$, $\beta_\phi = 0.112$, $\alpha_\phi = -0.052$, $m_\rho = 0.765$ GeV, $m_\omega = 0.784$ GeV, $m_\phi = 1.019$ GeV, and $\Gamma_\rho = 0.112$ GeV.

Note: Iachello discussed in his talk at JLab extending the fits to the timelike region.

He changed the OA factor $g(q^2)$. Does not affect polarizations.

(The other source of phase is the treatment of the rho widths. The phi and omega were approximated as zero width, but not the rho.)

Results

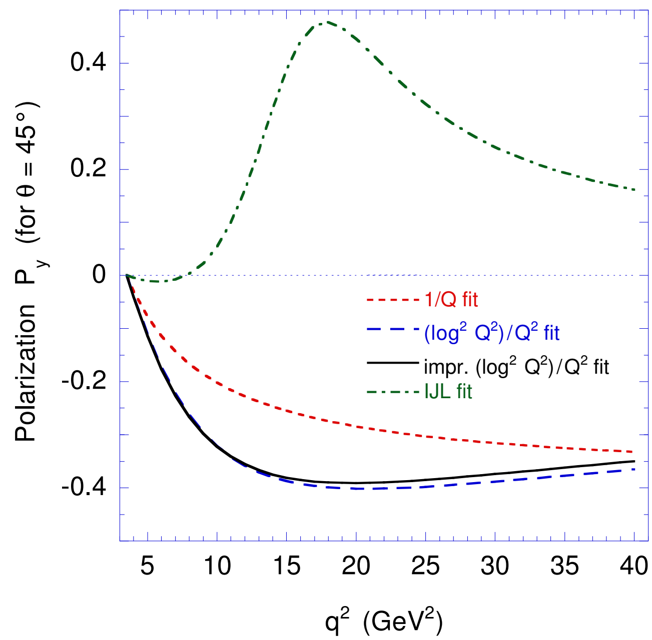

FIG. 2: Predicted polarization $\mathcal{P}_y$ in the timelike region for selected form factor fits.

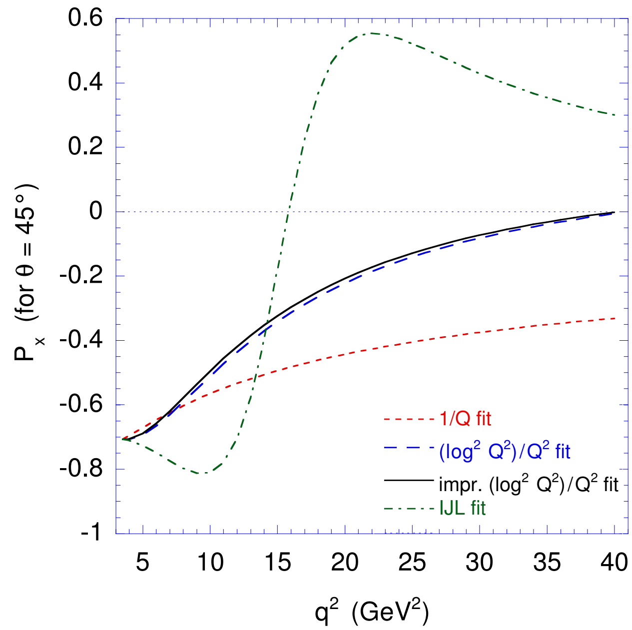

FIG. 3: Predicted polarization $\mathcal{P}_x$ in the timelike region for $\theta = 45°$ and $P_e = 1$.

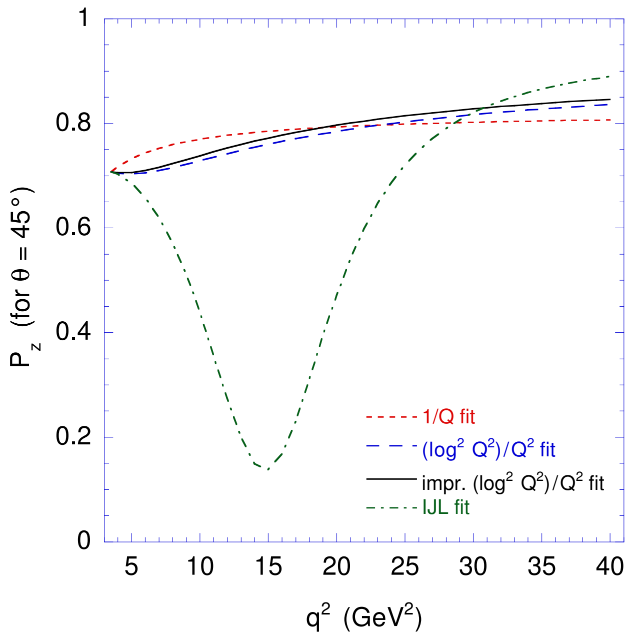

FIG. 4: The predicted polarization $\mathcal{P}_z$ in the timelike region for $\theta = 45°$ and $P_e = 1$.

Notes:

• The polarizations are not small. And: they are very distinct from a purely polynomial fit to the spacelike data, which gives zero $\mathcal{P}_y$.

• The magnetic form factor in the IJL model is very small in the 10 to 20 GeV^2 region (taking the dipole form for comparison) and has a zero in the complex plane near $q^2 = 15$ GeV^2. This accounts for much of the different behavior of the IJL model seen in the polarization plots. That the IJL ratio for G_E/G_M is strikingly large even by the standard set by the other three models also strongly affects the angular behavior of the differential cross section. This is witnessed by Fig. 5.

•The lower three models are also showing significant contributions from G_E; at $90°$, the difference between the curves shown and the value 0.5 is entirely due to $|G_E|^2$.

Differential cross section

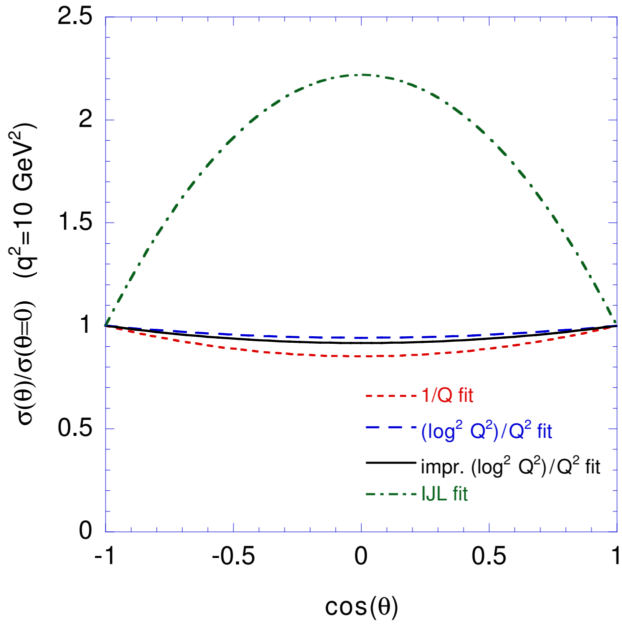

FIG. 5: The predicted differential cross section $\sigma(\theta) \equiv d\sigma/d\Omega$.

Conclusions

- Polarization needed to find phases of G_E and G_M in timelike region—and to solve for F_1 and F_2.
- The normal polarization $\mathcal{P}_y$ is a single-spin asymmetry. Example of time-reversal-odd observable. Non-zero if there is a phase difference between G_E and G_M.
- Predicted polarizations are large and distinctive.

Topic II:

Radiative corrections in spacelike region

- Consider correction from HO electromagnetic effects in elastic ep scattering for
  - Cross sections
  - Polarizations
- Implications for elastic form factor measurements

Recall for one-photon exchange,

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{NS}}{\epsilon(1+\tau)} \left(\epsilon G_E^2 + \tau G_M^2\right) \tag{14}$$

where

$$\sigma_{NS} = \frac{4\alpha^2 \cos^2 \frac{\theta}{2}}{q^4} \frac{E_2^3}{E_1} \tag{15}$$

and

$$\frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2 \frac{\theta}{2} \tag{16}$$

and also

$$\tau = \frac{Q^2}{4M^2} \ . \tag{17}$$

The Rosenbluth separation involves using the differential cross section measurements at fixed q^2 but varying ϵ $(0 < \epsilon < 1)$ to separate G_E. *If* G_E and G_M both scale the same way, then at $Q^2 = 6$ GeV^2, one has

$$\frac{G_E^2}{\tau G_M^2} = \frac{4M^2}{Q^2 \mu_p^2} = 7.6\% \tag{18}$$

The radiative corrections are a few percent (a few times α, in this case, rather than α/π), and ϵ dependent. One does not usually have the full range of ϵ, and the radiative corrections are of the same size as the measurement needed to determine G_E.

Some diagrams for radiative corrections

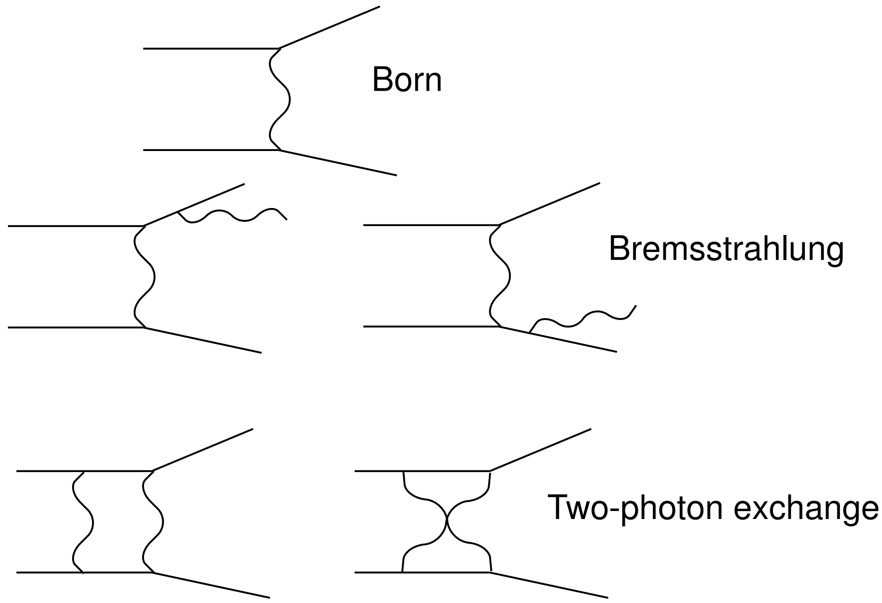

FIG. 6: Some contributions to radiative corrections.

Two-photon corrections.

• Included in classic Mo-Tsai calculations (1969); also calculated by Maximon-Tjon (2000) in soft-photon approximation. Soft-photon approximation gives no polarization dependence. Also gives correction of − (1–5) %, depending on ϵ or on scattering angle.

• If it is doubled, then Rosenbluth and polarization data will be reconciled.

‡ Recent calculations from Blunden, Melnitchouk, and Tjon [arXiv:nucl-th/0306076] including just proton intermediate states produced additional about − 2 % correction to difference in Rosenbluth slope. Partly reconciled the two data sets.

• Consider partonic calculation of two-photon exchange contributions (Vdh, and Afanasev, Brodsky, CC).

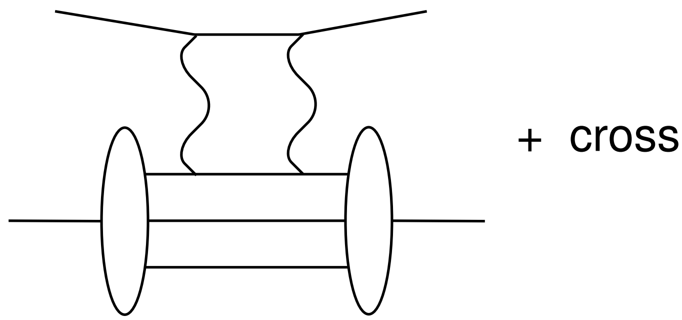

FIG. 7: Partonic box diagrams.

The electron-muon radiative corrections, and hence the electron-quark ones, are completely known. Must embed these corrections in a proton.

Correction is given in terms of δ, where

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{BORN}}{d\Omega}(1+\delta) \tag{19}$$

and the corrections (including bremsstrahlung) that involve both electron and quark are

$$\begin{aligned} \delta \;=\; & -e_q \frac{\alpha}{\pi}\Big\{ 4\ln(1-y)\ln\frac{2m\Delta}{Q^2} + \frac{y/2}{1+(1-y)^2} \times \\ & \times \Big[(y-2)\left\{3\ln^2(1-y) - 2\ln(1-y)\ln y + \pi^2\right\} \\ & \qquad -2(1-2y)\ln(1-y) - 2y\ln y \Big]\Big\} \end{aligned} \tag{20}$$

where Δ is the energy resolution in the CM frame, and $y = -t/s$. (Formula from Bardin and Kalinovskaya.)

• The log with the mass come from soft photon, long range, contributions. The remainder come from short range interactions.

• The photons are interacting with particles that have only a fraction x of the full energy of the whole proton (working in CM, or else using light front momentum fraction). Results in multiplying the short range parts by a factor $1/\langle x \rangle$, increasing their contribution by a factor of about 3.

• Serious problem: wrong sign. Biggest contributor is the term "π^2". (Added note: But careful subtracting out of "soft" radiative corrections already included in the analysis of the data may leave just extra "hard" parts with the necessary sign.)

Polarization effects

- Soft photon give no polarization effects. The box does.
- The box contributions (added to the Born diagram) can always be written as—after some Dirac matrix manipulation—

$$\mathcal{M} = \bar{u}(k_2)\gamma^\mu u(k_1) \;\; \bar{u}(p_2)\left[\gamma_\mu \tilde{G}_M - \frac{(p_1+p_2)_\mu}{2M}\tilde{F}_2\right]u(p_1)$$
$$+ \; \bar{u}(k_2)\gamma^\mu \gamma_5 u(k_1) \;\; \bar{u}(p_2)\left[\gamma_\mu \gamma_5 G_A\right]u(p_1) \qquad (21)$$

- G_A begins with terms of $O(\alpha)$, whereas the other form factors have lowest order terms plus corrections of $O(\alpha)$.
- One has in this language

$$\sigma = \sigma_{NS}\left(\tau|\tilde{G}_M|^2 + \epsilon|\tilde{G}_E|^2 + 2\sqrt{\tau(1+\tau)}\sqrt{1-\epsilon^2}\ ReG_M G_A^*\right) \tag{22}$$

which we have discussed before, and

$$\frac{\mathcal{P}_x}{\mathcal{P}_z} = \frac{Re\tilde{G}_E\tilde{G}_M^*\sqrt{\tau}\sqrt{2\epsilon(1-\epsilon)} - 2Re\tilde{G}_E G_A^*\sqrt{1+\tau}\sqrt{2\epsilon(1+\epsilon)}}{|\tilde{G}_M|^2\tau\sqrt{1-\epsilon^2} + 2ReG_M G_A^*\sqrt{\tau(1+\tau)}} \tag{23}$$

• The polarization does not have a small term that we want to measure that gets, in general, a correction of the same size as itself. Do note, however, that in the forward direction or $\epsilon \to 1$ the terms without correction are suppressed and the answer comes entirely from higher order terms. (We do not think the experimenters are at this forward an angle, but it could be probed.)

Closing comments regarding radiative corrections

- Corrections to cross section could be large on scale of G_E^2 terms, and currently more model dependent than we would like.

- Polarizations at moderate angles get smaller corrections than G_E^2 terms in cross section.

- Radiative corrections could be tested in

  - charge asymmetry in $e^- p$ vs. $e^+ p$ elastic scattering.

  - measuring polarization $\mathcal{P}_y$, which is not zero when radiative corrections are included, and which is a single spin asymmetry.

  - measuring $\mathcal{P}_x$ or $\mathcal{P}_z$ in the very forward direction.