Form of QED Elastic Cross Sections

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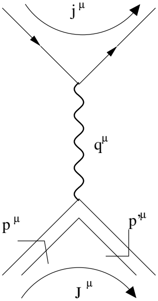
- Want to find most general form of cross section with QED considerations
- Transition amplitude from $i \rightarrow f$:

$$T_{fi} = -e \int j^{fi}_{\mu} A^{\mu} d^4x = -e \int j^{fi}_{\mu} \frac{J^{\mu}}{q^2} d^4x \qquad (1)$$

- First, we must describe general e.m. current J^{μ} for:



Four vectors available:

$$p^\mu, p'^\mu, \gamma^\mu \tag{2}$$

where

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{3}$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \tag{4}$$

Tensors:

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right) \tag{5}$$

Have to consider all linear combinations

For 4×4 matricies, 16 linearly independent quantities:

$$\mathbb{I}, \gamma^{\mu}, \gamma^{5}, \sigma^{\mu\nu}, \gamma^{\mu}\gamma^{5} \tag{6}$$

where

$$\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} \tag{7}$$

Parity operator can be represented $P = \gamma^{0}$, so:

$$\{\gamma^{5}, P\} = 0 \tag{8}$$

so currents with γ^{5} ruled out by parity conservation.

- p and p' are the only available 4-vectors
- q^2 is the only independent scalar as all combinations of p and p' can be reduced to m_N and q.
- The most general form of J^μ is then:

$$J^\nu = e\bar{u}(p')\big[K_1(q^2)\gamma^\nu + (p^\nu + p'^\nu)K_2(q^2) + (p^\nu - p'^\nu)K_3(q^2) + \\ + i\sigma^{\nu\mu}(p_\mu - p'_\mu)K_4(q^2) + i\sigma^{\nu\mu}(p_\mu + p'_\mu)K_5(q^2)\big]u(p)e^{i(p-p')\cdot x} \quad (9$$

- Gordon decomposition identity:

$$\bar{u}\gamma^{\mu}u = \frac{1}{2M}\bar{u}\left(p^{\mu} + p'^{\mu} + i\sigma^{\mu\nu}\left(p'_{\nu} - p_{\nu}\right)\right)u \tag{10}$$

eliminating $p + p'$ terms.

- Current conservation says $\partial_{\mu}J^{\mu} = q_{\mu}J^{\mu} = 0$, so terms that do not vanish must have K $= 0$
- For K_3 term:

$$q^{\mu}q_{\mu}K_3 = q^2K_3 \neq 0 \tag{11}$$

so $K_3(q^2) = 0$.

- Rewriting J^{μ} in different terms:

$$J^{\nu} = e\bar{u}(p')\left[F_1(q^2)\gamma^{\nu} + \frac{\kappa}{2M}q_{\mu}\sigma^{\nu\mu}F_2(q^2)\right] \qquad (12)$$

- Transition amplitude from state $i \longrightarrow f$:

$$T_{fi} = -i \int j^{\mu} \frac{1}{q^2} J_{\mu} d^4 x \tag{13}$$

- Transition rate:

$$W_{fi} = \frac{|T_{fi}|^2}{TV} \tag{14}$$

- relates to differenatial cross section $d\sigma$:

$$d\sigma = \frac{W_{fi}}{\Phi} dQ \tag{15}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\left(F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} (F_1 + \kappa F_2)^2 \sin^2 \frac{\theta}{2} \right] \tag{16}$$

In terms of Mandelstam variables:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2st^2}\left[(F_1+F_2)^2\left[2\left(s-M^2\right)^2+2st+t^2\right]\right.$$
$$\left.-2F_2\left[F_2\left(1+\frac{t}{4M^2}\right)+2F_1\right]\left[\left(s-M^2\right)^2+ts\right]\right] \quad (17)$$

References:

- My thesis
- Halzen and Martin, *Quarks & Leptons: An Introductory Course in Modern Particle Physics*, Wiley, 1984