On the Uncertainty of Simulations

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The uncertainty of the hamc results is determined by the acceptance of the spectrometer and the number of successful trials. If from the simulation the asymmetry (or any other physical quantity) looks like Fig. 1, this distribution would somewhat describe the probability: $P(A=a)$ where a is the value of the asymmetry result. For simplicity, one can use a step function for $P(A=a)$:

$$P(A=a) = \frac{1}{A_2 - A_1} = \frac{1}{\Delta A} \tag{1}$$

where ΔA is the width of the step function. For PREX the HAMC results of the asymmetry are shown in Fig. 1 where there are 220 trials passing all acceptance cuts (starting # of trials was 10K). One can assume a step function with width 1 ppm a good approximation of this distribution, i.e. $\Delta A = 1$ ppm. Note that using the step function eliminate any complication from the arbitrary acceptance and cross-section weighting [2]. If we run the simulation twice with 1 successful trial per simulation, then the probability distribution of the difference between the two simulated asymmetries, $x \equiv A_{trial\#2} - A_{trial\#1}$, would be

$$\begin{aligned} P(x) &= \int_{A_1}^{A_2-x} \left[P(A_{trial\#1}=a_1)\times P(A_{trial\#1}=a_1+x)\right] dA_{trial\#1} \quad \text{if } x>0 \\ &\quad \int_{A_1-x}^{A_2} \left[P(A_{trial\#1}=a_1)\times P(A_{trial\#1}=a_1+x)\right] dA_{trial\#1} \quad \text{if } x<0 \\ &= \frac{A_2 - A_1 - |x|}{(A_2-A_1)^2} \\ &= \frac{\Delta A - |x|}{(\Delta A)^2}\,. \end{aligned} \tag{2}$$

The average value is $\langle x\rangle = 0$ and the standard deviation is

$$\delta x = \sqrt{\int_{-\Delta A/2}^{+\Delta A/2} \left(x - \langle x\rangle\right)^2 P(x)dx} = \sqrt{\frac{1}{3}}\Delta A\,. \tag{3}$$

If we run the simulation twice (two iterations) with N successful trials per simulation, they would give the following asymmetries:

$$\begin{aligned} A^{it\#1} &= \frac{1}{N}\sum_{i=1}^{N} A^{it\#1}_{trial\#i} \\ A^{it\#2} &= \frac{1}{N}\sum_{i=1}^{N} A^{it\#2}_{trial\#i} \end{aligned}$$

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[2] one can certainly make fancier probability distribution functions based on the histogram, but the result will be similar.

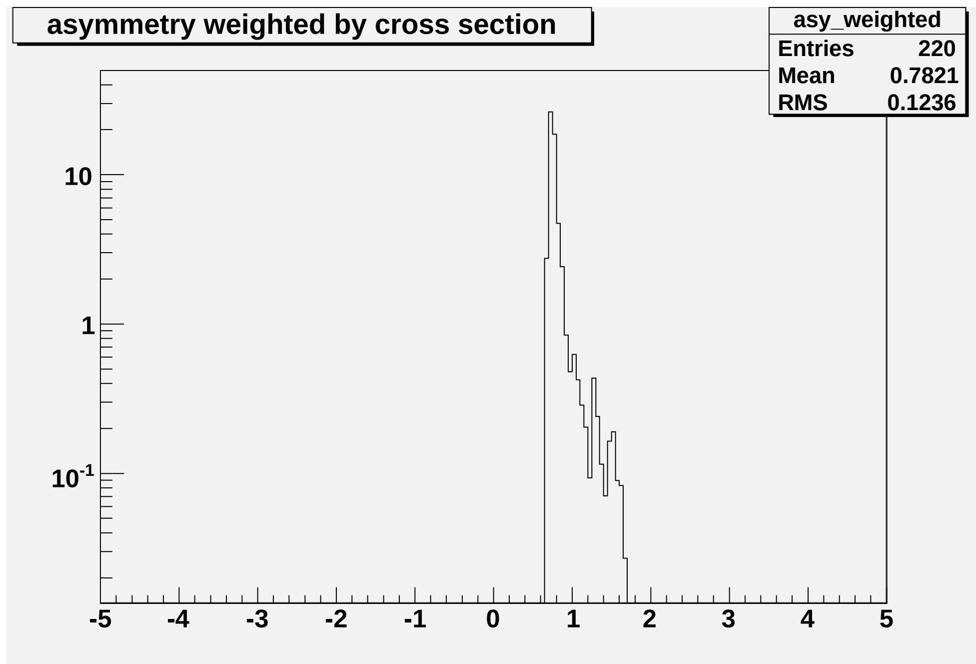

Figure 1: Simulated asymmetries for PREX in ppm with 10K starting trials and 220 trials successfully passing all acceptance cuts.

and the difference between the two iterations is

$$\eta \;\; \equiv \;\; A^{it\#2} - A^{it\#1} = \frac{1}{N}\sum_{i=1}^{N}\left(A^{it\#2}_{trial\#i} - A^{it\#1}_{trial\#i}\right) \; . \tag{4}$$

Since each pair of simulation can be considered independent from other pairs, the standard deviation of η would be

$$\delta\eta \;\; = \;\; \sum_{i=1}^{N}\left(\frac{1}{N}\right)^2 (\delta x_i)^2 = \frac{\Delta A}{\sqrt{3N}} \; . \tag{5}$$

When studying derivatives of the asymmetry w.r.t. beam parameters, for example, the sensitivity is limited by this uncertainty of the simulation itself. For example, using 2M starting trials (which corresponds to roughly 40K trials passing all acceptance cuts), the uncertainty from the simulation is $\frac{1\ \mathrm{ppm}}{\sqrt{3\times 40000}} = 0.0029$ ppm. Change in the asymmetry with a change in the beam y position is shown in Fig. 2. One can see that below 1 mm we are losing the linearity because the expected change in the asymmetry is below 0.003 ppm. To get the sensitivity to smaller "kicks" in the beam y position one would need to run more trials for the simulation, for example $4M$

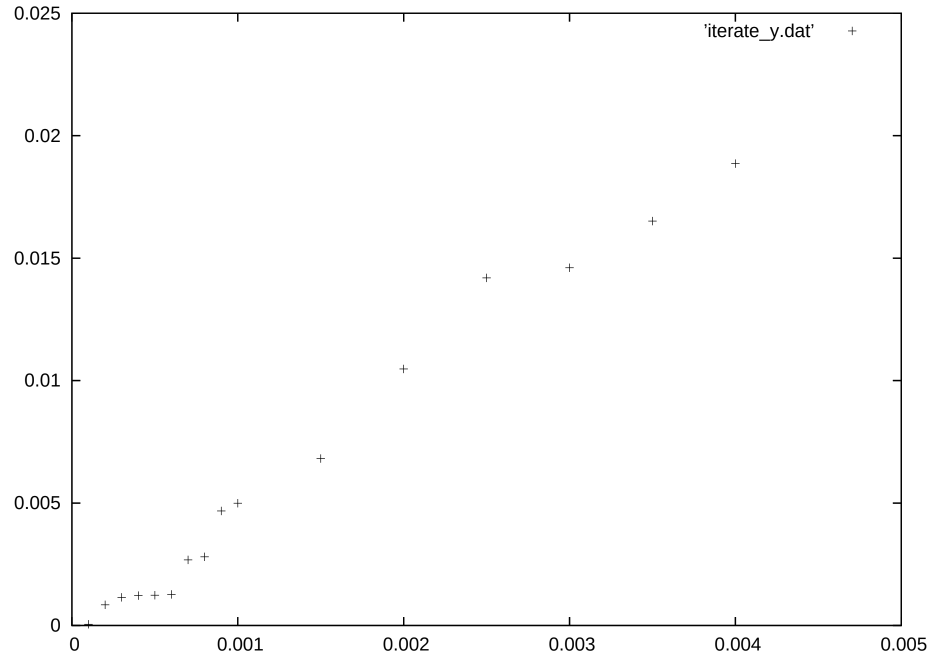

Figure 2: Simulated results for the change in the PREX asymmetry in ppm w.r.t. change in the beam y position in meters. The simulation was run twice with y changed in the second iteration. Each iteration has 2M starting trials.

successful trials to get to 0.1 mm, $400M$ to get to 0.01 mm, $4E10$ to get to 1 μm, etc. But I think the linearity at the mm level indicate that higher order derivatives are small and the linearity can be safely used at μm level.