E08-011 Technical Note on Asymmetry Results, Radiative Corrections, and Extractions of the Quark Axial Charge (Working Draft)

(Dated: July 4, 2012)

Abstract

During the 6 GeV PVDIS experiment (E08-011) [1] we measured the parity violation asymmetry in scattering cross sections using the JLab polarized electron beam and a unpolarized detuerium target. There were two deep inelastic measurements at $Q^2 = 1.085$ and 1.901 (GeV/c)^2, and four measurements in the nucleon resonance region. As this draft is written, the majority of the data analysis is completed but we are still missing certain pieces of radiative corrections, and this is the last piece we need for writing up the PRL. Here I will summarize PVDIS formulism (Sec. I), our preliminary asymmetry results (Sec. II), what corrections we calculated through our Monte-Carlo and applied to the asymmetry (Sec. III), how we extracted the C_{2q} coupling from these asymmetries (Sec. IV), Give an overview of radiative corrections (Sec V A), summarize the procedure used for E158 (Sec. V B), and present a list of remaining questions and a to do list (Sec. V C). The goal is to understand fully where we are in the radiative correction for PVDIS and hopefully complete these calculations in a short term.

I. FORMULAS FOR PVDIS ASYMMETRIES

In electron scattering, the parity violation (PV) asymmetry is related to [2]:

$$C_{1u} = 2g_A^e g_V^u \approx -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \approx -0.18 \quad (1)$$

$$C_{1d} = 2g_A^e g_V^d \approx +\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \approx 0.34 \quad (2)$$

$$C_{2u} = 2g_V^e g_A^u \approx -\frac{1}{2} + 2\sin^2\theta_W \approx -0.03 \quad (3)$$

$$C_{2d} = 2g_V^e g_A^d \approx +\frac{1}{2} - 2\sin^2\theta_W \approx 0.03\ , \quad (4)$$

where the relationship to $\sin^2\theta_W$ is shown at the tree-level approximation and $\sin^2\theta_W = 0.237$ is used. This is based on the $\overline{MS}$ value of $\sin^2\theta_W$ at our measured $Q^2 = 1.085$ and 1.901 (GeV/c)^2 both round to 0.237 (J.E.), while a value of 0.235 was used in the original proposal PR05-007. Here g_V^f and g_A^f are the fermion vector and axial neutral weak couplings.

The PV asymmetry of electron deep inelastic scattering (DIS) off a nuclear target is

$$\begin{aligned} A_{PV}^{DIS} &= -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[2g_A^e Y_1(y)\frac{F_1^{\gamma Z}}{F_1^Z} + g_V^e Y_3(y)\frac{F_3^{\gamma Z}}{F_1^Z}\right] \\ &= -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[a_1(x)Y_1(y) + a_3(x)Y_3(y)\right]\ , \end{aligned} \quad (5)$$

where G_F is the Fermi constant, α is the fine structure constant, x is the Bjorken scaling variable, $y = \nu/E$ is the fractional energy loss of the electron with E the incident electron energy. With $r^2 = 1 + \frac{Q^2}{\nu^2}$ and $R^{\gamma,\gamma Z}$ the ratio of the longitudinal and transverse virtual photon electromagnetic absorption and the $\gamma - Z^0$ interference cross sections, respectively:

$$Y_1 = \left[\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right]\frac{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma Z}}\right] - xy\frac{M}{E}}{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma}}\right] - xy\frac{M}{E}} \quad (6)$$

and

$$Y_3 = \left[\frac{r^2}{1+R^{\gamma}}\right]\frac{1+(1-y)^2}{1+(1-y)^2 - y^2\left[1-\frac{r^2}{1+R^{\gamma}}\right] - xy\frac{M}{E}}\ . \quad (7)$$

To a good approximation one has $R^{\gamma} \approx R^{\gamma Z}$ and $Y_1(y) \approx 1$.

The $a_{1,3}$ terms are related to the structure functions:

$$a_1(x) = 2g_A^e\frac{F_1^{\gamma Z}}{F_1^Z}\ , \quad (8)$$

$$a_3(x) = g_V^e\frac{F_3^{\gamma Z}}{F_1^Z}\ . \quad (9)$$

Equivalently, one can also use F_2 structure functions, which are related to F_1 via R:

$$F_2 \;=\; \frac{2xF_1(1+R)}{r^2} \tag{10}$$

or

$$F_1 \;=\; \frac{r^2F_2}{2x(1+R)\,.} \tag{11}$$

In the simple quark parton model (QPM),

$$F_1^{\gamma}(x) \;=\; \frac{1}{2}\sum Q_q^2\left[q(x)+\bar{q}(x)\right], \tag{12}$$

$$F_1^{\gamma Z}(x) \;=\; \sum g_V^q\left[q(x)+\bar{q}(x)\right], \tag{13}$$

$$F_3^{\gamma Z}(x) \;=\; 2\sum Q_q g_A^q\left[q(x)-\bar{q}(x)\right], \tag{14}$$

$$F_2^{\gamma}(x) \;=\; 2xF_1^{\gamma}(x) = x\sum Q_q^2\left[q(x)+\bar{q}(x)\right] \tag{15}$$

$$F_2^{\gamma Z}(x) \;=\; 2xF_1^{\gamma Z}(x) = 2x\sum g_V^q\left[q(x)+\bar{q}(x)\right] \tag{16}$$

where the summation is over the quark flavor $q = u, d, s\cdots$, Q_q is the corresponding quark electric charge, $q(x)$ and $\bar{q}(x)$ are the parton distribution functions (PDF). Defining $q^{\pm}(x) \equiv q(x) \pm \bar{q}(x)$, in the QPM we have

$$a_1(x) \;=\; 2\frac{\sum C_{1i}Q_i q_i^+(x)}{\sum Q_i^2 q_i^+(x)}\,, \tag{17}$$

$$a_3(x) \;=\; 2\frac{\sum C_{2i}Q_i q_i^-(x)}{\sum Q_i^2 q_i^+(x)}\,, \tag{18}$$

For an isoscalar target such as the deuteron, neglecting effects from heavier quark flavors and assuming that $u^p = d^n$, $d^p = u^n$ [$u, d^{p(n)}$ are the up and down quark PDF in the proton (neutron)], $s = \bar{s}$, and $c = \bar{c}$, the functions $a_{1,3}(x)$ simplify to

$$a_1(x) \;=\; \frac{6\left[2C_{1u}(1+R_c) - C_{1d}(1+R_s)\right]}{5+R_s+4R_c}\,, \tag{19}$$

$$a_3(x) \;=\; \frac{6\left(2C_{2u} - C_{2d}\right)R_v}{5+R_s+4R_c}\,, \tag{20}$$

where the PDF's give $R_c \equiv [2(c+\bar{c})]/(u+\bar{u}+d+\bar{d})$, $R_s \equiv [2(s+\bar{s})]/(u+\bar{u}+d+\bar{d})$ and $R_V \equiv (u-\bar{u}+d-\bar{d})/(u+\bar{u}+d+\bar{d})$.

The main goal of the experiment (at the 6 GeV precision) is to investigate the possibility of extracting C_{2q} from the measured DIS asymmetry. So how does the uncertainty in the structure functions affect this extration? This can be answered by looking into the "no structure" expression

of $a_{1,3}$, i.e, assuming the nucleon is simply made of valence u and d quarks. This results in the expressions given in the PDG [2] summary:

$$a_1(x) = \frac{6}{5}\left(2C_{1u} - C_{1d}\right) , \tag{21}$$

$$a_3(x) = \frac{6}{5}\left(2C_{2u} - C_{2d}\right) , \tag{22}$$

where there is no requirement that $d/u = 1/2$. Values of $a_{1,3}$ using different PDF fits are presented in Sec. IV and compared to these "no structure" values.

II. OUR PRELIMINARY ASYMMETRY RESULTS

Table I and II show the current preliminary results on the asymmetries along with the kinematics for all DIS and resonance measurements from this experiment. The electromagnatic radiative correction calculated in our Monte Carlo program includes internal and external bremstralung and ionization loss of the incoming and the outgoing electrons, and will be described in details in the next section. We applied this correction ONLY to the DIS results shown below and corrections to the resonance measurement are in progress.

TABLE I: Summary of E08-011 kinematics and preliminary asymmetry results for DIS measurements. The x_{bj} and Q^2 values are averaged over the acceptance of the spectrometer and weighted by the cross section, i.e., are the true kinematics of the measurement. EM radiative corrections have been applied to correct for electron energy losses due to bremstralung and ionization effects.

Setting	DIS 1	DIS 2
Beam energy E_0 (GeV)	6.0674	6.0674
Central scattering angle θ_0	12.9°	20.0°
Central momentum setting E_0' (GeV)	3.66	2.63
$\langle x \rangle$	0.241	0.295
$\langle Q^2 \rangle$ (GeV/c)^2	1.085	1.901
A_{PV}^{DIS} (measured, ppm)	-92.27 ± 3.15(stat.)± 2.77(syst.)	-163.60 ± 6.48(stat.)± 3.05(syst.)

TABLE II: Summary of E08-011 Kinematics and Preliminary Asymmetry Results for resonance measurements. The Q^2 and W values are calculated from the central spectrometer settings and we will provide the acceptance- and cross-section average values. We have not corrected the effect of electron energy losses but will do so for the publication. Systematic uncertainties are being analyzed too.

Setting	RES 3	RES 4	RES 5	RES 7
Beam energy E_0 (GeV)	4.8674	4.8674	4.8674	6.0674
Central θ_0	12.9°	12.9°	12.9°	15.0°
Central E_0' (GeV)	4.0	3.55	3.10	3.66
Q^2 (GeV/c)^2	0.983	0.872	0.762	1.513
W (GeV/c^2)	1.235	1.575	1.853	1.971
A_{PV}^{res} (measured, ppm)	-66.26 ± 7.77(stat.)	-73.4 ± 6.9(stat.)	-60.9 ± 5.15(stat.)	-118.8 ± 16.9(stat.)

III. RADIATIVE CORRECTIONS THROUGH MONTE-CARLO (HAMC) – COMPLETED

As mentioned previously, the only electromagnatic radiative correction calculated in our Monte Carlo program includes internal and external bremstralung and ionization loss of the incoming and the outgoing electrons. The detailed procedure is described below.

For each kinematic setting, a full simulation taking into account the actual target geometry and spectrometer acceptance is done. For each event, one starts from the (fixed) beam energy E and a randomly selected scattering angle θ and momentum of the scattered electron E', where E' is the scattered momentum at the vertex (referred to as E_v' hereafter) but E is not associated with the vertex. The energy loss of incoming and outgoing electrons δE and $\delta E'$ are then calculated using the formula given on page 5-7 of Ref. [3], which includes external bremstralung, internal bremstralung (effective radiator formula), and ionization loss. Then the incoming electron's energy at the vertex is calculated as $E_v = E - \delta E$ and the final momentum of the scattered electron is $E_d' = E' - \delta E'$. If θ and E_d' falls within the spectrometer acceptance, the cross section and the PV asymmetry are calculated using the vertex values E_v and E_v' and are stored. Figure 1 illustrates these definitions:

The vertex kinematics (E_v, θ, E_v') of an event could fall into one of the following categories: $e-^2H$ elastic, quasi-elastic, nucleon resonance, and DIS:

1. For $e-^2$H elastic, we use code "deut_elastic" from E. Beise (SAMPLE collaboration):

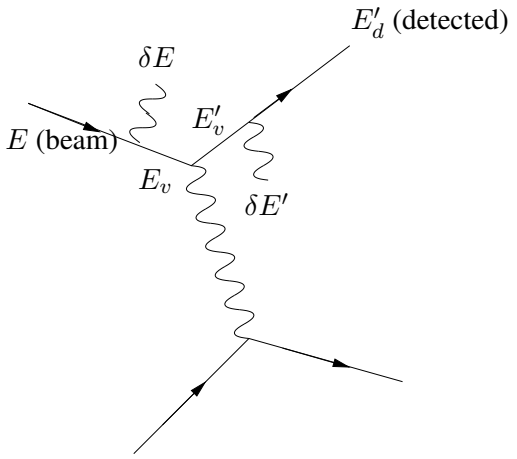

FIG. 1: Kinematics used in HAMC to correct the energy loss of incoming and outgoing electrons.

- Cross section is based on parameterization of A and B by Jacques Ball (D. Abbott et al, Eur. Phys. Jour A7 (2000) 421 (citation copied from the code).
- Asymmetry is based on simple model that compares well to calculation of S. Pollock, PRD 47 (1990) 3010 (citation copied from the code).
- Form factors in the deuteron code are using the J. Kelly parameterization of the nucleon form factors, which worked better for the backward angle measurement of the SAMPLE experiment (E.B.).
- The code gives the deuteron elastic PV asymmetry in the form of $a_0 + a_2 G_M^s$ where G_M is taken to be zero in our calculation.

2. For quasi-elastic:

- Cross section is calculated using the usual elastic formula for the neutron and the proton, then smeared by the smearing algorithm of P. Bosted (extracted from source code of the fit).
- Asymmetry for $e - p$ elastic is calculated using the HAPPEX formula (embeded in HAMC);
- Asymmetry for $e - n$ elastic is calculated using the same HAPPEX formula as the proton but with the neutron form factors;
- Currently the average of $e - p$ and $e - n$ asymmetries is taken as the quasi-elastic asymmetry. This will be corrected when we are ready for another round of HAMC

simulations.

3. For nucleon resonances ($W < 2$):

   - Cross section is based on P. Bosted's fits [4];
   - Asymmetries are calculated from three models: two "theoretical models" from H. Lee [5] and M. Gorshteyn [6], respectively, and one "toy model" where we used $A_{res} = \frac{\sigma_{res}}{\sigma_{dis}} A_{dis}$ where A_{dis} is calculated from the DIS formula (see below), σ_{dis} is from ???, and σ_{res} is from P. Bosted's fit [4].

4. For DIS ($W > 2$):

   - The cross section is calculated using Bosted's fits [4];
   - The PVDIS asymmetry is calculated using Eqs. (5-7), MSTW2008 NLO (or NNLO) 3-flavor PDFs and quark-parton model formula Eqs. (11), (14), (15), and (16). For R in Eq. (11) again Bosted's fit is used. This is the same prescription as described in the next section for the C_{2q} extraction.

A full simulation therefore can deliver an acceptance- and cross-section-averaged asymmetry $\langle A(x_v, Q^2_v)\rangle$. This is compared with a "point" calculation of $A(\langle x_d\rangle, \langle Q^2_d\rangle)$ using the same DIS prescription described above, where $\langle x_d\rangle$ and $\langle Q^2_d\rangle$ are the acceptance averaged "detected" kinematics, i.e, the x and Q^2 we can get using the measured beam energy and the measured scattered electron's momentum and angle. The difference between the two is quoted as a "radiative correction", which of course is not the "full" correction but only include effects from Bremstralung loss as well as acceptances. The acceptance correction is traditionally not referred to as radiative correction but nevertheless is a separate correction that should be applied to data, and we applied them together here.

The corrections obtained from the above procedure is applied (added) to the measured asymmetries, resulting in the preliminary results shown in Table I. The corrections are $(2.1 \pm 2.0)\%$ for $Q^2 = 1.085$ and $(1.9 \pm 0.43)\%$ for $Q^2 = 1.901$, i.e., increase the asymmetry about about 2%. The uncertainties of the correction are estimated based on how well the measured resonance asymmetries agree with the three resonance asymmetry models. Since the $Q^2 = 1.085$ point sits closer to the resonances, it has a larger uncertainty.

Note:

- Ideally one should calculate the cross section and the PV asymmetry consistently in the Monte-Carlo. We used two different methods: cross sections from Bosted's fit of structure functions [4] and PVDIS asymmetries from MSTW PDFs and Eq. (5), but this does not matter much, neither does exactly which structure function or PDF we used in the Monte Carlo as long as they are "close enough" to the true cross section or asymmetry. The reason is because as you can see from the procedure described above, all what we have corrected is the effect of kinematic shift from (x_d, Q^2_d) to (x_v, Q^2_v) due to energy loss of electrons. Because of this reason, I think whether $\gamma\gamma$ box (i.e. TPE) is included in Bosted's or MSTW's fits (yes for Bosted per J. Arrington, i.e. Bosted's fit did not correct for TPE; unknown for MSTW) is irrelavant, TPE corrections still need to be evaluated and applied to the asymmetries.

- The depolarization effect of the electron from bremstralung and/or ionization effects are not yet implemented in HAMC.

IV. EXTRACTING C_{2q}

To extract $2C_{2u} - C_{2d}$ from the measured asymmetries, we calculate the contribution from the $F_1^{\gamma Z}$ and $F_3^{\gamma Z}$ terms of Eq. (5). The inputs to Eq. (5) are described below:

1. The α_{EM} is evaluated at our measured Q^2 using $\alpha_{EM}|_{Q^2=0} = 1/137.036$ and the running code of W. Melnichouk et al. [PDG2010 value in Section 10.2 is $\alpha = 1/137.035999084(51)$]. This seems to be the "on-shell" value (not sure about this since I can't find the word "on-shell" associate with this value in PDG). Jens commented that because QED corrections involve real photons, the on-shell value is the natural one to use rather than the $\overline{MS}$ value $\hat{\alpha}$, also available from PDG. Using $\alpha_{EM}(Q^2)$ takes into account purely EM vacuum polarization, which cannot be corrected otherwise.

2. The Fermi constant $G_F = 1.1663787(6) \times 10^{-5}$ GeV^{-2} was given by J. Erler, to appear in PDG 2012;

3. The value of $\sin^2\theta_W$ does not go directly into the asymmetry calculation;

4. The $C_{1,2}^{u,d}$ are evaluated at our measured Q^2 by J. Erler in the $\overline{MS}$ scheme, using a fixed $M_H = 124.5$ GeV and the rest of the parameters determined by the global fit (Jens email

2012/2/26). This calculation includes the "charge radius effect" and interference between γ-exchange and γZ box.

- K. Kumar commented it seems to include all EW RC, plus a preliminary estimation of the interference between γ-exchange and γZ box.
- Minor tweaks might need to be done for the $\gamma - (\gamma Z)$ term.

The exact values are shown below. These can be compared to Table 10.3 of PDG2012 [2] which defines them at $Q^2 = 0$ (more valid for Atomic PV than PVDIS).

TABLE III: Values of $C_{1,2q}$ calculated at our measured Q^2 values. Note that $Q^2 = 1.925$ and 1.10 are central kinematics using the central angle and momentum settings of the spectrometer, while $Q^2 = 1.901$ and 1.085 are the actual (x-section- and acceptance-averaged) kinematics of our measurement. The values for $Q^2 = 1.901$ and 1.085 used the best fit of $m_H = 102$ GeV.

Q^2 (GeV/c)^2	1.925	1.10	1.901	1.085
ρ'	0.98899	0.98899	0.9891	0.9891
ρ	1.00066	1.00066	1.0007	1.0007
κ'	0.99520	0.99667	0.9943	0.9958
κ	1.02076	1.02222	1.0298	1.0298
λ'			-1.8×10^{-5}	-1.8×10^{-5}
λ_{2u}	-0.01095	-0.01104	-0.0110	-0.0104
λ_{2d}	0.00055	0.00079	0.0006	0.0008
C_{1u}	-0.19104	-0.19059	-0.1913	-0.1908
C_{1d}	0.34279	0.34257	0.3429	0.3427
C_{2u}	-0.03886	-0.03827	-0.0388	-0.0382
C_{2d}	0.02846	0.02802	0.0284	0.0280
$2C_{1u} - C_{1d}$	-0.72487	-0.72374	-0.7255	-0.7244
$2C_{2u} - C_{2d}$	-0.10618	-0.10457	-0.1061	-0.1045

The relevant formula from PDG2012 [2] Table 10.3 are copied below:

$$C_{1u} = \rho'_e \left(-\frac{1}{2} + \frac{4}{3}\hat{\kappa}'_e \hat{s}^2_Z\right) + \lambda', \tag{23}$$

$$C_{1d} = \rho'_e \left(\frac{1}{2} - \frac{2}{3}\hat{\kappa}'_e \hat{s}^2_Z\right) - 2\lambda', \tag{24}$$

$$C_{2u} = \rho_e \left(-\frac{1}{2} + 2\hat{\kappa}'_e \hat{s}^2_Z\right) + \lambda_u, \tag{25}$$

$$C_{2u} = \rho_e \left(\frac{1}{2} - 2\hat{\kappa}'_e \hat{s}^2_Z\right) + \lambda_d. \tag{26}$$

5. Interference between Z-exchange and $\gamma\gamma$ box is NOT calculated anywhere, although it is estimated to be the same order as the $\gamma - (\gamma Z)$ interference.

6. Effects of the $\gamma\gamma$ box, which should go into the denominator of Eq. (5) (Andrei A., hep-ph/0502128), is NOT calculated anywhere.

7. Calculation of structure functions based on "PDF+QPM": For the structure functions in Eqs. (17-18) ideally we should use full calculations at the NLO or N^{2}LO level. However, almost all PDF fits widely available to the community do not include codes to evaluate structure functions. The MSTW structure function code is available online but it does not include $Z-$exchange or $\gamma - Z$ interference terms. Therefore if we use CTEQ [7, 8] or MSTW [9, 10] PDFs we have to use the simple parton model to construct the structure functions: In this "PDF+QPM" approach we used Eqs. (15-16) to calculate $F_2^{\gamma,\gamma Z}$ from PDFs (the PDFs can be from LO, NLO or N^{2}LO fits) and then used Eq. (11) to calculate $F_1^{\gamma,\gamma Z}$ where the latest experimental fit of R was used [4].

8. Full calculation of structure functions: The only code we have for full structure function calculations that include both $F_2^{\gamma,\gamma Z}$ and $F_3^{\gamma Z}$ is the CTEQ/JLab (CJ) fit, but it only works down to $Q^2 = 1.7$ (GeV/c)^2 and does not include uncertainty evaluation. We carefully compared the CJ full calculation with the "PDF+QPM" approach and found the variation to be small at $Q^2 = 1.901$ (GeV/c)^2, with the "MSTW2008 NLO PDF+QPM" to be the closest to the full CJ fit. This is perhaps due to the fact that the effect of structure functions largely cancel in the asymmetries, as can be seen in Table IV. Therefore currently we are using the "MSTW2008 NLO PDF+QPM" calculation to extract C_{2q}.

9. Additional EM radiative corrections, such as vertex and (some?) loop corrections still need to be studied, see next section.

10. Effect of possible difference between $R^{\gamma Z}$ and R^{γ} were studied: To account for a shift of 1 ppm in the asymmetry, 7.7% and 4.5% differences between $R^{\gamma Z}$ and R^{γ} are needed, for $Q^2 = 1.085$ and 1.901 (GeV/c^2), respectively. This uncertainty is not included in the current calculation for the asymmetry shown in Table IV.

11. The higher twist effects introduce a Q^2-dependence to the structure functions in addition to the perturbative QCD evolution. The HT effects on the a_3 term was estimated using neutrino structure function data H_3^{ν} [11] and are found to shift the asymmetry by $+0.70$ ppm and $+1.2$ ppm for the lower and the higher Q^2. This correction is currently NOT applied in the calculation because I am not sure if the neutrino data (on structure function) applies to our a_3 term (ratio of structure functions $F_3^{\gamma Z}$ and F_1^{γ}), although the quark content of H_3^{ν} is almost identical to $F_3^{\gamma Z}$.

12. The higher twist effects on R^{γ} were estimated in Ref. [11] and the effect on the asymmetry is negligible.

13. One more note on Prescott's result (from J. Erler): With $2C_{1u} - C_{1d}$ fixed to the SM, $a_2 = (-1.4 \pm 2.4) \times 10^{-5}$ which corresponds to $2C_{2u} - C_{2d} = -0.17 \pm 0.30$. Without SM C_{1q} the values are $a_1 = (-9.5 \pm 1.8) \times 10^{-5}$ and $a_2 = (5.0 \pm 6.5) \times 10^{-5}$. SLAC quotes: $a_1 = (-9.7 \pm 2.6) \times 10^{-5}$ and $a_2 = (4.9 \pm 8.1) \times 10^{-5}$ where errors were added linearly. Calculations here added all errors in quadrature. (email 2012/3/28)

14. Once the appropriate a_1 and a_3 terms for the asymmetry are evaluated, we compare them to our measured asymmetry and extract C_{2q} as:

$$\boxed{[2C_{2u} - C_{2d}]\,(measured) = [2C_{2u} - C_{2d}]^{SM\ value} \times \frac{A_{PV}^{measured} - A_{a_1 term}^{calculated}}{A_{a_3 term}^{calculated}}} \qquad (27)$$

Table IV summarizes our calculation of the SM value for the asymmetry.

TABLE IV: Comparison of asymmetry calculation using different structure functions. The value for $\alpha_{EM}(Q^2)$ (run from $1/137.036$ at $Q^2 = 0$) and $C_{1,2}^{u,d}(Q^2)$ are also shown.

	$\langle Q^2\rangle = 1.085$, $\langle x\rangle = 0.241$	$\langle Q^2\rangle = 1.901$, $\langle x\rangle = 0.295$
Physical couplings		
$\alpha_{EM}(Q^2)$	$1/134.45$	$1/134.20$
C_1^u, C_1^d	-0.19059, 0.34257	-0.1913, 0.3429
$2C_{1u} - C_{1d}$	-0.72375	-0.7255
C_2^u, C_2^d	-0.03827, 0.02802	-0.0388, 0.0284
$2C_2^d - C_2^d$	-0.10456	-0.1060
$A(a_1), A(a_3)$ terms in ppm		
"no structure"	-83.21, -5.57	-145.77, -14.56
CTEQ/JLab (CJ) full fit	NA	-147.86, -13.60
"PDF+QPM" MSTW2008 LO	-83.75, -4.598	-146.71,-13.10
"PDF+QPM" MSTW2008 NLO	-84.39, -4.735	-147.22,-13.39
"PDF+QPM" MSTW2008 NNLO	-84.41, -4.774	-147.17,-13.48

V. INVESTIGATION OF RADIATIVE CORRECTIONS FOR PVDIS

A. All Radiative Corrections

At tree level we have only one Feynman diagram for electron DIS off a nuclear or nucleon target, as shown in Fig. 2:

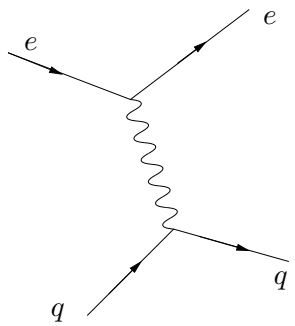

FIG. 2: Tree diagram for electron DIS off a nuclear or nucleon target.

But for higher orders we have plenty: 1) Bremstralung and ionization loss; 2) loop; 3) vertex;

and 4) box diagrams. These are shown in Figs. 3-6:

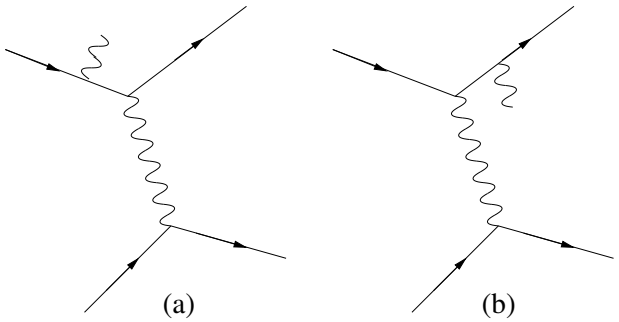

FIG. 3: Diagrams for bremstralung loss of (a) incoming (a) and (b) outgoing electrons. Same diagrams can be used for ionization loss.

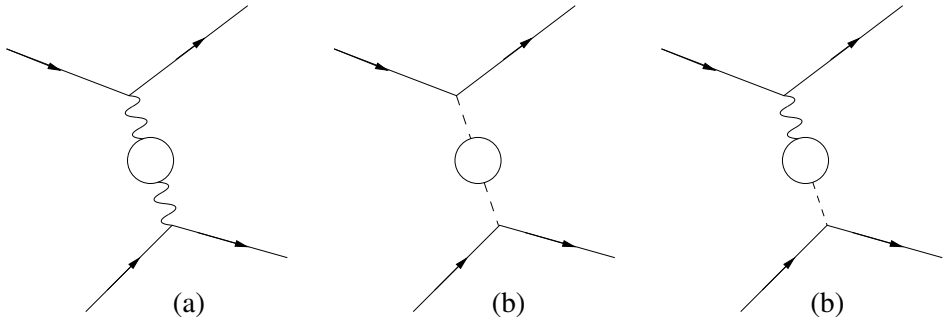

FIG. 4: Loop diagrams for the exchanged boson, either (a) γ, (b) Z (this gives the ρ parameter), or (c) mixed (this gives the running $\sin^2\theta_W$).

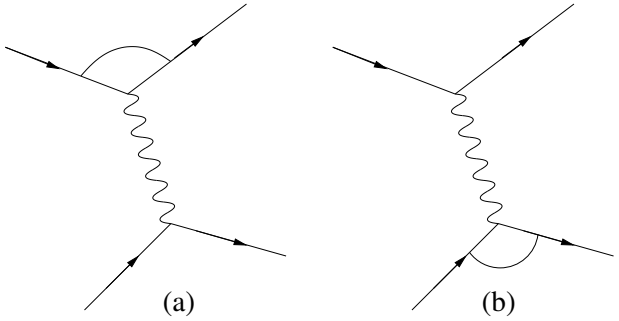

FIG. 5: Vertex diagrams for (a) electrons vertex and (b) quark vertex.

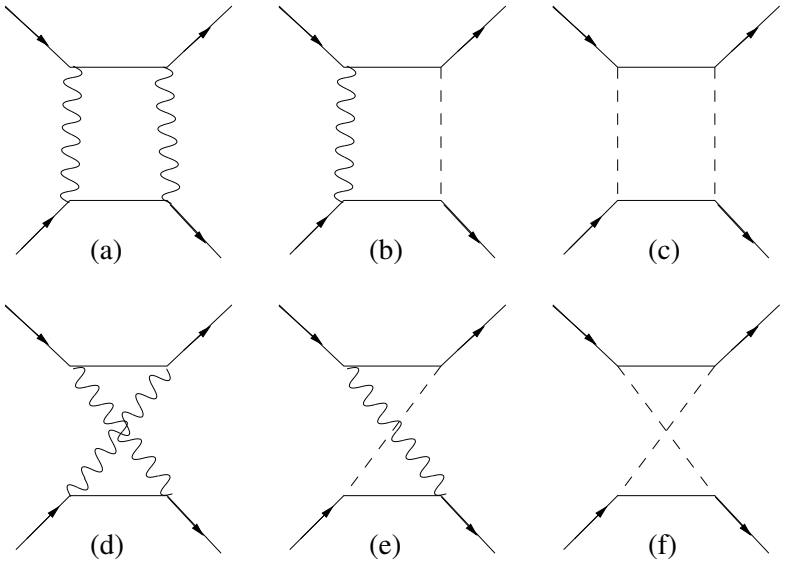

FIG. 6: Box diagrams: (a) $\gamma\gamma$ box, (b) γZ box, (c) ZZ and WW box (J.E.: these are NOT doubly-Fermi-suppressed. They are calculated and included in the running $C_{1,2}$ of Table III), (d) $\gamma\gamma$ box cross digram, (e) γZ box cross diagram, (f) ZZ and WW box cross diagrams (doubly Fermi suppressed).

My understanding is:

1. Bremstralung and ionization losses have already been corrected through the procedure of Section III;

2. Loop diagram of the exchanged γ, Fig. 4(a): is accounted for by using the running on-shell $\alpha(Q^2)$;

3. Loop diagram of the exchanged Z, Fig. 4(b): is accounted for in J. Erler's calculation of $C_{1,2}$ in the $\overline{MS}$ scheme;

4. Loop diagram of the exchanged γ changing to Z or vice versa, Fig. 4(c): is accounted for in J. E.'s calculation of $C_{1,2}$ in the $\overline{MS}$ scheme;

5. Vertex diagrams Fig. 5: At least part of these are automatically included in Bremstralung corrections to cancel the IR-divergent part of Bremstralung. But I do NOT know if there are any "left-over" and IR-finite vertex corrections;

6. Box diagrams in Fig. 6: The $\gamma\gamma$ box diagrams (a) and (d) have not been calculated; J.E.'s calculation of $C_{1,2}$ in the $\overline{MS}$ scheme included a crude estimation of (b) and (e) adapted from the Atomic PV results and should account for the dominant logarithmic correction

necessary to go to DIS. Further calculation on (b) and (e) would be needed at least at the SOLID precision. Corrections related to (c) and (f) have also been fully accounted for in J.E's calculation.

B. Discussions: How It was Done for E158 and Suggestions for PVDIS

Yury's summary:

First, let me clarify what was done for E158. The QED radiative corrections are based on the following paper: http://inspirehep.net/record/688152?ln=en

We used internal bremstralung plus kinematics-dependent parts of $\gamma\gamma$ and γZ boxes and vertex corrections from this calculation – this is needed to cancel out IR divergence of the brem. All of these corrections are computed in the OS scheme, but they are factorizable, since they affect the denominator of the asymmetry. This is the factor F in the E158 paper. Other EW corrections (summarized as κ and Δ in the paper) were taken from Marciano-Czarnecki paper, and are computed in $\overline{MS}$ scheme. For consistency with that calculation (which does not include $\gamma-\gamma$ vacuum polarization IIRC), $\alpha(Q)$ was used, though for E158 this doesn't matter.

So now a couple of comments about PVDIS situation. In principle, Jens is correct that it does not matter what scheme you use for the γZ boxes and other $O(\alpha)$ corrections – formally, the differences are of $O(\alpha^2)$. (Note: I am confused how we can use OS for some and $\overline{MS}$ for other parameters in the calculation). However, it's good to be consistent to avoid confusion. Also, some choices introduce larger $O(\alpha^2)$ than others. For example, using OS for vacuum polarization gives larger corrections than $\overline{MS}$ (simply because difference between tree-level and effective couplings is larger). I would advocate using $\overline{MS}$ for anything that has to do with the electroweak contributions if it's practical – for no other reason than to be able to extract $\sin^2\theta_W(\overline{MZ})$ in $\overline{MS}$ scheme from the results. If such self-consistency is not practical, OS is fine for γZ boxes – but then it should be clear that you are measuring effective couplings only.

The other question important for experimentalists is what value of α to use in front of the asymmetry. This depends on what is included in the effective coupling. If you include only corrections to the numerator of the asymmetry (e.g γZ vacuum polarization) and not to the denominator (total cross section), then you should use $\alpha(Q)$, clearly. If somehow $\gamma\gamma$ vacuum polarization is included, then using alpha(Q) would double-count that correction – but that's a strange choice.

It sounds to me that what was done for E158 is still possible for PVDIS:

1) Divide the corrections into multiplicative (ρ, α, and F factor due to bremstralung and IR-infinite parts of the boxes) and additive (Δ – e.g. EW boxes and vertices).

2) Use a self-consistent scheme to define Δ and ρ – e.g. $\overline{MS}$.

3) Use a self-consistent scheme to define $\alpha(Q)$ and F – e.g. OS if this is the most practical – this is mainly QED multiplicative corrections.

The only question that I could not figure out from the thread below is whether gamma-gamma boxes are included in the MC that computes brem contributions. At least some part of them have to be included – otherwise brem is not IR-safe. Before you add a term for $\gamma\gamma$ boxes to F, you need to know how much was included in the MC.

Comments from KK on Yuri's Procedure:

My assessment is that you are indeed following Yury's 3 steps. Step 2 is completely done by using Jen's numbers, including γZ box (though Jens says he might do further minor tweaks). For step 3, you have computed 2 pieces of the F factor, the brem part and the acceptance part. What is not known is the treatment (or lack thereof) of $\gamma\gamma$ boxes. For that, you really need to know what is in the Monte Carlo, as Yury points out in his last sentence. Jens also effectively said the same thing.

XZ's comments: I do not think any $\gamma\gamma$ is included in MC, as explained at the end of Section III.

C. Questions and To-Dos

From months of email threads and discussions, I think the questions/tasks remain are:

1. Are my accessments in Sec. V A correct? If yes, questions raised there stand and are repeated here:

   (a) Vertex diagrams Fig. 5: At least part of these are automatically included in Bremstralung corrections to cancel the IR-divergent part of Bremstralung. But I do NOT know if there are any "left-over" vertex corrections; – note this seems to be answered, see Addendum JP's comments.

   (b) Box diagrams in Fig. 6: None of these except $\gamma - \gamma Z$ interference are accounted for. They need to be studied one by one.

2. Are there IR-divergent component of loop and vertex diagrams and do they all cancel out? (Or perhaps we don't want to go into this and just assume they do cancel out, otherwise we have to throw away the whole quantum field theory?) – note this seems to be answered, see Addendum JP's comments.

3. Is $\gamma\gamma$ box diagram the same as the $\gamma\gamma$ loop in Yuri's summary?

   XZ: I had a swift doubt that these are the same thing but now I think no. $\gamma\gamma$ loop is really part of the QED vacuum polarization.

4. Does it make sense to use on-shell $\alpha(Q^2)$ and $\overline{MS}$ of $C_{1,2}$ in the asymmetry calculation, when comparison between the calculated and the measured asymmetry gives us C_{2q} using Eq. (27)? Is the α I am using, run from $\alpha_{EM}|_{Q^2=0} = 1/137.036$ using W. Melnitchouk's running code, really the "on-shell" value? Perhaps we should use on-shell $C_{1,2}$ if they exist?

5. Is $\gamma\gamma$ box (TPE) already corrected in the denominator of the asymmetry?

   XZ: As explained at the end of Section III, I think TPE correction still need to be applied to the asymmetry (denominator).

6. We need to calculate interference between Z and $\gamma\gamma$ box and apply this correction to (the numerator of) the asymmetry.

7. To account for depolarization effect of the electron in bremstralung and ionization effects. Once this is studied, it can be plugged into to HAMC and we will redo the simulation.

   XZ: Bob Michaels promised to do this :) – see addendum below.

8. Correct quasi-elastic asymmetry calculation in HAMC.

   XZ: D. Wang will do this once we have identified all modifications to HAMC.

I can use some further discussions on 1-4) (if there is any addition or doubt to my answers). I also would like to confirm that P. Blunden and A. Afanasev will work on 5) and 6) on a timely manner (within a month), and that R. Michaels wil work on 7), also in a timely manner. The goal is to finalize the PVDIS PRL draft by mid August (which means we must have some preliminary corrections by mid July to allow time for discussions and circulating the draft).

VI. ADDENDUM

(6/21/2012) The running code of α_{EM} used here was originally written by P. Blunden with minor tweaks from W. Melnichouk.

(6/22/2012) (J.P. Chen): The internal breamstralung code of HAMC takes care of all internal breamstralung and vertex corrections already. There is no "left-over" vertex corrections to be made as far as the electron vertex is concerned.

A. Formulism for depolarization effect in Bremstralung

(6/28/2012)

The depolarization of electron from Bremstralung radiation was presented in Ref. [12], Eq.(9.11):

$$D(\vec{p}_1, \vec{\zeta}_1) \;=\; \frac{k^2\left[\psi_1 - \zeta_{1z}^2(\psi_1 - \frac{2}{3}\psi_2)\right]}{(\epsilon_1^2+\epsilon_2^2)\psi_1 - \frac{2}{3}\epsilon_1\epsilon_2\psi_2} \tag{28}$$

where in the "complete screening" limit, [12] Eq.(9.13) gives

$$\psi_1 \;=\; 4\ln(111Z^{-1/3}) + 2 - 4f(Z) = 4[\ln(183Z^{-1/3}) - f(Z)], \tag{29}$$

$$\psi_2 \;=\; 4[\ln(183Z^{-1/3}) - f(Z)] - \frac{2}{3}. \tag{30}$$

The function $f(Z)$ is given by [12] Eq.(6.20):

$$f(Z) \;=\; a^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+a^2)} \;, \tag{31}$$

with $a \;=\; (Ze^2/\hbar/c)$ given on the left side of page 888, just above the footnotes. From [12] Eq.(2.1), $\epsilon_{1,2}$ are the energy of the electron before and after the Bremstralung in unit of mc^2 (note I think the electron mass should be used here for m). The meaning of k can be deduced from Ref. [13] where $\vec{k}$ is the "photon vector" so k should be the photon energy in unit of mc^2 (again we should use electron mass). $\vec{\zeta}$ is the polarization vector of the electron and $\zeta_{1z} = 1$ for longitudinally polarized electrons.

The "complete screening" limit is defined as $\beta_i \xi/\delta \gg 1$ just below Eq.(6.32) in Ref. [12]. $\beta_i = (Z^{1/3}/121)b_i$ with $b_1 = 6$, $b_2 = 1.2$ and $b_3 = 0.3$ in Ref. [12] Eq.(6.30). $\xi = 1/(1+u^2)$ is given in Eq.(3.8a) of Ref. [14] where $u = p_1\theta_1$. Here $\vec{p}_1, \vec{p}_2$ are momentum of the electron before and after Bremstralung in unit of mc, and θ_1, θ_2 are the angle between $\vec{p}_1, \vec{p}_2$ and the photon $\vec{k}$, respectively. Because for high energy electrons θ_1 is very small, $u \approx 0$ and $\xi \approx 1$. The quantity $\delta = q_{min} = k/(2\epsilon_1\epsilon_2)$ is given in Ref. [14] Eq.(3.9). Putting everything together we have

$$\frac{\beta_i \xi}{\delta} \;=\; \frac{\frac{Z^{1/3}}{121} b_i}{(1+\epsilon_1^2\theta_1^2)\frac{k}{2\epsilon_1\epsilon_2}} \approx \frac{\frac{Z^{1/3}}{121} b_i}{\frac{k}{2\epsilon_1\epsilon_2} + \frac{1}{2}k\theta_1^2} \gg 1 \tag{32}$$

where the approximation is valid if $k \ll \epsilon_1$ and the complete screening condition is satisfied if $\epsilon_1 \gg 1$ (Our 6 GeV beam has $\epsilon \approx 12000$).

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