The GDH Sum Rule, the Spin Structure of Helium-3 and the Neutron using Nearly Real Photons

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Gerasimov-Drell-Hearn Sum Rule

For cirularly polarized real photons ($Q^2 = 0$):

$$I_{\rm GDH} = \int_{\nu_0}^{\infty} \left[\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)\right] \frac{{\rm d}\nu}{\nu} = -2\pi^2\alpha\left(\frac{\kappa}{M}\right)^2$$

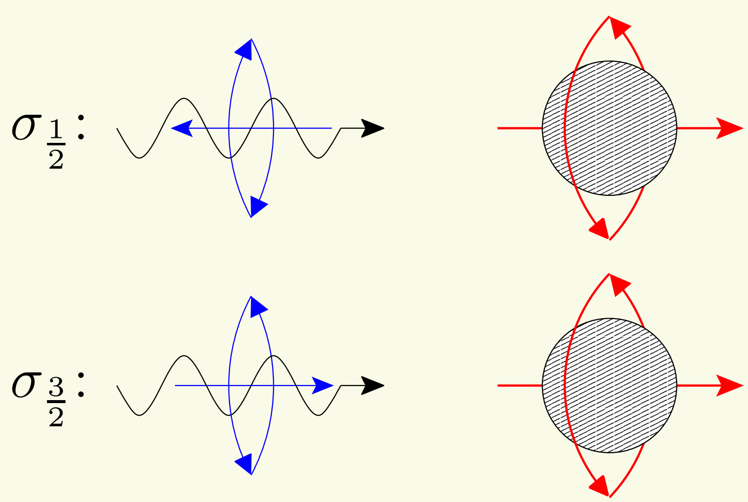
$$I^{\rm n}_{GDH} = -233.2\ \mu{\rm b}\quad (\kappa_{\rm n} = -1.916)$$

Relates real photoabsorption cross section to anomalous part of target magnetic moment κ

Based on fundamental physical arguments:

1. Lorentz and Gauge Invariance (Low Energy Theorem [*Phys. Rev.* **96**, 1428 (1954)])
2. Conservation of Probability (Optical Theorem)
3. Causality (Dispersion Relations)

The integral is dominated by the lowest energy resonant excitation: $\Delta(1232)$, $S = \frac{3}{2}$



Generalized Integral for $S = 1/2$

Generalized to *virtual* photoabsorption and $Q^2 > 0$:

$$I = \int_{\nu_0}^{\infty} \left[\frac{K(\nu, Q^2)}{\nu}\right] \left[\sigma_{\frac{1}{2}}(\nu, Q^2) - \sigma_{\frac{3}{2}}(\nu, Q^2)\right] \frac{\mathrm{d}\nu}{\nu}$$
$$K(\nu, 0) = \nu$$

K is the (convention dependant) virtual photon flux.

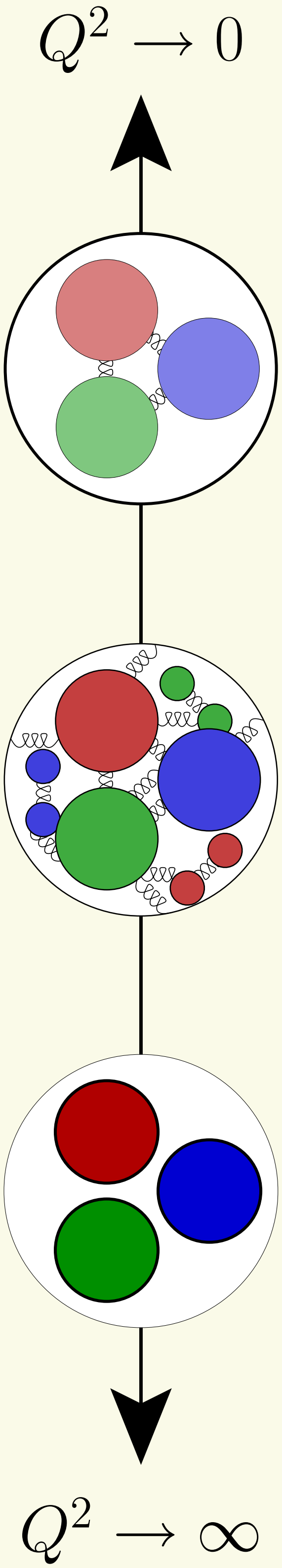
An alternative was suggested by X. Ji & J. Osbourne [*J. Phys. G: Nucl. Part. Phys.* **27**, 127 (2001)]:

$$I_{\mathrm{JO}} = \int_{\nu_0}^{\infty} G_1(\nu, Q^2) \frac{\mathrm{d}\nu}{\nu} = \frac{1}{4}\bar{S}_1(0, Q^2)$$

This generalization:

1. Reproduces the GDH sum rule when $Q^2 \to 0$
2. Reproduces the Bjorken sum rule when $Q^2 \to \infty$
3. Forms a sum rule when set equal to the virtual photon Compton Amplitude $S_1(\nu, Q^2)$, **which can be calculated using theoretical tools.**

Theoretical Tools



Chiral Pertubation Theory

Effective field theory
with hadronic degrees of freedom

Predictions for slope of
GDH Integral at $Q^2 = 0$

Lattice QCD

"Full" QCD on a spacetime lattice

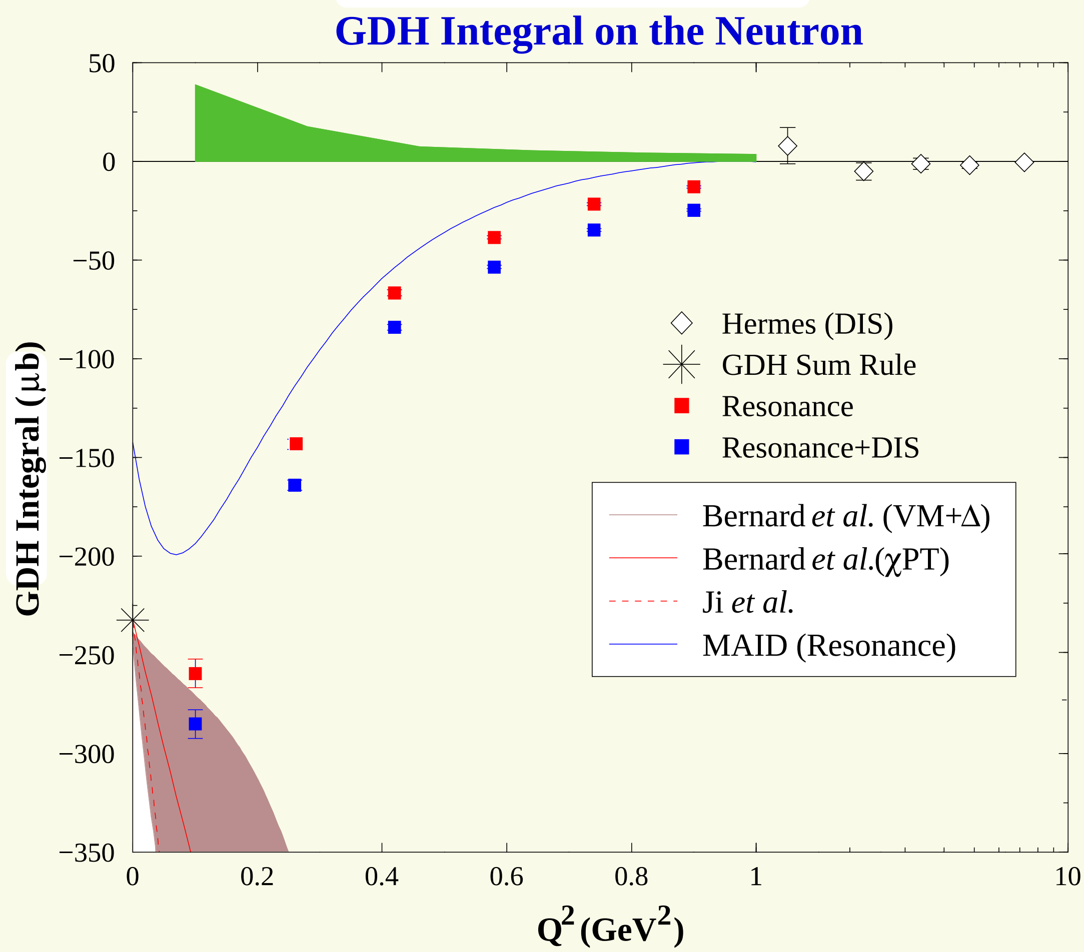
Will eventually be able to calculate
virtual forward Compton amplitude

pQCD

Perturbative expansions
due to Asymptotic Freedom

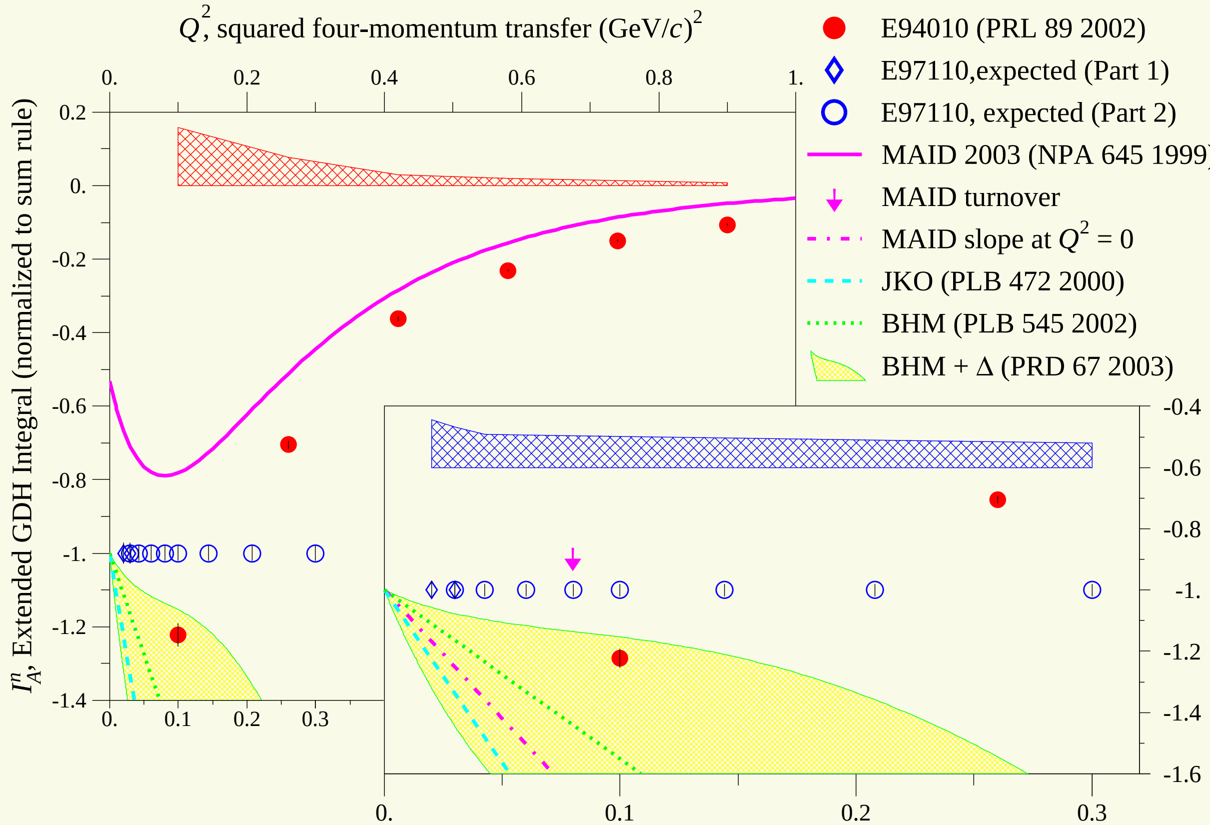
Predictions in the form of
the Bjorken Sum Rule

GDH Integral for $Q^2 > 0.1$ GeV^2



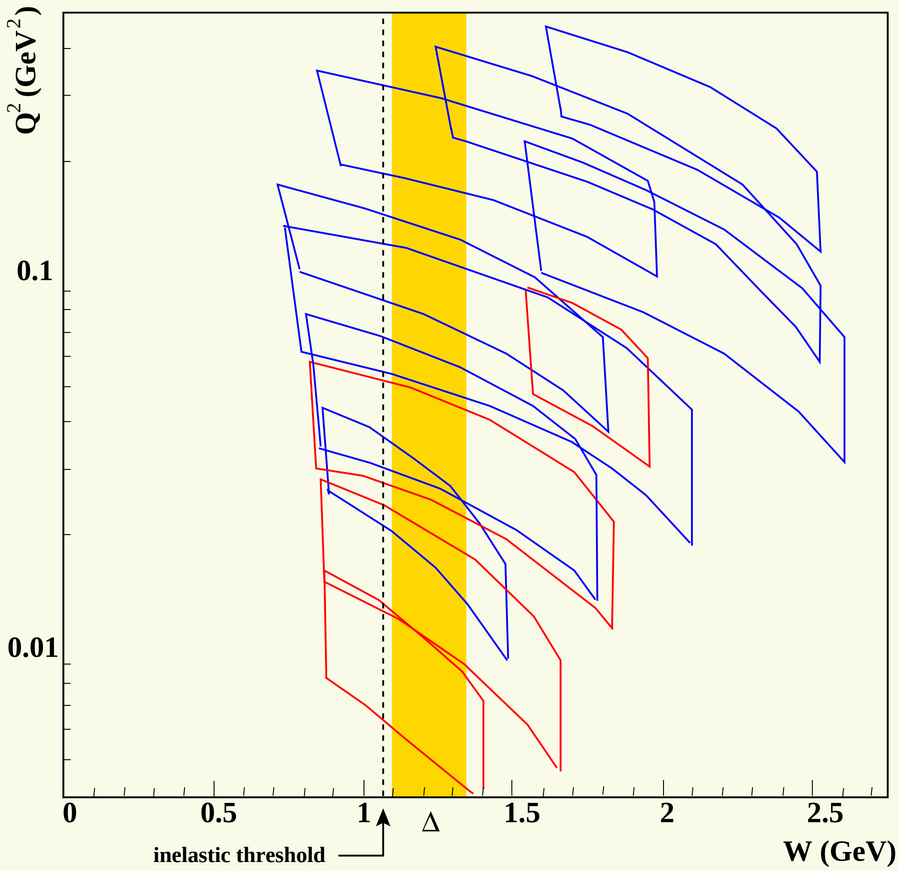
1. For $Q^2 > 1.0$ GeV^2, the integral is **very close to zero.** (HERMES [*Eur. Phys. J.* **C26**, 527 (2003)])
2. For 0.1 GeV$^2 < Q^2 < 1.0$ GeV^2, the integral **drops dramatically.** (JLAB [*PRL* **89**, 242301 (2002)])

Goals for JLAB E97110



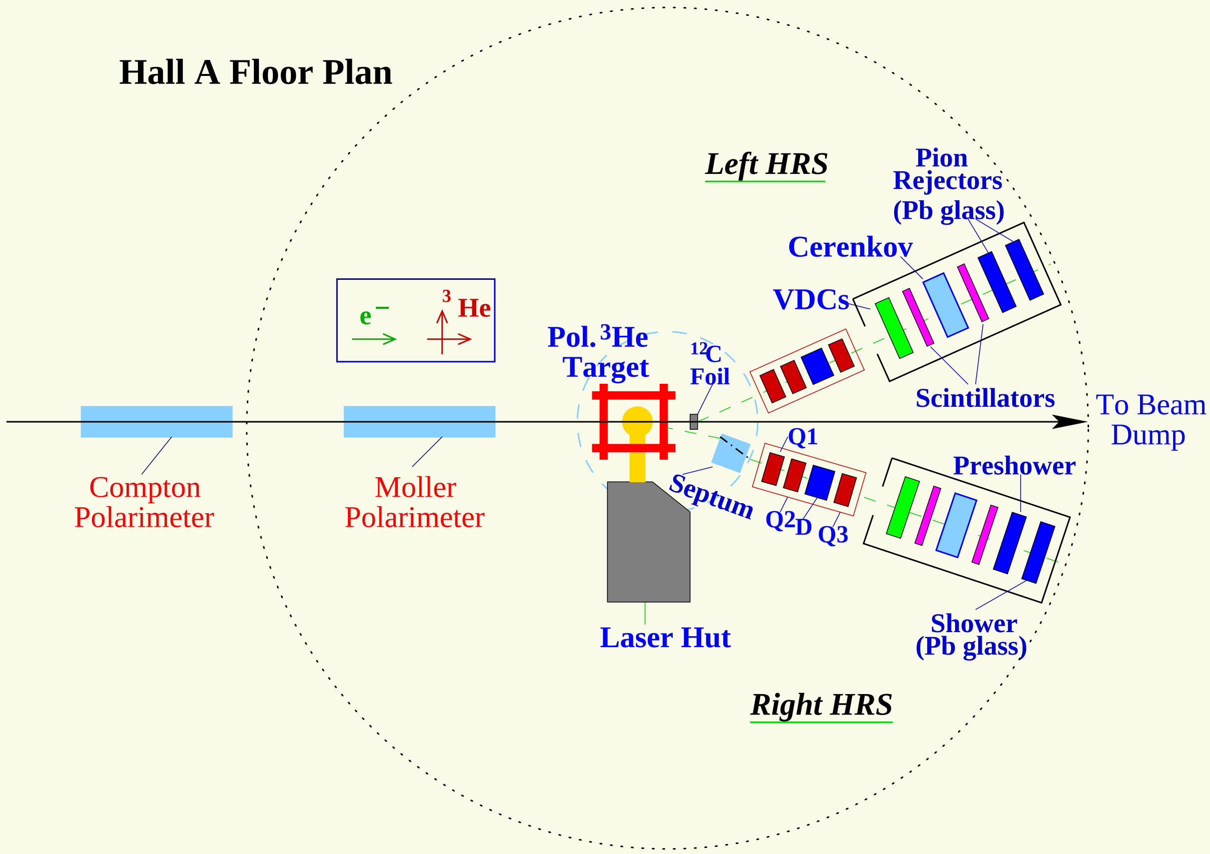
1. Determine the slope of the generalized GDH Integral at low Q^2
2. Extrapolate to $Q^2 = 0$ and compare with the GDH sum rule
3. Extract moments of the spin structure functions
4. Extract generalized forward spin polarizabilties

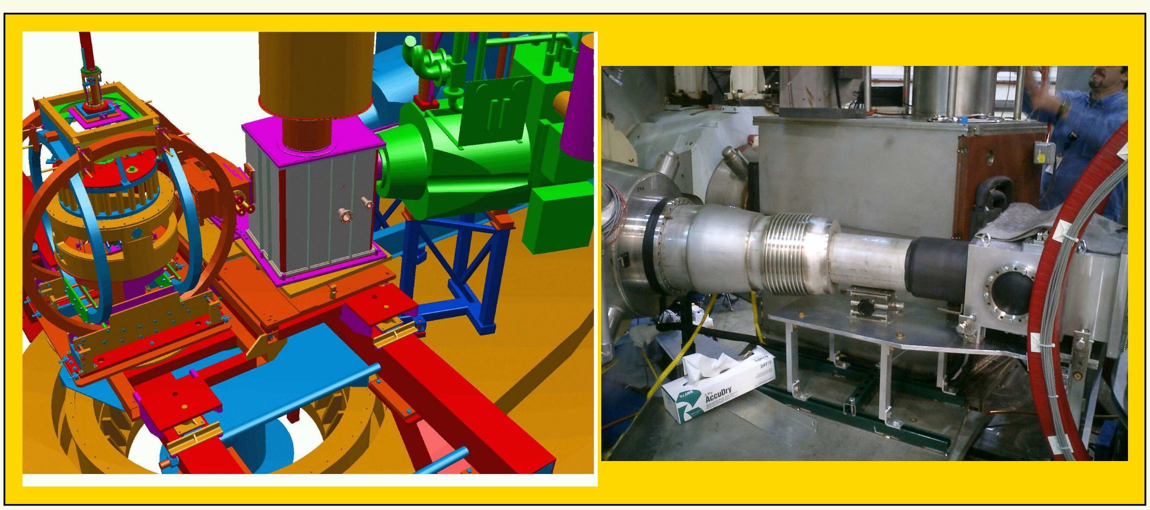
Kinematic Coverage



1. Longitudinal polarized electron beam with energies from 1.1 GeV to 4.4 GeV
2. Longitudinal/transverse polarized ^{3}He target
3. Inclusive electron scattering
4. To reach low Q^2, scattered electrons were detected at 6° and 9°.
5. Nominal spectrometer momentum setting was scanned from 0.5 GeV to 3.1 GeV.

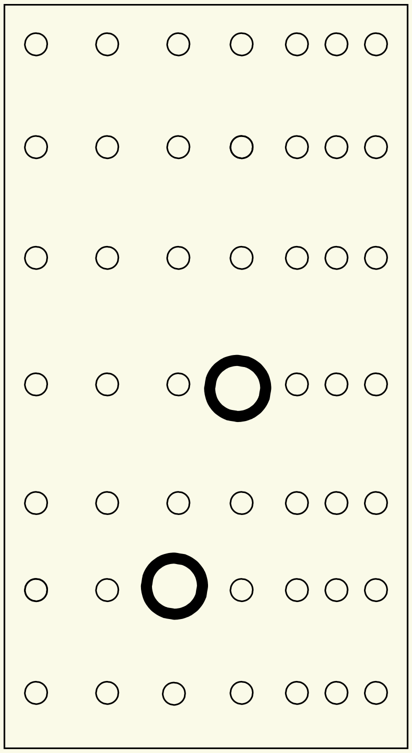
Hall A with Septum Magnet



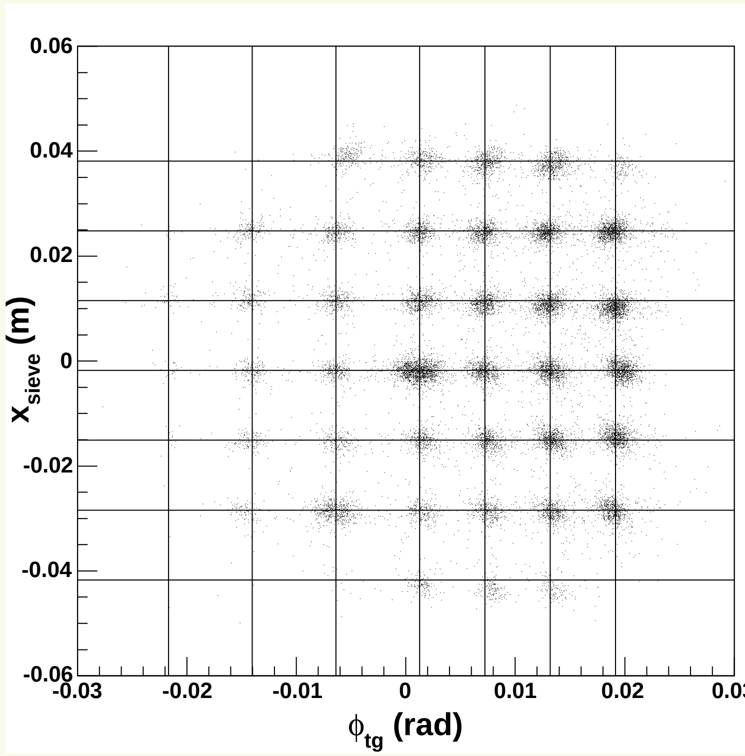


Some Analysis Results

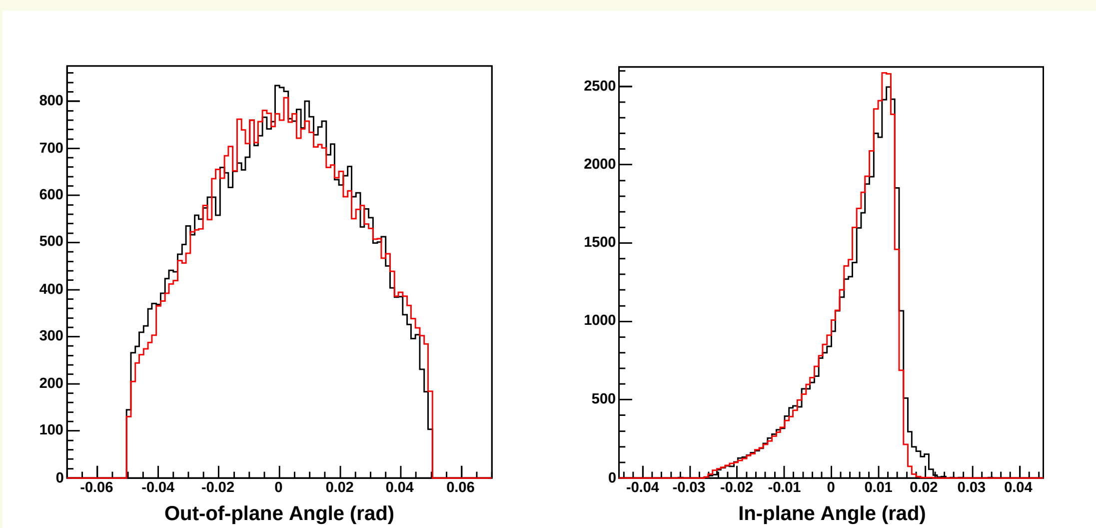
Angular Reconstruction



Sieve Slit



Angular Acceptance



Data **Simulation**

Analysis Status

1. Detector Calibrations
2. Magnet Optics Optimization
3. **Spectrometer Acceptance**
4. **Beam & Target Analysis (polarimetry, density, charge calibration, etc.)**
5. Generating Cross Sections and Asymmetries
6. Radiative Corrections
7. Extract Integrals

The addition of a brand new spectrometer magnetic element (septum magnet) has made the magnet optics optimization and spectrometer acceptance very challenging.

Summary

1. The Generalized GDH Integral allows us to study the nucleon over the full Q^2 range.
2. Low Q^2 measurements of the integral test the dynamics of Chiral Pertubration Theory.
3. E97110 ran successfully in the summer of 2003.
4. This represents a precise measurement of the generalized GDH Integral at low Q^2 ($0.02\ \mathrm{GeV}^2 < Q^2 < 0.30\ \mathrm{GeV}^2$) for the neutron and ^{3}He nucleus.
5. Addition of the Septum magnet has made the analysis very challenging.
6. We're hopefull to have preliminary results by the end of the summer!

Collaboration List

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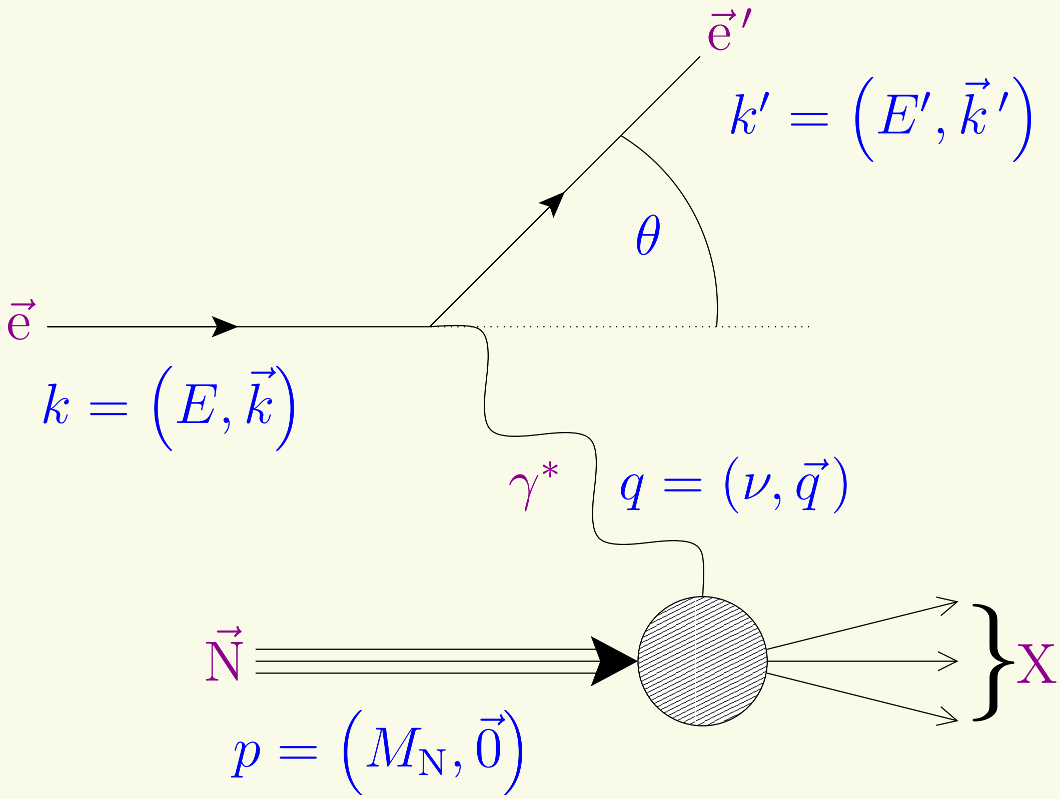
Spokesperson **Thesis Student**

Extra Slides

Polarized Inclusive Electron Scattering



Electron beam is longitudinally polarized.
Target is longitudinally or transversely polarized.

Four-Momentum Lost by Incident Electron:

$$Q^2 = -q^2 = -\left(k - k'\right)^2 \approx 4EE' \sin^2\left(\frac{\theta}{2}\right)$$

Energy Lost by Incident Electron:

$$\nu = E - E'$$

Invariant Mass of the Hadron Decay Products:

$$W_{\mathrm{X}} = |p + q| = \sqrt{M_{\mathrm{N}}^2 + 2\nu M_{\mathrm{N}} - Q^2}$$

Experimental Observsvables

We want $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$, which is related to the spin structure functions g_1 and g_2 in the following way:

$$\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}} = -2\sigma'_{TT}$$
$$= \frac{8\pi^2\alpha}{MK}\left[g_1 - \left(\frac{Q^2}{\nu^2}\right) g_2\right]$$

g_1 and g_2 are related to the measured cross section differences in the following way:

$$\Delta\sigma_{\parallel} = A\left[\Big(E + E'\cos(\theta)\Big)\, g_1 - \left(\frac{Q^2}{\nu}\right) g_2\right]$$
$$\Delta\sigma_{\perp} = A\frac{E'\sin(\theta)}{\nu}\left[(\nu)\, g_1 + (2E)\, g_2\right]$$
$$A = \frac{4\alpha^2}{M\nu Q^2}\frac{E'}{E}$$

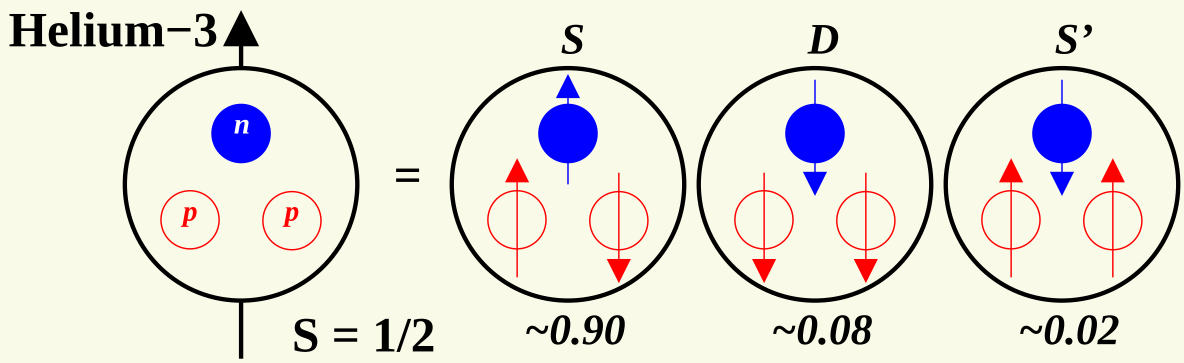
The measured cross section differences are defined as:

$$\Delta\sigma_{\parallel} = \frac{d^2\sigma^{\downarrow\Uparrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\Uparrow}}{dE'd\Omega}$$
$$\Delta\sigma_{\perp} = \frac{d^2\sigma^{\downarrow\Rightarrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\Leftarrow}}{dE'd\Omega}$$

$\downarrow\uparrow$ labels the beam polarization and $\Uparrow\Rightarrow$ labels the target. To access the GDH integrand for $Q^2 > 0$, we need a longitudinally and transversly polarized target.

Effective Polarized Neutron Target

Largest contribution to the ^{3}He wave function is a neutron and two antialigned protons [J.L. Friar *et al*, *Phys. Rev.* **C42**, 2310 (1990)] :



Traditionally neutron quantities have been extracted from ^{3}He quantities using the "effective polarization" prescription following C. Ciofi degli Atti & S. Scopetta [*Phys. Lett. B* **404**, 223 (1997)], for example:

$$\underbrace{I^n(Q^2)}_{\text{neutron}} = \frac{1}{p_n}\left[\underbrace{I^3(Q^2)}_{\text{helium}-3} - \underbrace{2p_p I^p(Q^2)}_{2\times\text{proton}}\right]$$