The Generalized GDH Sum Rule

Measuring the Spin Structure of He-3 and the Neutron using Nearly Real Photons

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For the Hall A Collaboration

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GDH Sum Rule $\left(Q^2 = 0\right)$

For circularly polarized real photons ($Q^2 = 0$):

$$I_{\mathrm{GDH}} = \int_{\nu_0}^{\infty} \left[\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)\right] \frac{\mathrm{d}\nu}{\nu} = -2\pi^2\alpha\left(\frac{\kappa}{M}\right)^2$$

$$I^{\mathrm{n}}_{GDH} = -233\ \mu\mathrm{b}\ \ \&\ \ I^{^3\mathrm{He}}_{GDH} = -498\ \mu\mathrm{b}$$

This sum rule relates the real photoabsorption cross section difference to the anomalous part of the target magnetic moment κ.

Causality $\longrightarrow$ Dispersion Relation

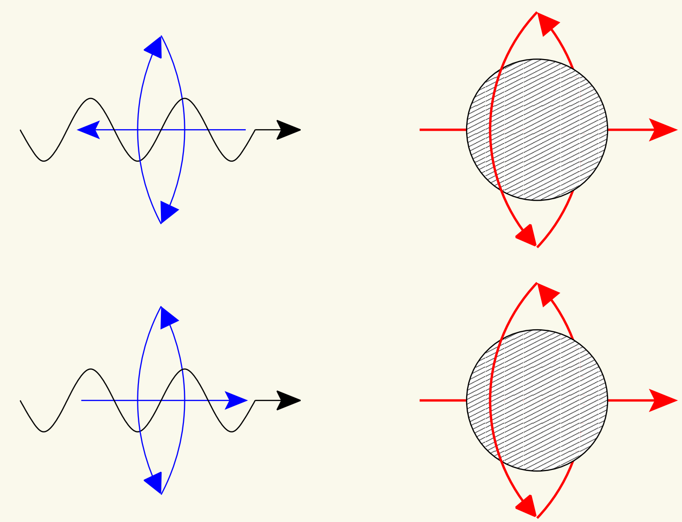
Unitarity $\longrightarrow$ Optical Theorem

Lorentz & Gauge Invariance $\longrightarrow$

$\longrightarrow$ Low Energy Theorem

$\sigma_{\frac{1}{2}}$:

$\sigma_{\frac{3}{2}}$:



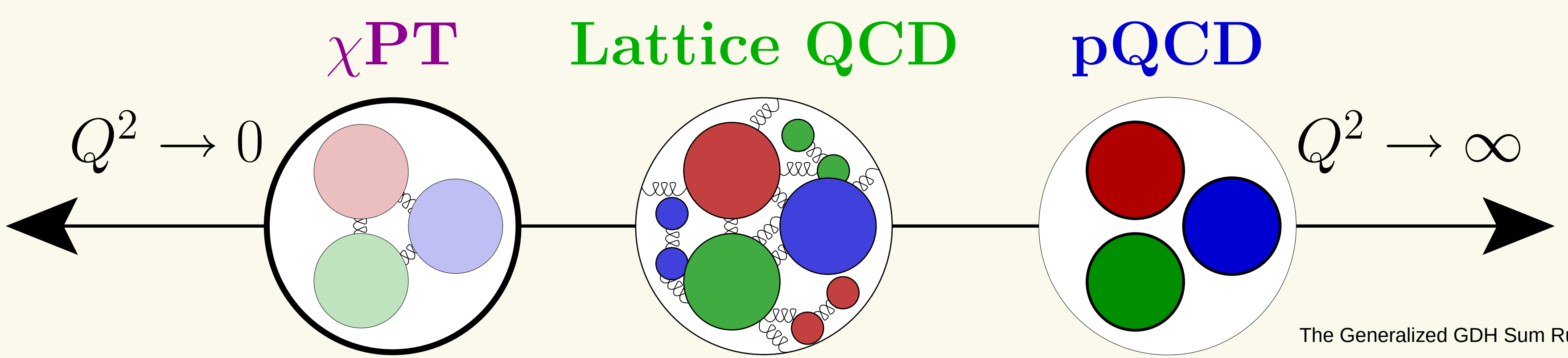
Generalized Integral for $S = 1/2$

When the integrand is generalized to $Q^2 > 0$:

$$I = \int_{\nu_0}^{\infty} \left[\frac{K(\nu,Q^2)}{\nu}\right] \left[\sigma_{\frac{1}{2}}(\nu,Q^2) - \sigma_{\frac{3}{2}}(\nu,Q^2)\right] \frac{\mathrm{d}\nu}{\nu}$$

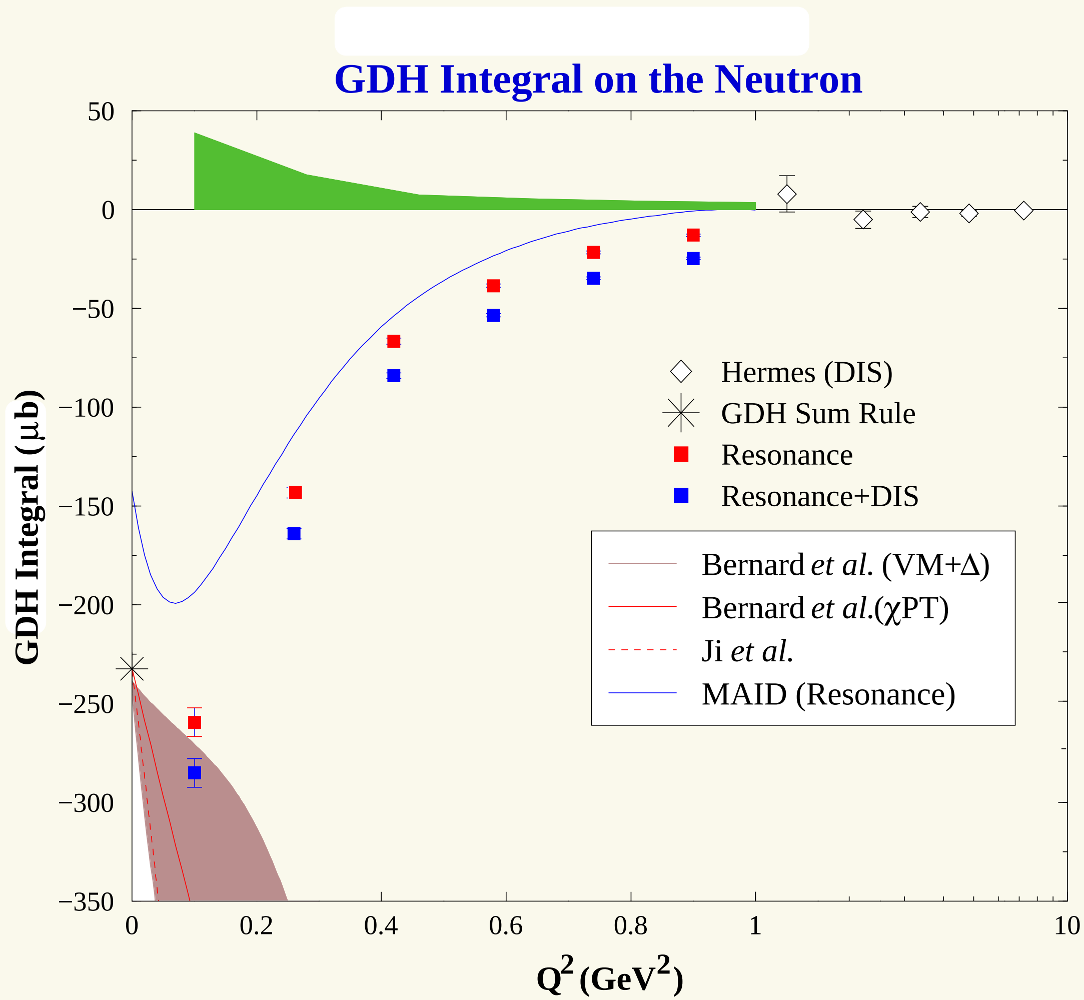
$$K(\nu,0) = \nu$$

...the integral can form a sum rule proportional to the virtual photon Compton Amplitude $S_1(\nu, Q^2)$ [see for example: X. Ji & J. Osbourne J. Phys. G: Nucl. Part. Phys. 27, 127 (2001)], which can be calculated over the full Q^2 range using different theoretical tools.



GDH Integral for $Q^2 > 0.1\ \mathrm{GeV}^2$

1. At high Q^2, the integral is **very close to zero.** (HERMES [*Eur. Phys. J.* **C26**, 527 (2003)])
2. At intermediate Q^2, the integral **drops dramatically.** (JLAB [*PRL* **89**, 242301 (2002)])
3. At low Q^2, the integral must "turn over" in order to satisfy the sum rule.



E97110: small angle GDH

- A polarized ^{3}He nucleus “stands in” as a polarized neutron.
- Detected only the scattered electron at 6^o and 9^o using the right septum magnet and the standard Hall A HRS package.
- ^{3}He target cells were specifically designed and constructed to minimize radiative corrections.
- We have data for both longitudinal (parallel) and tranverse (perpendicular) target polarizations.
- Contamination from the glass and Nitrogen are subtracted using data from reference cell runs for each kinematic.
- Measured “double” polarized cross sections and asymmetries for inclusive electron scattering from a polarized ^{3}He target.

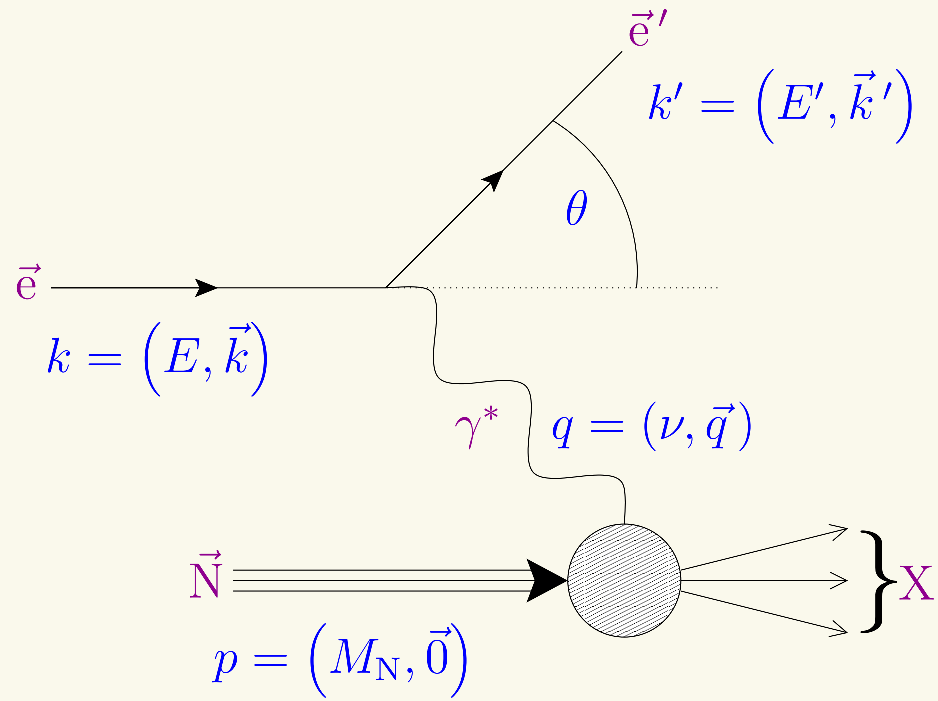
Polarized Inclusive Electron Scattering

Energy Lost by Incident Electron:

$$\nu = E - E'$$

4-Momentum Transferred:

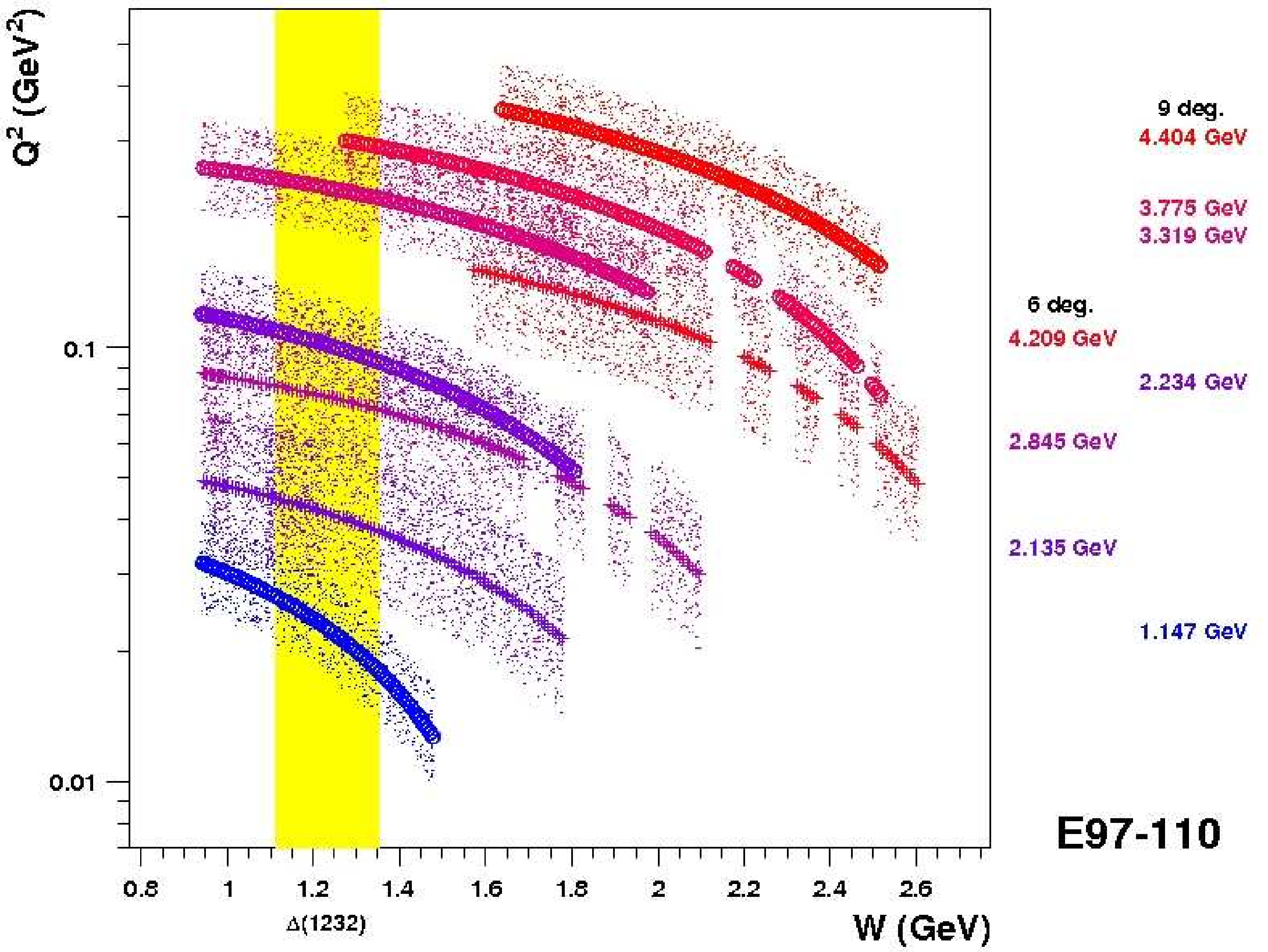
$$Q^2 = -q^2 \approx 4EE' \sin^2\left(\frac{\theta}{2}\right)$$



Invariant Mass of the Hadron Decay Products:

$$W_{\mathrm{X}} = |p + q| = \sqrt{M_{\mathrm{N}}^2 + 2\nu M_{\mathrm{N}} - Q^2}$$

Kinematic Coverage



Experimental Observables

The measured cross section differences are:

$$\Delta\sigma_{\parallel} = \frac{4\alpha^2}{MQ^2}\frac{E'}{E}\left[\left(\frac{E+E'\cos(\theta)}{\nu}\right)g_1 - \left(\frac{Q^2}{\nu^2}\right)g_2\right]$$

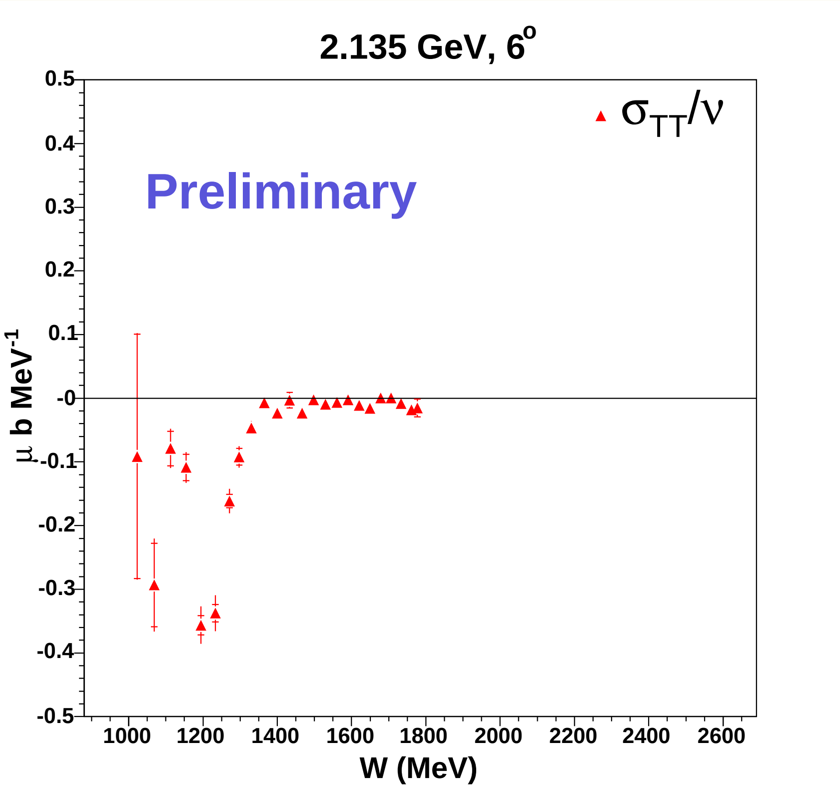
$$\Delta\sigma_{\perp} = \frac{4\alpha^2}{MQ^2}\frac{E'}{E}\left(\frac{E'\sin(\theta)}{\nu}\right)\left[g_1 + \left(\frac{2E}{\nu}\right)g_2\right]$$

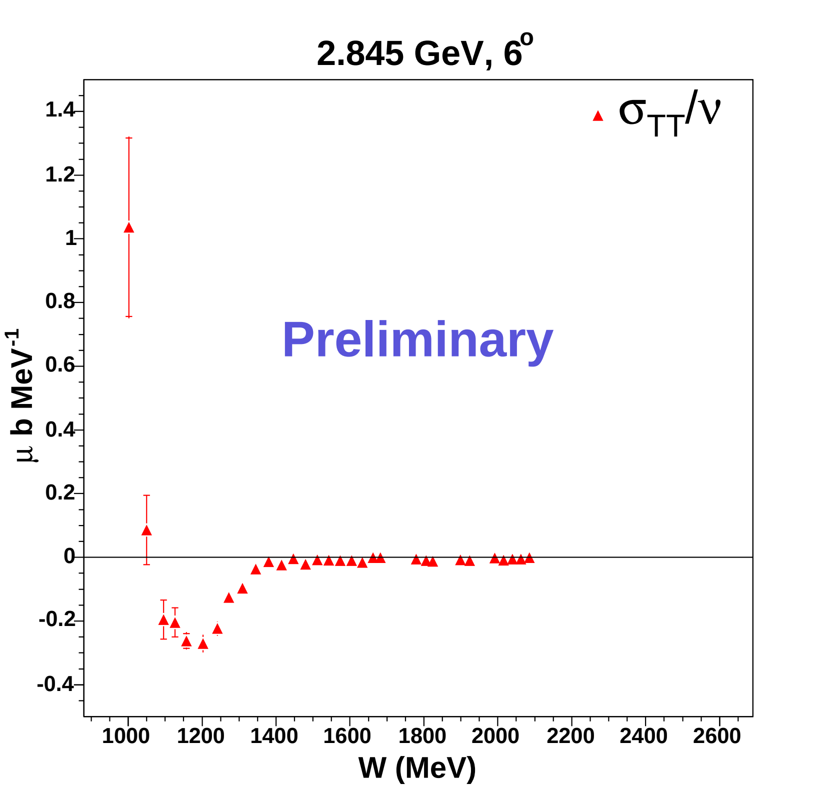
The GDH Integrand is given by:

$$\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}} = -2\sigma'_{TT} = \frac{8\pi^2\alpha}{MK}\left[g_1 - \left(\frac{Q^2}{\nu^2}\right)g_2\right]$$

To access the GDH integrand for $Q^2 > 0$, we need a longitudinally and transversely polarized target.

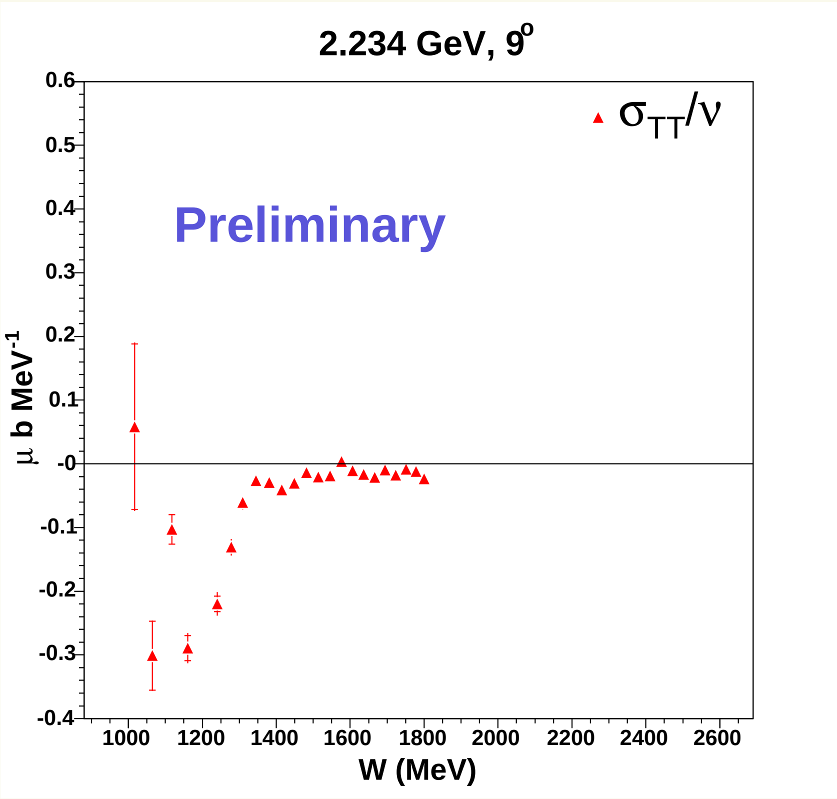
Preliminary Results

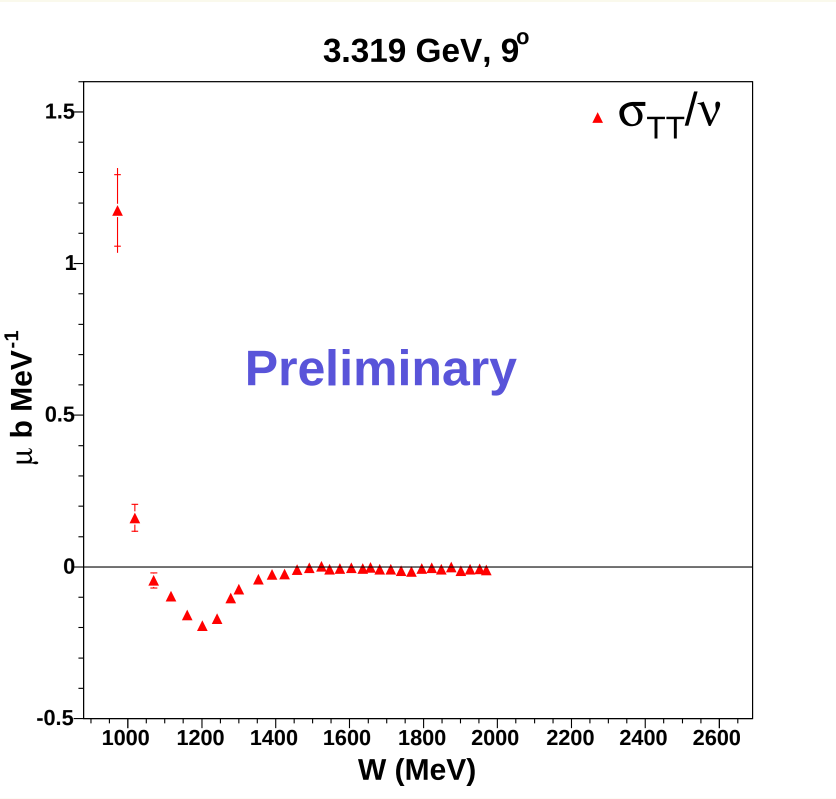




Non-convention dependant part of GDH integrand

Preliminary Results





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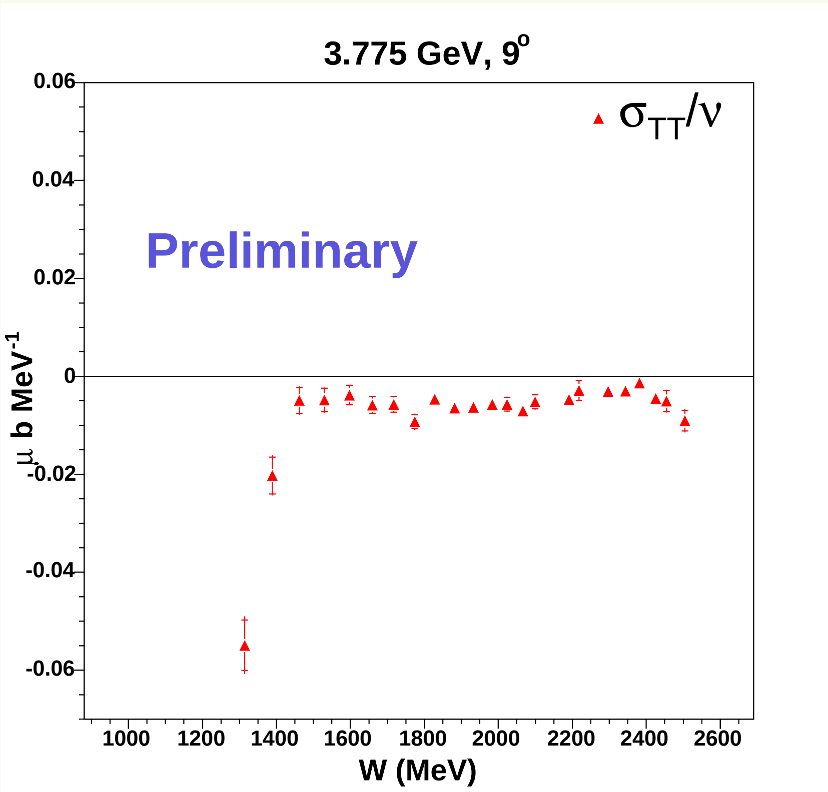
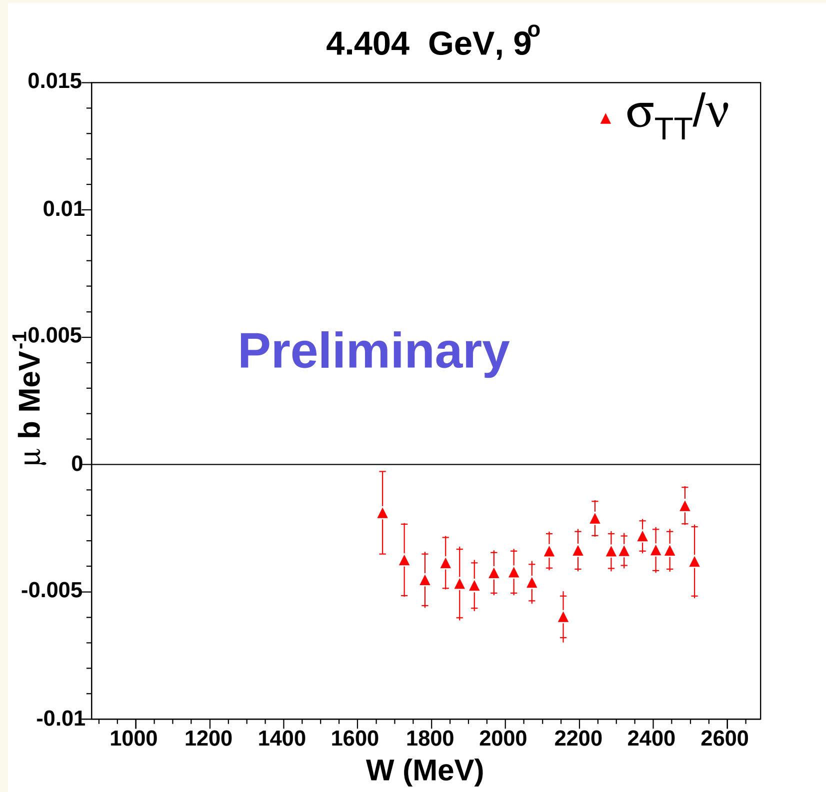


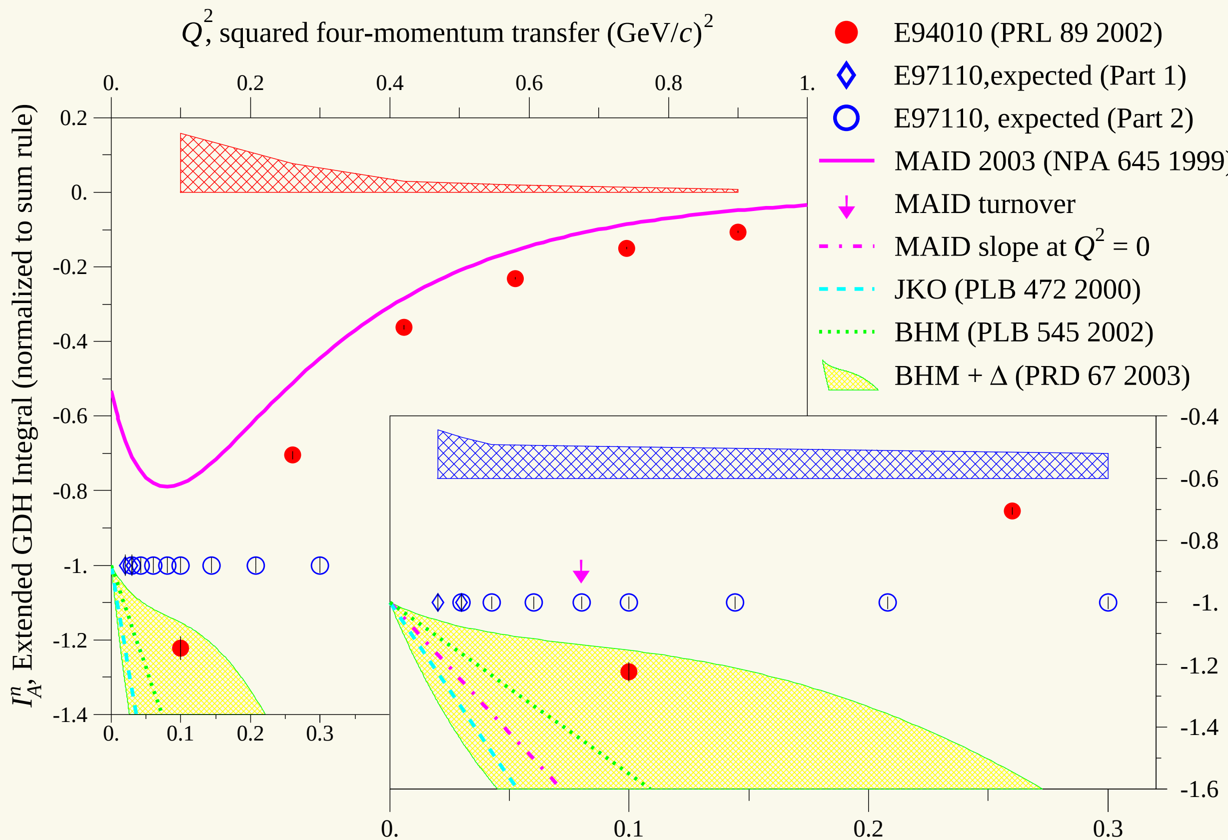


Non-convention dependant part of GDH integrand

Summary: Expected Results

This data set complements the E94010 data set below $Q^2 = 0.10\ \mathrm{GeV}^2$ with improved precision.

1. Turnover?
2. Slope at low Q^2?
3. Extrapolation to the real photon point ($Q^2 = 0$)?



In addition, we will also extract the moments of the spin structure functions and forward spin polarizabilities.

Collaboration List

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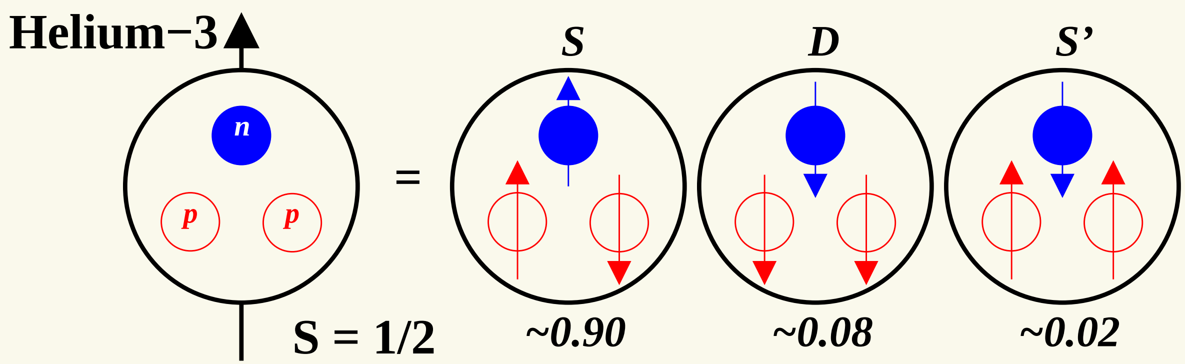
Spokesperson **Thesis Student**

Effective Polarized Neutron Target

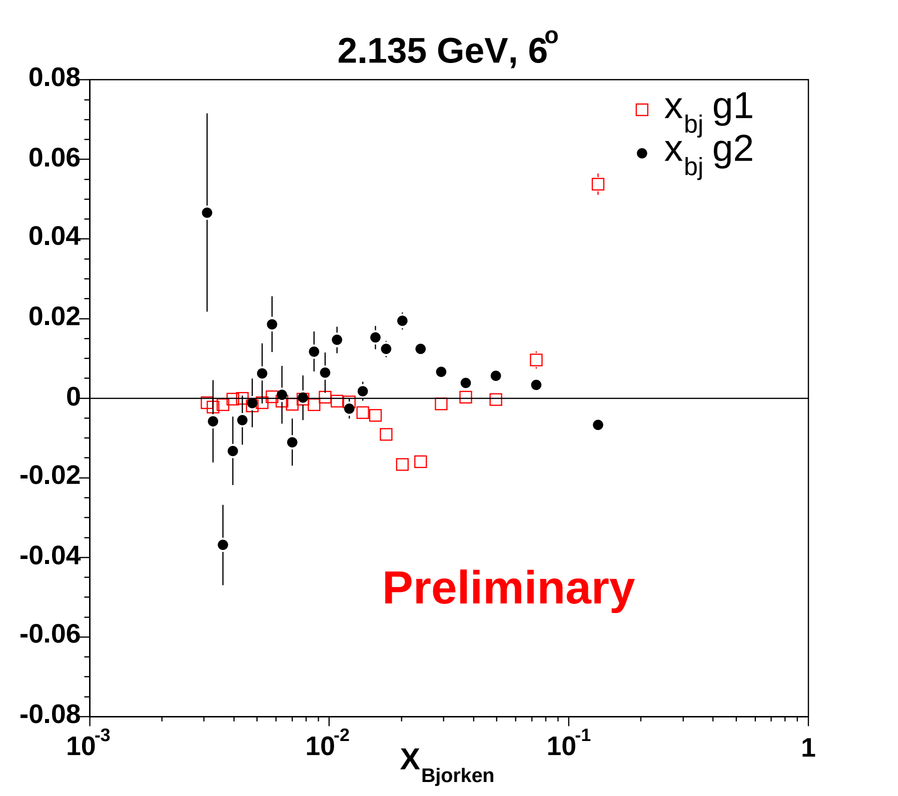
Largest contribution to the $^3\mathrm{He}$3 wave function is a neutron and two antialigned protons [J.L. Friar *et al, Phys. Rev.* **C42**, 2310 (1990)] :

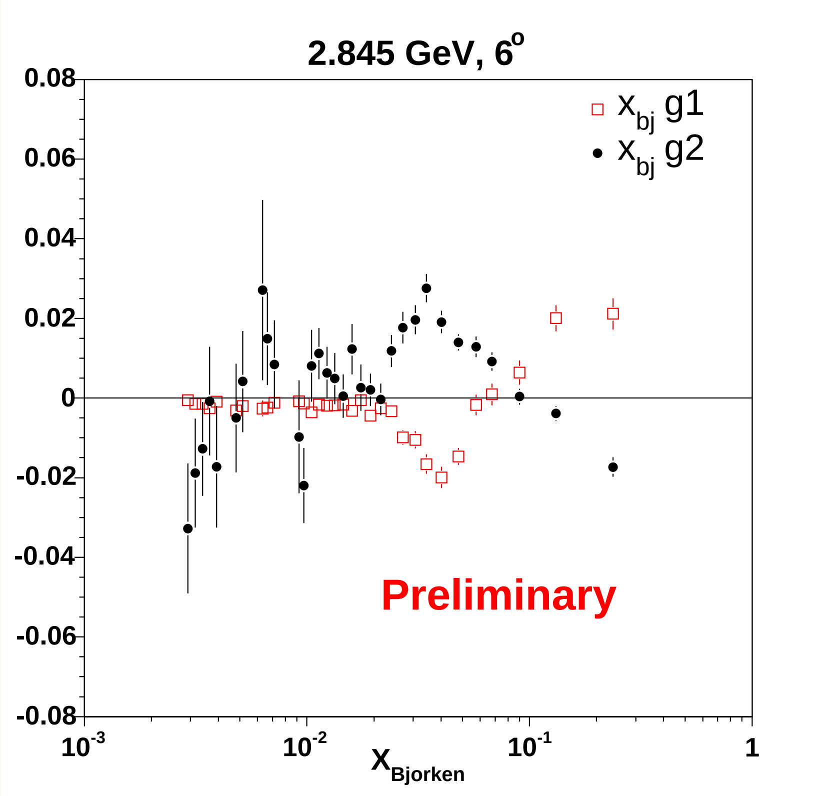


Traditionally neutron quantities have been extracted from $^3\mathrm{He}$3 quantities using the “effective polarization” prescription following C. Ciofi degli Atti & S. Scopetta [*Phys. Lett. B* **404**, 223 (1997)], for example:

$$\underbrace{I^n(Q^2)}_{\text{neutron}} = \frac{1}{p_n}\left[\underbrace{I^3(Q^2)}_{\text{helium}-3} - \underbrace{2p_pI^p(Q^2)}_{2\times\text{proton}}\right]$$

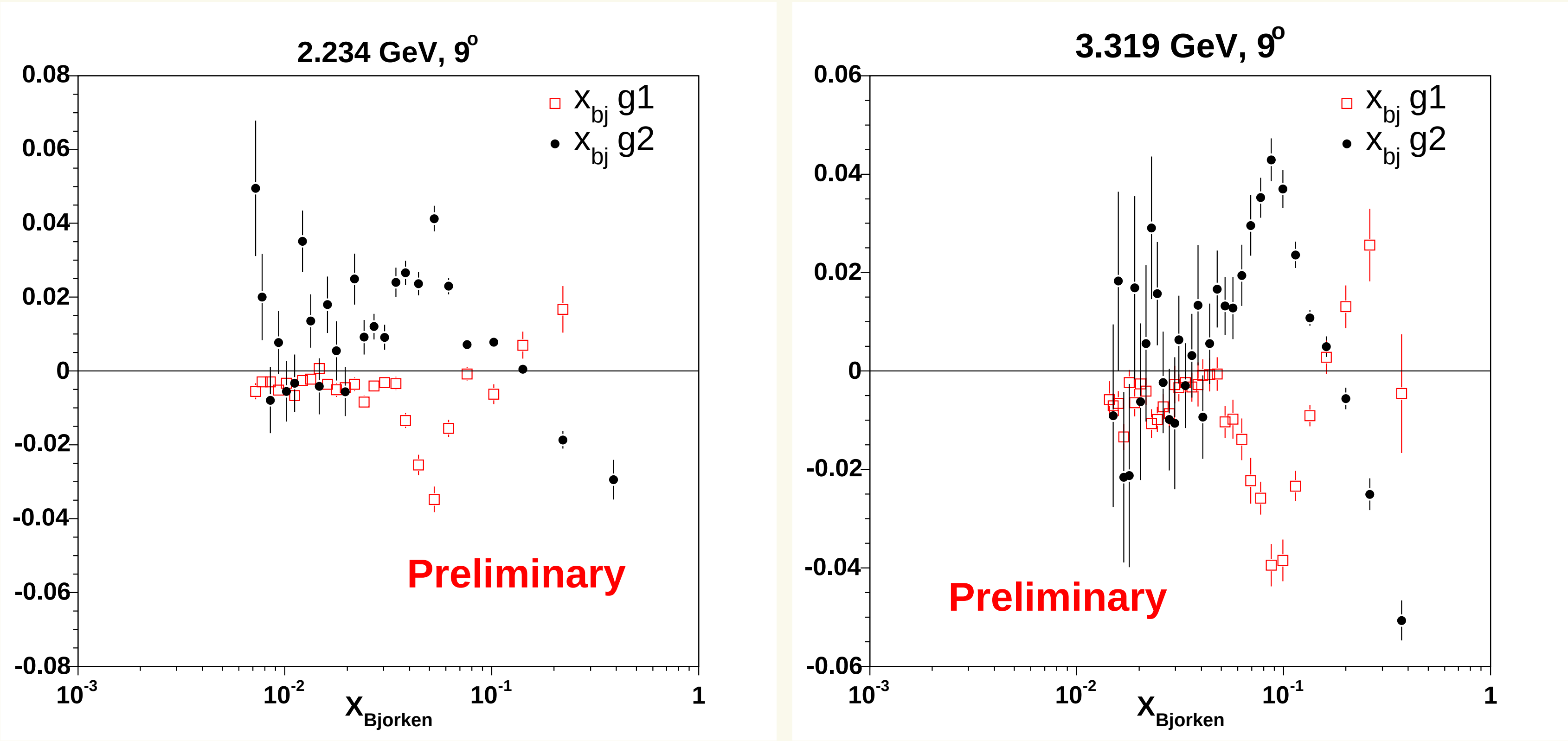
Preliminary Results





Spin Structure Functions weighted by x

Preliminary Results



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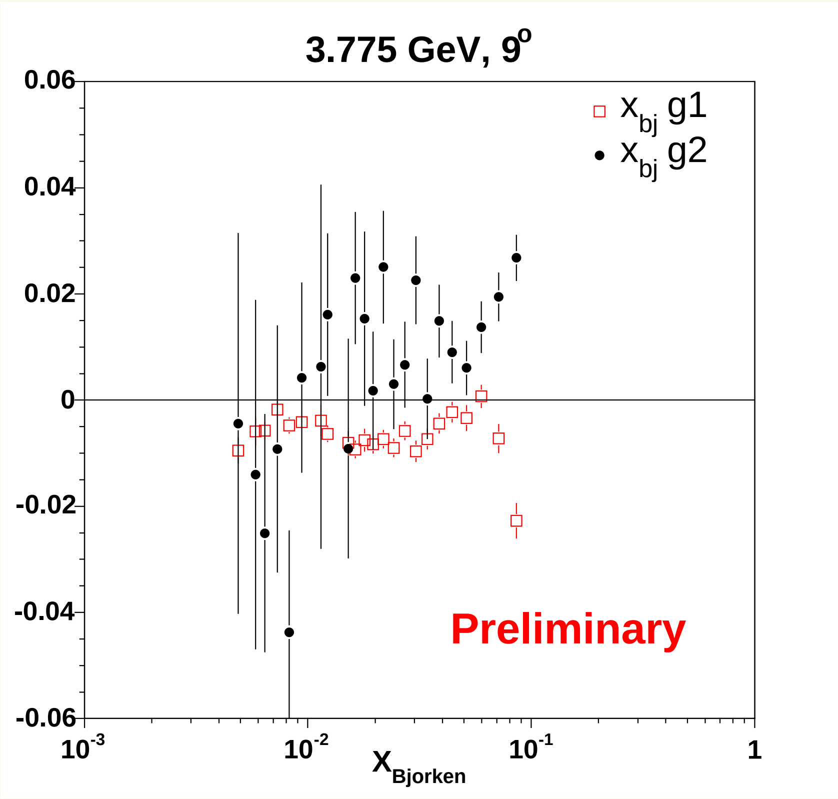

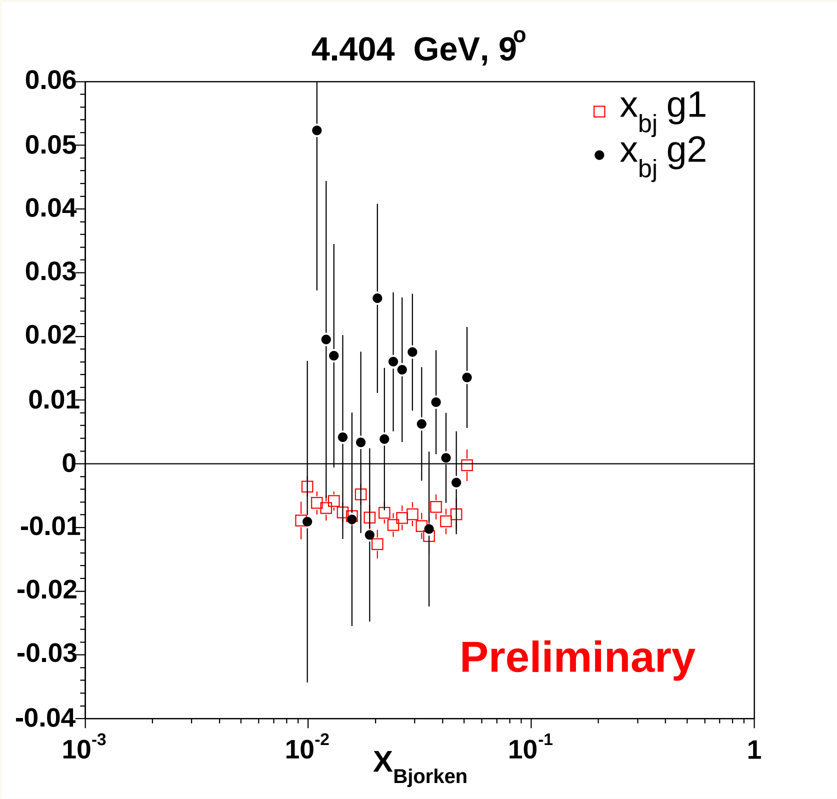

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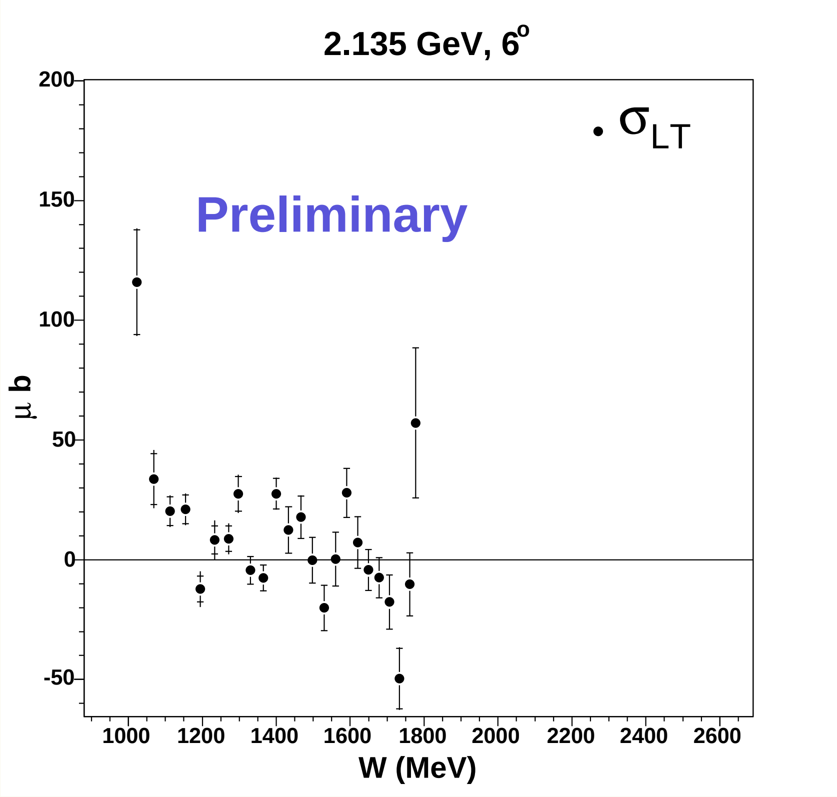

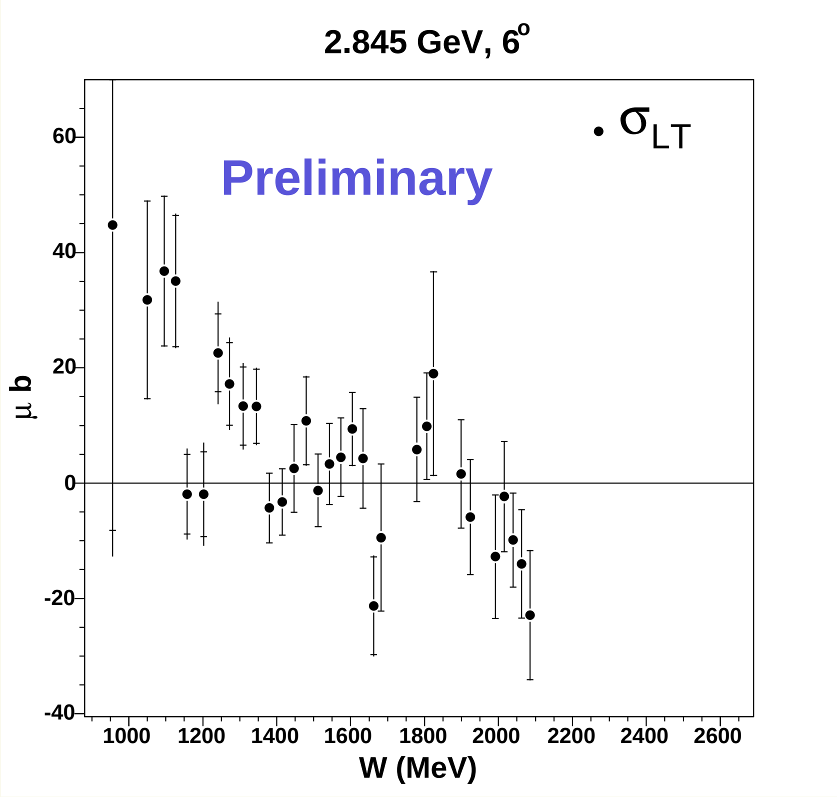

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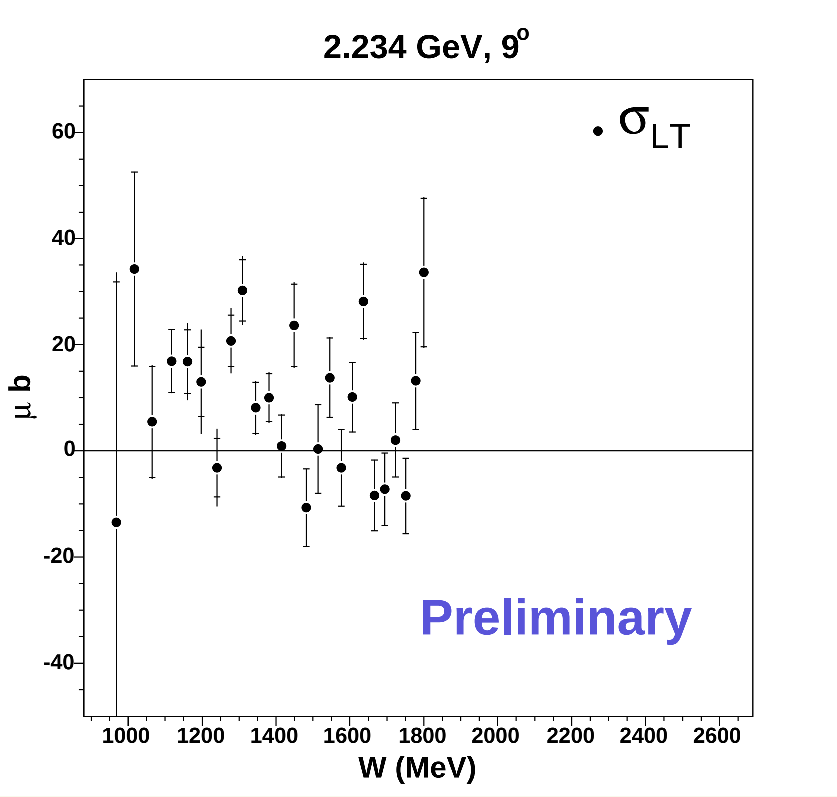

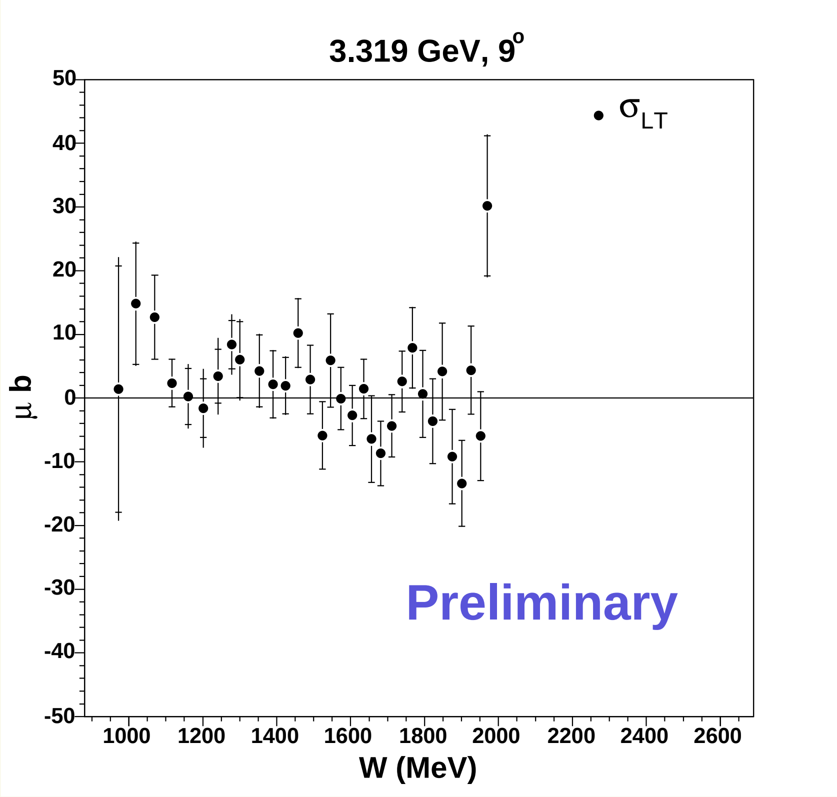

Preliminary Results