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*E97-110: Small Angle GDH  
Experimental Status Report*

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Jefferson Lab

on the behalf of the Spokespersons: J.P. Chen, A. Deur and F. Garibaldi

Ph. D. Students: J. Singh and J. Yuan

Recently Graduated: V. Sulkosky

Hall A Collaboration Meeting

December 14<sup>th</sup>, 2007

# Gerasimov-Drell-Hearn (GDH) Sum Rule ( $Q^2 = 0$ )

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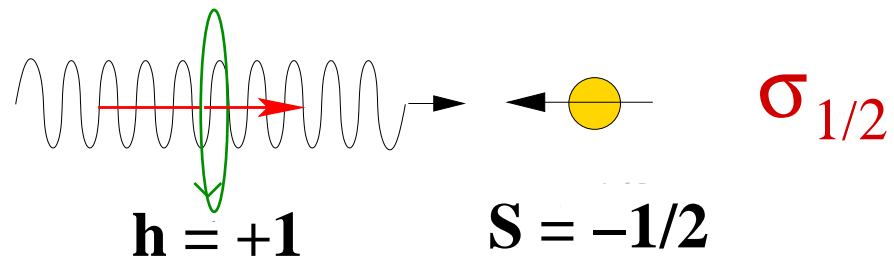
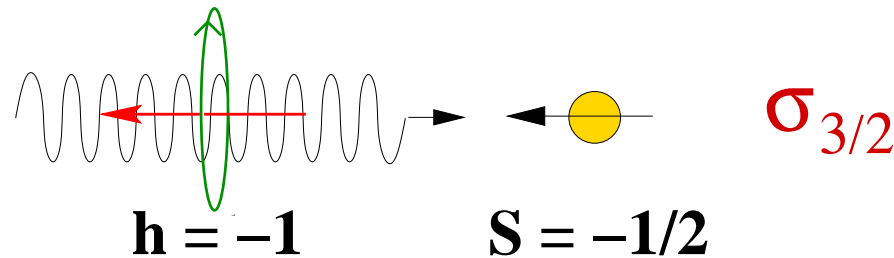
$$I_{\text{GDH}} = \int_{\nu_{\text{th}}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2 \alpha \left( \frac{\kappa}{M} \right)^2$$

- Circularly polarized photons incident on a longitudinally polarized spin- $\frac{1}{2}$  target.

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- $\sigma_{\frac{1}{2}}$  ( $\sigma_{\frac{3}{2}}$ ) photoabsorption cross section with photon helicity parallel (anti-parallel) to the target spin.



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- Circularly **polarized photons** incident on a longitudinally polarized spin- $\frac{1}{2}$  target.
- $\sigma_{\frac{1}{2}}$  ( $\sigma_{\frac{3}{2}}$ ) **photoabsorption cross section** with photon helicity parallel (anti-parallel) to the target spin.
- The sum rule is related to the **target's mass  $M$**  and **anomalous part of the magnetic moment  $\kappa$** .
- The sum rule is **valid for any spin-S target**.

# Generalized GDH Integral ( $Q^2 > 0$ )

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$$I(Q^2) = \int_{\nu_{\text{th}}}^{\infty} \left[ \sigma_{\frac{1}{2}}(\nu, Q^2) - \sigma_{\frac{3}{2}}(\nu, Q^2) \right] \frac{d\nu}{\nu}$$

$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{MK} \left[ g_1(\nu, Q^2) - \left( \frac{Q^2}{\nu^2} \right) g_2(\nu, Q^2) \right]$$

- Replace **photoproduction cross sections** with the corresponding **electroproduction cross sections**.
- The integral is related to the Compton scattering amplitudes:  $S_1(Q^2)$  and  $S_2(Q^2)$ .

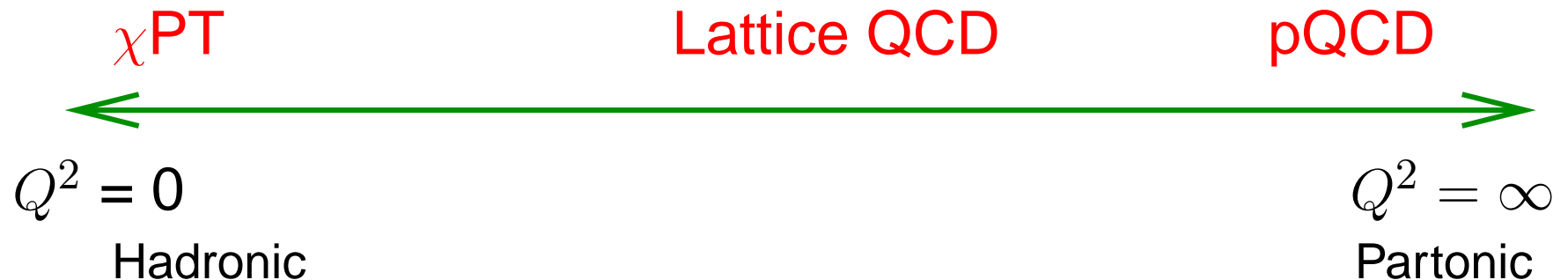
$$S_1(Q^2) = \frac{8}{Q^2} \int_0^1 g_1(x, Q^2) dx = \frac{8}{Q^2} \Gamma_1(Q^2)$$

X.-D. Ji and J. Osborne, J. Phys. **G27**, 127 (2001)

At  $Q^2 = 0$ , the **GDH sum rule is recovered**.

# Importance of the Generalized GDH Sum Rule

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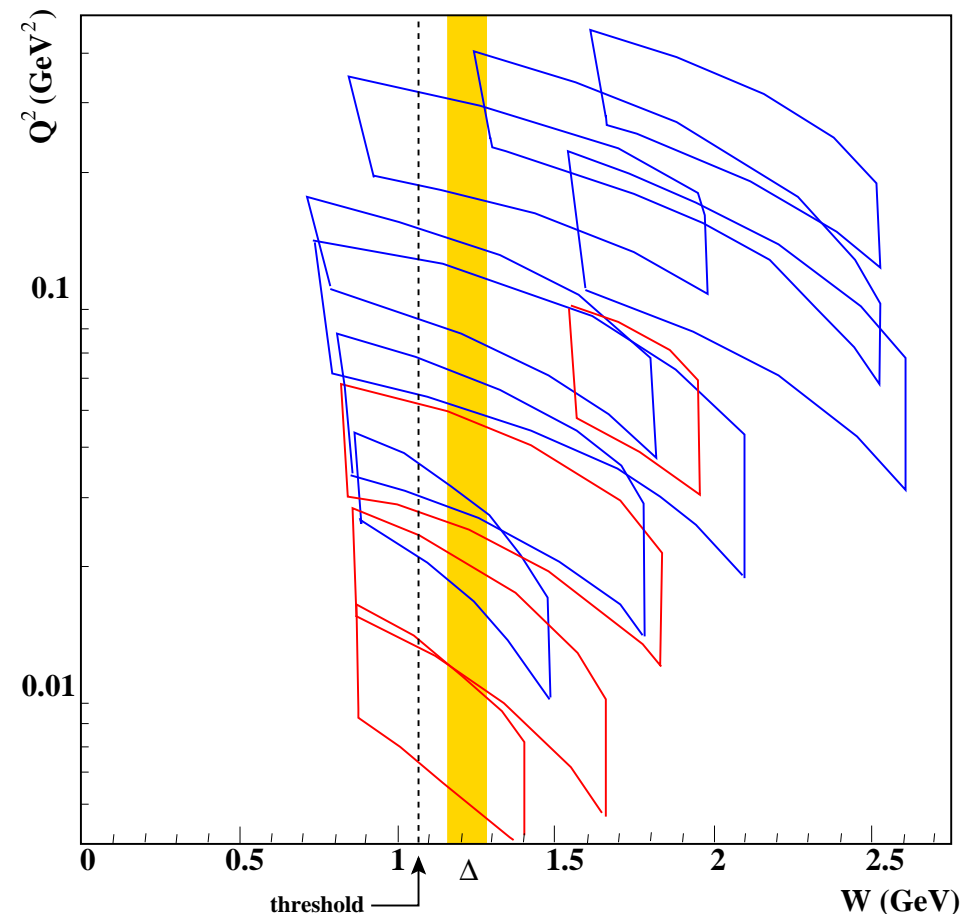


- Constrained at the two ends of the  $Q^2$  spectrum by known sum rules.
- $S_1$  and  $S_2$  are **calculable at any  $Q^2$** .
- Compare theoretical predictions to experimental measurements over the **entire  $Q^2$  range**.
- Provides a bridge from the **non-perturbative region** to the **perturbative region of QCD**.

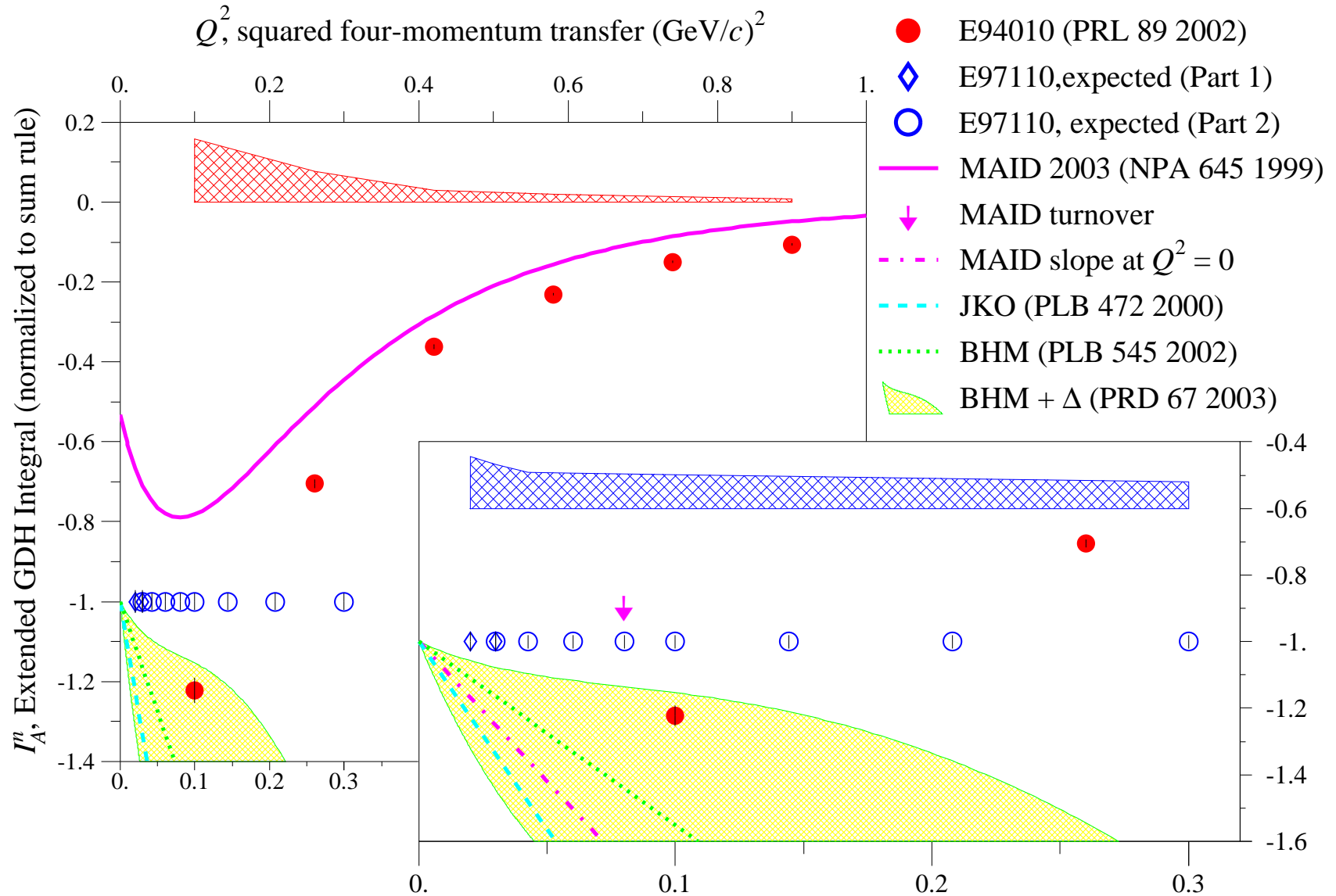
# Experiment E97-110

Precise measurement of **generalized GDH integral** at low  $Q^2$ , 0.02 to 0.3  $\text{GeV}^2$

- Ran in spring and summer 2003
- Inclusive experiment:  ${}^3\text{He}(\vec{e}, e')X$ 
  - ⇒ Scattering angles of  $6^\circ$  and  $9^\circ$
  - ⇒ Polarized electron beam:  
 $65\% < P_{\text{beam}} < 78\%$
  - ⇒ Pol.  ${}^3\text{He}$  target (para & perp):  
 $\langle P_{\text{targ}} \rangle = 40\%$
- Measured polarized cross-section differences



# Expected Neutron Results





# Analysis Progress

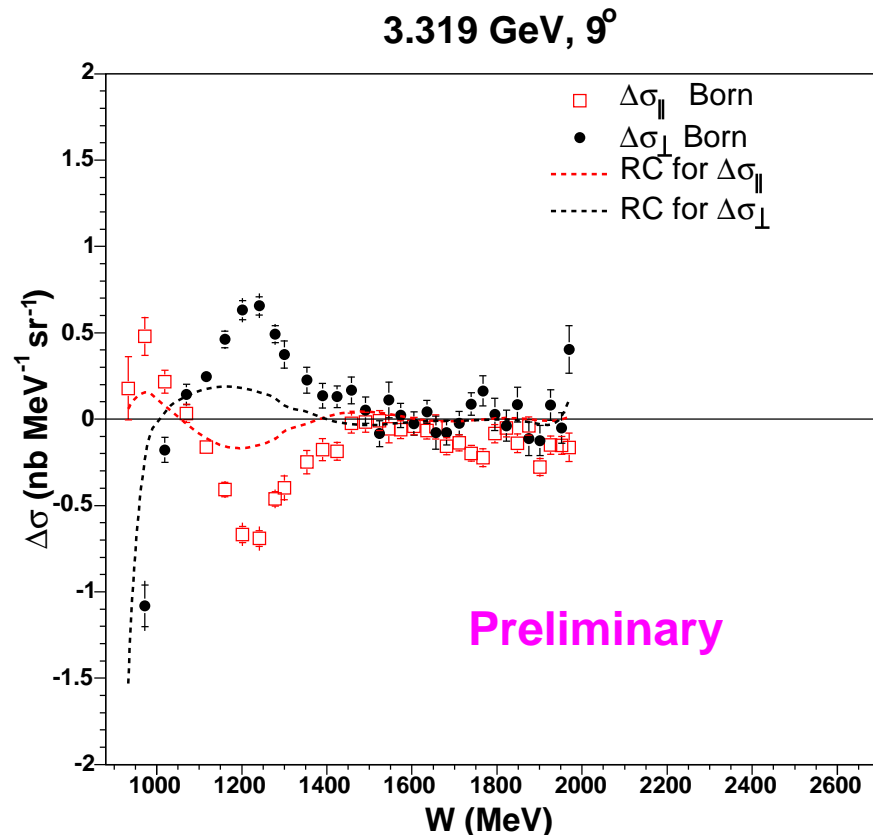
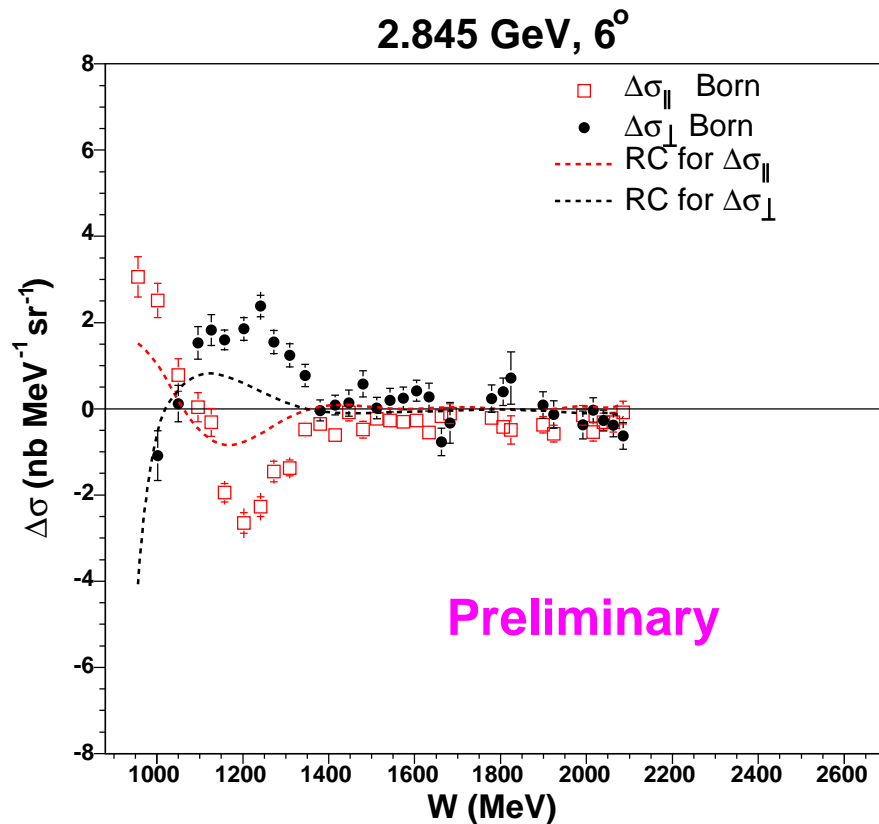
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- Preliminary asymmetries and unpolarized cross sections have been generated.
- Issues that still need to be addressed:
  - Beam polarization: **check bleedthrough correction with Compton where available.**
  - Target polarization: **track down 15% relative difference between NMR and EPR calibrations.**
  - Acceptance: **some issues need to be worked out, especially for 3.775 GeV 9° data.**
  - Collimator background.
  - Elastic analysis as a cross check of systematics.

# Systematic Uncertainties

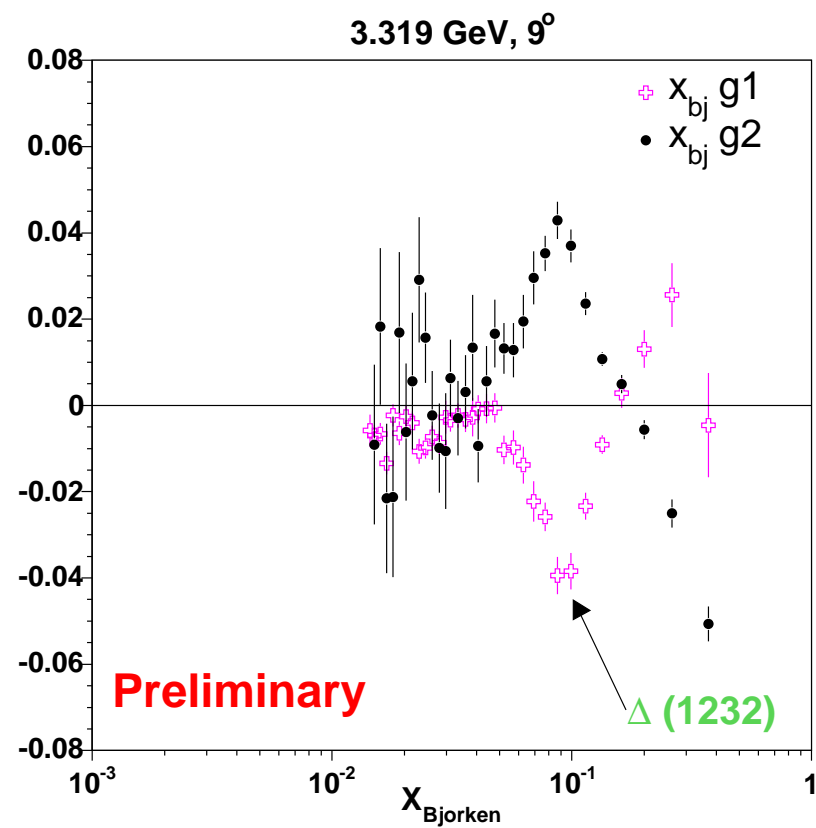
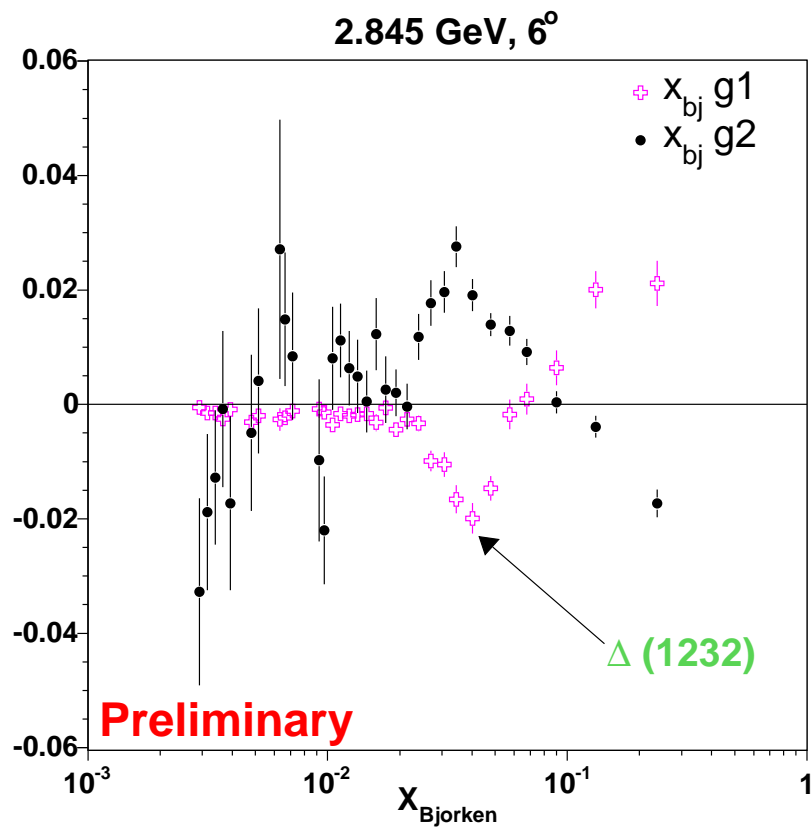
Source	Systematic Uncertainty		
	6°	9°	3.775 GeV, 9°
Angle	6°	9°	3.775 GeV, 9°
Target density		2.0%	
Acceptance/Effects	5.0%	5.0%	15.0%
VDC efficiency	3.0%	2.5%	2.5%
Charge		1.0%	
PID Detector and Cut effs.		< 1.0%	
$\delta\sigma_{\text{raw}}$	6.4%	6.2%	15.5%
Nitrogen dilution		0.2–0.5%	
$\delta\sigma_{\text{exp}}$	6.5%	6.3%	15.5%
Beam Polarization		3.5%	
Target Polarization		7.5%	
Radiative Corrections		5–10% in $\Delta$ region	
Total on $\Delta\sigma$	11.6–14.5%	11.5–14.4%	18.3–20.2%

# Cross Section Differences

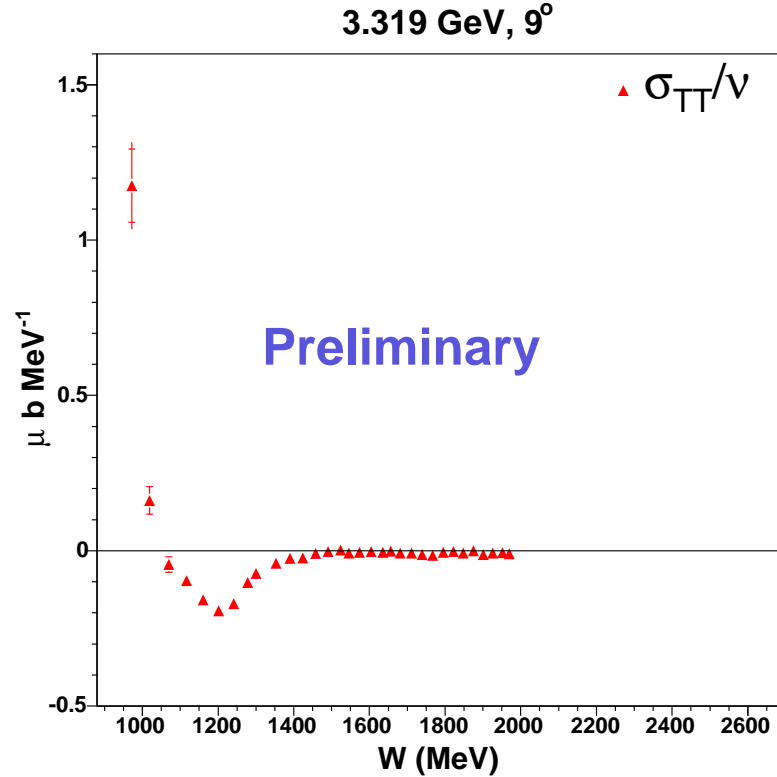
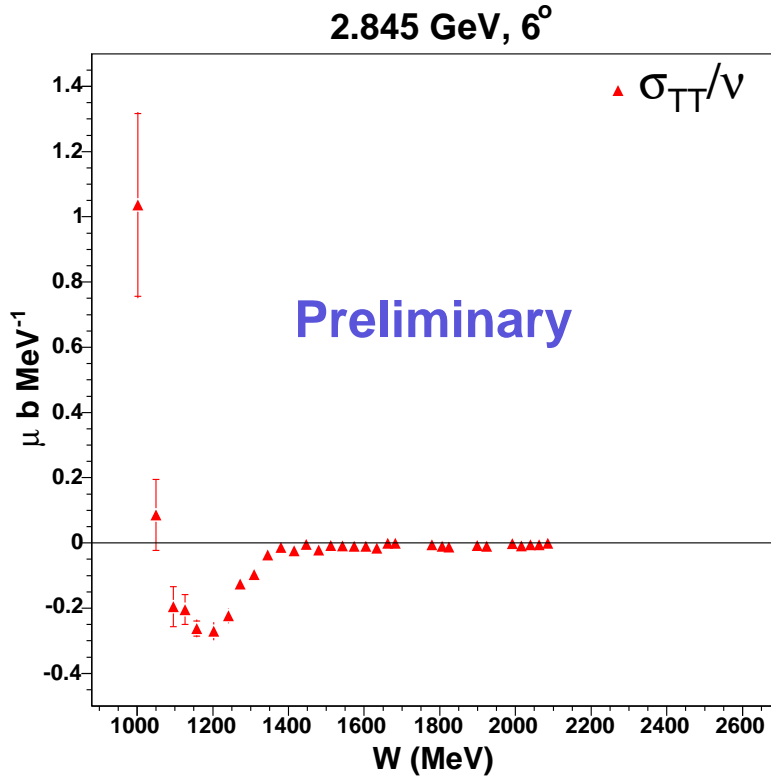


Radiative corrections: formalism of L. Mo and Y. Tsai (unpolarized) and POLRAD (polarized), work done by J. Singh.

# $^3\text{He}$ Spin Structure Functions



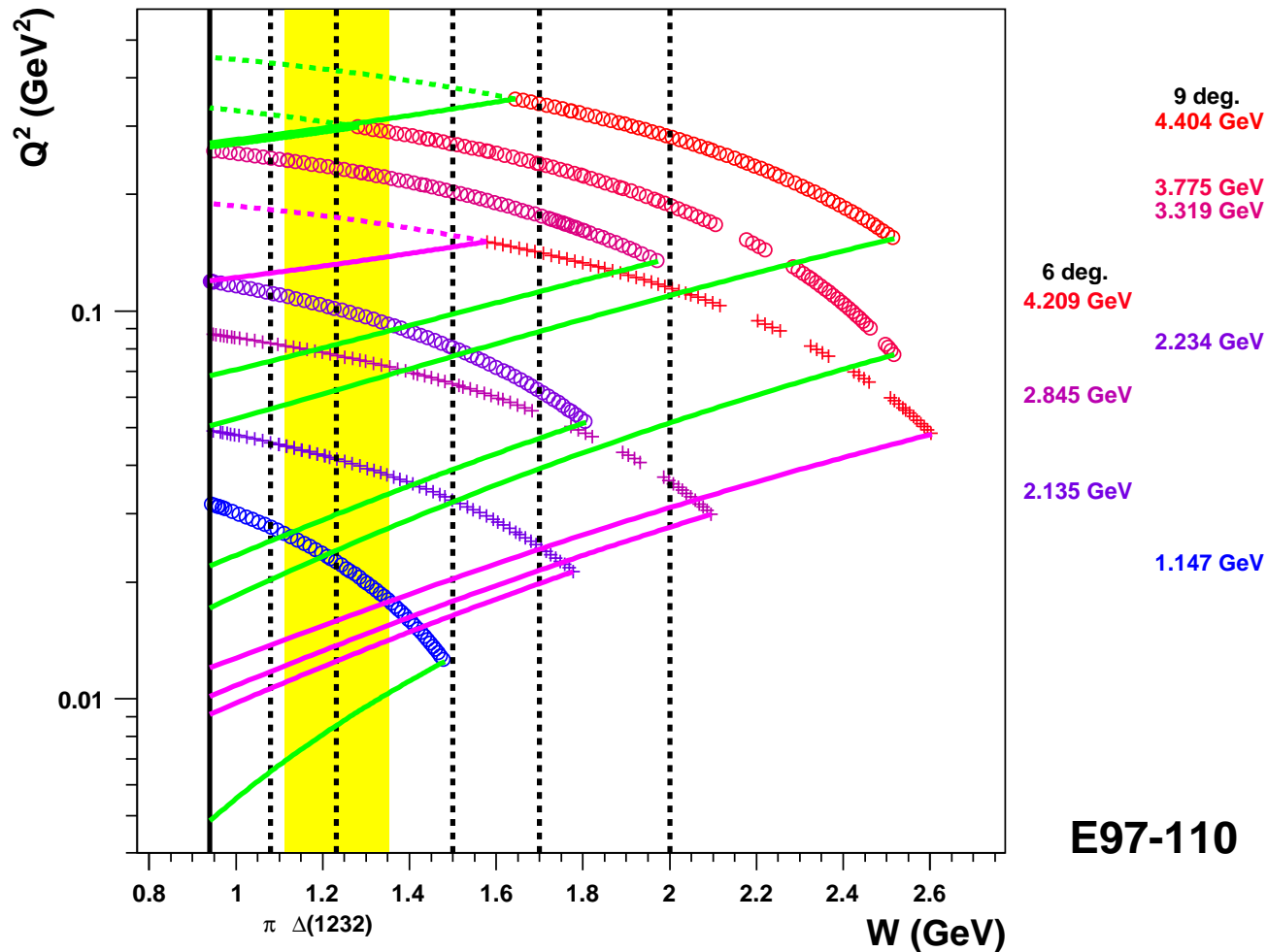
# The GDH Integrand: $\sigma_{\text{TT}}$



$$I(Q^2) = \int_{\nu_{\text{th}}}^{\infty} \frac{2\sigma_{\text{TT}}}{\nu} d\nu; \quad 2\sigma_{\text{TT}} = \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)$$

$$\sigma_{\text{TT}} = \frac{4\pi^2\alpha}{MK} \left[ g_1(\nu, Q^2) - \left( \frac{Q^2}{\nu^2} \right) g_2(\nu, Q^2) \right]$$

# Kinematic Coverage and Interpolation



Six evenly spaced points 0.04–0.24 GeV<sup>2</sup> with steps of 0.04 GeV<sup>2</sup>.

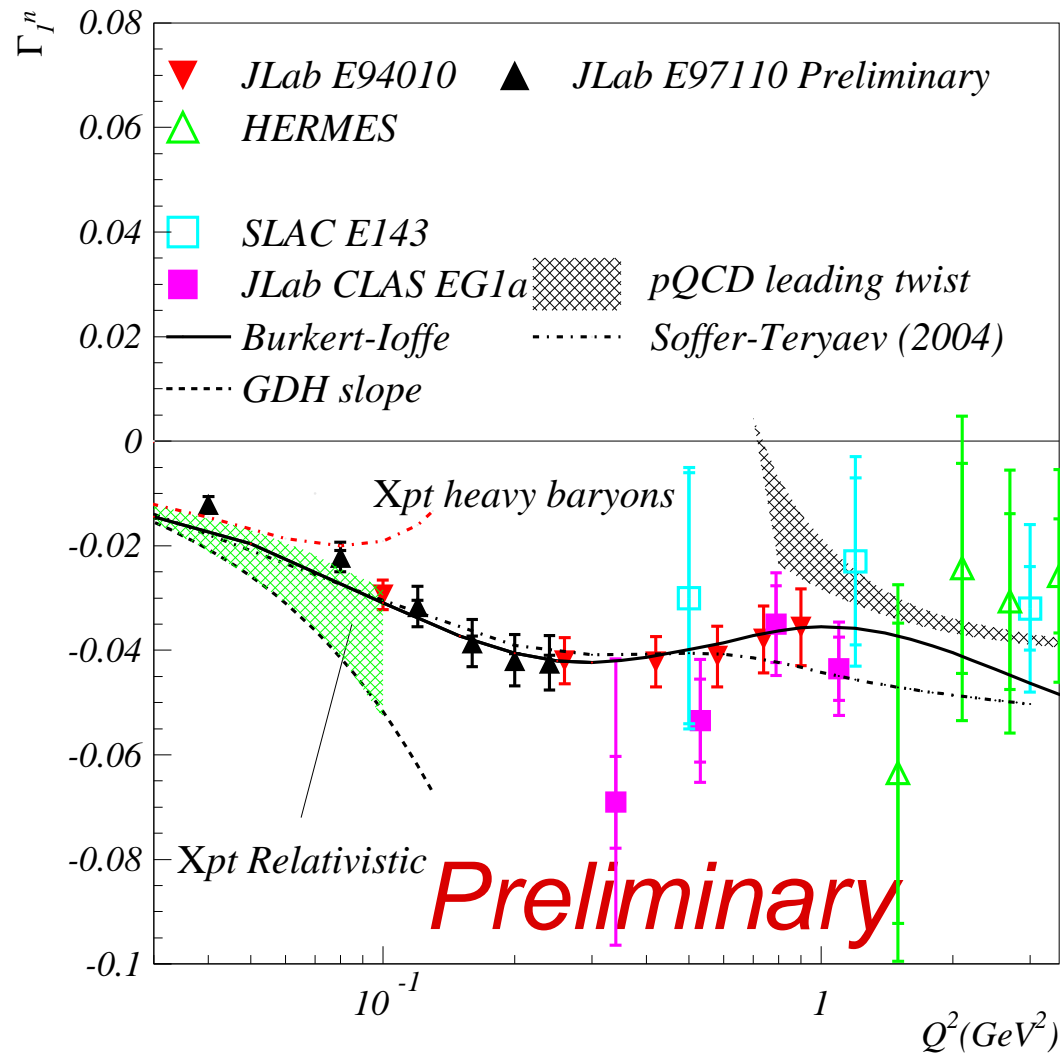
# Constant $Q^2$ Interpolation and Integral Extraction

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Procedure:

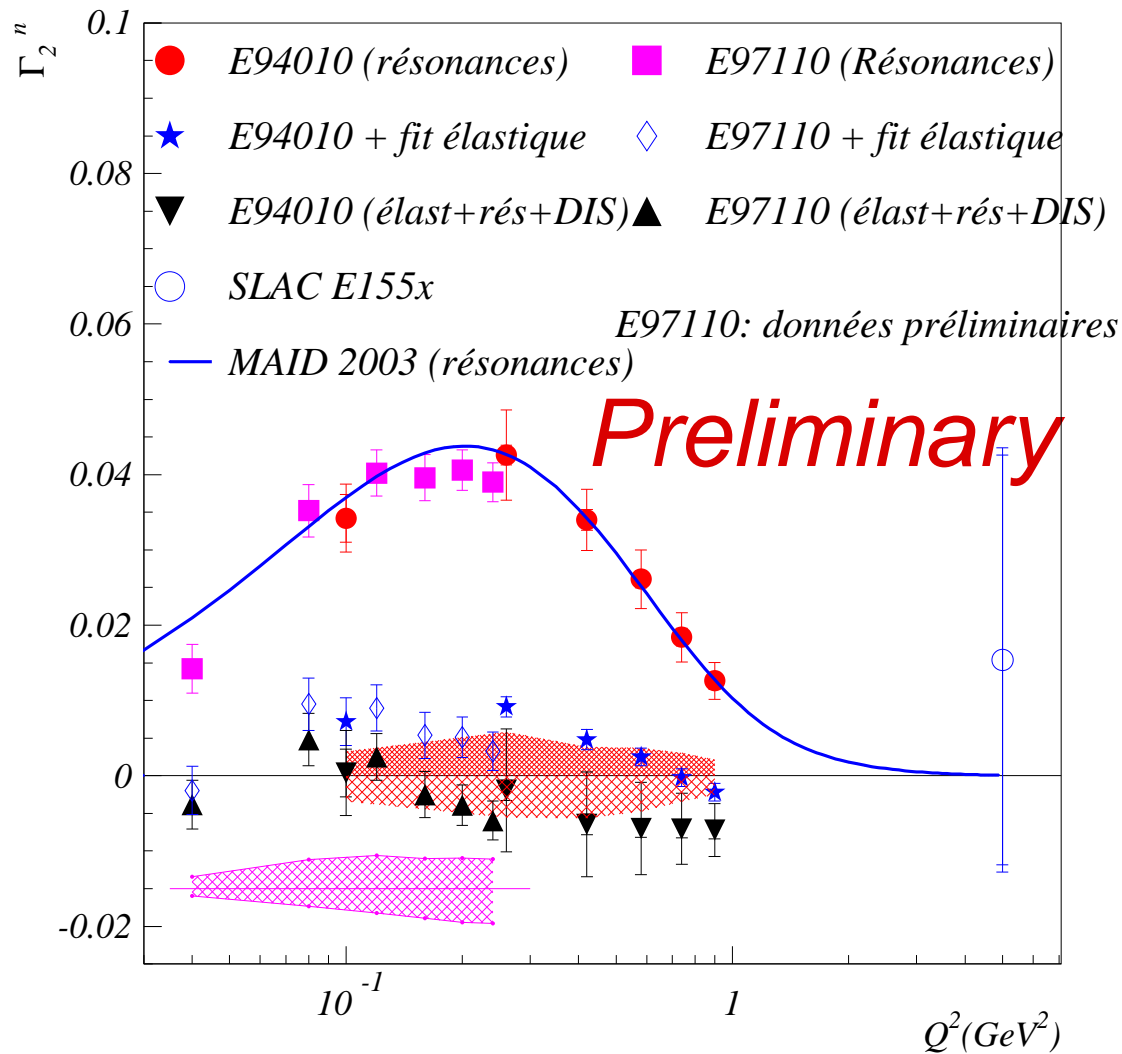
- First interpolate to constant  $W$  for each energy.
- Second interpolation with respect to  $Q^2$ .
- Integrals formed from  $W = 1073$  GeV to 2000 GeV.
- We could use our own data above  $W = 2000$  GeV.
- DIS contribution included up to  $W = \sqrt{1000}$  using Thomas and Bianchi parameterization.
- Neutron extraction performed using calculation from Scopetta and Ciofi degli Atti paper for  $Q^2 \geq 0.1$  GeV<sup>2</sup>.
- $Q^2 < 0.1$  GeV<sup>2</sup> use effective polarization technique (difference  $\sim$  5–10%).

# $\Gamma_1^n$ : First Moment of $g_1$





# $\Gamma_2^n$ : First Moment of $g_2$



# What Needs to be Done

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- Check and refine constant  $Q^2$  interpolation and integral extraction (V. Sulkosky).
- Second pass radiative corrections (J. Singh).
- Model and subtract QE contribution (V. Sulkosky).
- Collimator background (T. Holmstrom).
- Finalize target polarization (J. Singh).
- Elastic analysis (J. Singh, V. Sulkosky).
- Finalize acceptance for cross sections (V. Sulkosky).

# Summary and Conclusion

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- The GDH integral is an important tool that can be used to study nucleon spin structure over the full  $Q^2$  range.
- E97-110 provides precision data for the **generalized GDH integral at low  $Q^2$** , 0.02 to 0.3  $\text{GeV}^2$
- Preliminary results of the  **$^3\text{He}$  structure functions** and the **GDH integrand** are available.
- Extractions of the GDH integral and moments of the spin structure functions are in progress.
- These data allow us to **check  $\chi\text{PT}$  at very low  $Q^2$** .
- Final results available in about 6 months, then publications.

# The E97-110 Collaboration

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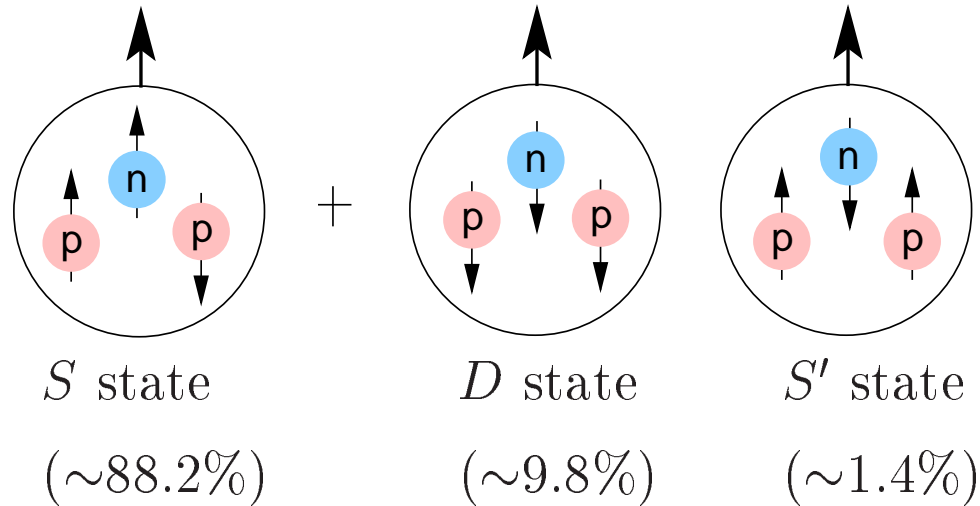
S. Abrahamyan, K. Aniol, D. Armstrong, T. Averett, S. Bailey,  
P. Bertin, W. Boeglin, F. Butaru, A. Camsonne, G.D. Cates,  
G. Chang, **J.P. Chen**, Seonho Choi, E. Chudakov, L. Coman,  
J. Cornejo, B. Craver, F. Cusanno, R. De Leo, C.W. de Jager,  
**A. Deur**, K.E. Ellen, R. Feuerbach, M. Finn, S. Frullani,  
K. Fuoti, H. Gao, **F. Garibaldi**, O. Gayou, R. Gilman,  
A. Glamazdin, C. Glashausser, J. Gomez, O. Hansen, D. Hayes,  
B. Hersman, D. W. Higinbotham, T. Holmstrom, T.B. Humensky,  
C. Hyde-Wright, H. Ibrahim, M. Iodice, X. Jiang, L. Kaufman,  
A. Kelleher, W. Kim, A. Kolarkar, N. Kolb, W. Korsch,  
K. Kramer, G. Kumbartzki, L. Lagamba, G. Laveissiere,  
J. LeRose, D. Lhuillier, R. Lindgren, N. Liyanage, B. Ma,  
D. Margaziotis, P. Markowitz, K. McCormick, Z.E. Meziani,  
R. Michaels, B. Moffit, P. Monaghan, S. Nanda, J. Niedziela,  
M. Niskin, K. Paschke, M. Potokar, A. Puckett, V. Punjabi,  
Y. Qiang, R. Ransome, B. Reitz, R. Roche, A. Saha, A. Shabetai,  
**J. Singh**, S. Sirca, K. Slifer, R. Snyder, P. Solvignon, R. Stringer,  
R. Subedi, **V. Sulkosky**, W.A. Tobias, P. Ulmer, G. Urciuoli,  
A. Vacheret, E. Voutier, K. Wang, L. Wan, B. Wojtsekhowski,  
S. Woo, H. Yao, **J. Yuan**, X. Zheng, L. Zhu

and the Jefferson Lab Hall A Collaboration

# *Extra Slides*

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# <sup>3</sup>He as an Effective Polarized Neutron Target



$$P_n = 86\% \text{ and } P_p = -2.8\%$$

J.L. Friar *et al.*, PRC **42**, (1990) 2310

## Extraction of Neutron Results

$$\Gamma_1^n(Q^2) = \frac{1}{P_n} \left[ \Gamma_1^{3\text{He}}(Q^2) - 2P_p \Gamma_1^p(Q^2) \right]$$

C. Ciofi degli Atti & S. Scopetta, PLB **404**, (1997) 223

# Inclusive Cross Sections

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- Unpolarized cross sections

$$\frac{d^2\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right]$$

- Polarized cross sections

$$\Delta\sigma_{\parallel} = \frac{d^2\sigma^{\downarrow\uparrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\uparrow}}{dE'd\Omega} = K \left[ (E + E' \cos \theta) g_1(x, Q^2) - \left( \frac{Q^2}{\nu} \right) g_2(x, Q^2) \right]$$

$$\Delta\sigma_{\perp} = \frac{d^2\sigma^{\downarrow\Rightarrow}}{dE'd\Omega} - \frac{d^2\sigma^{\uparrow\Rightarrow}}{dE'd\Omega} = K E' \sin \theta \left[ g_1(x, Q^2) + \frac{2E}{\nu} g_2(x, Q^2) \right]$$
$$K = \frac{4\alpha^2}{M\nu Q^2} \frac{E'}{E}$$

↓, ↑ are for electron spin  
↑↑, ⇒ are for target spin direction  
 $F_1, F_2, g_1, g_2$ : **structure functions**

# Spin- $\frac{1}{2}$ Targets

$$I_{\text{GDH}} = \int_{\nu_{\text{th}}}^{\infty} \frac{\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)}{\nu} d\nu = -2\pi^2 \alpha \left( \frac{\kappa}{M} \right)^2$$

The sum rule is **valid for any target**.

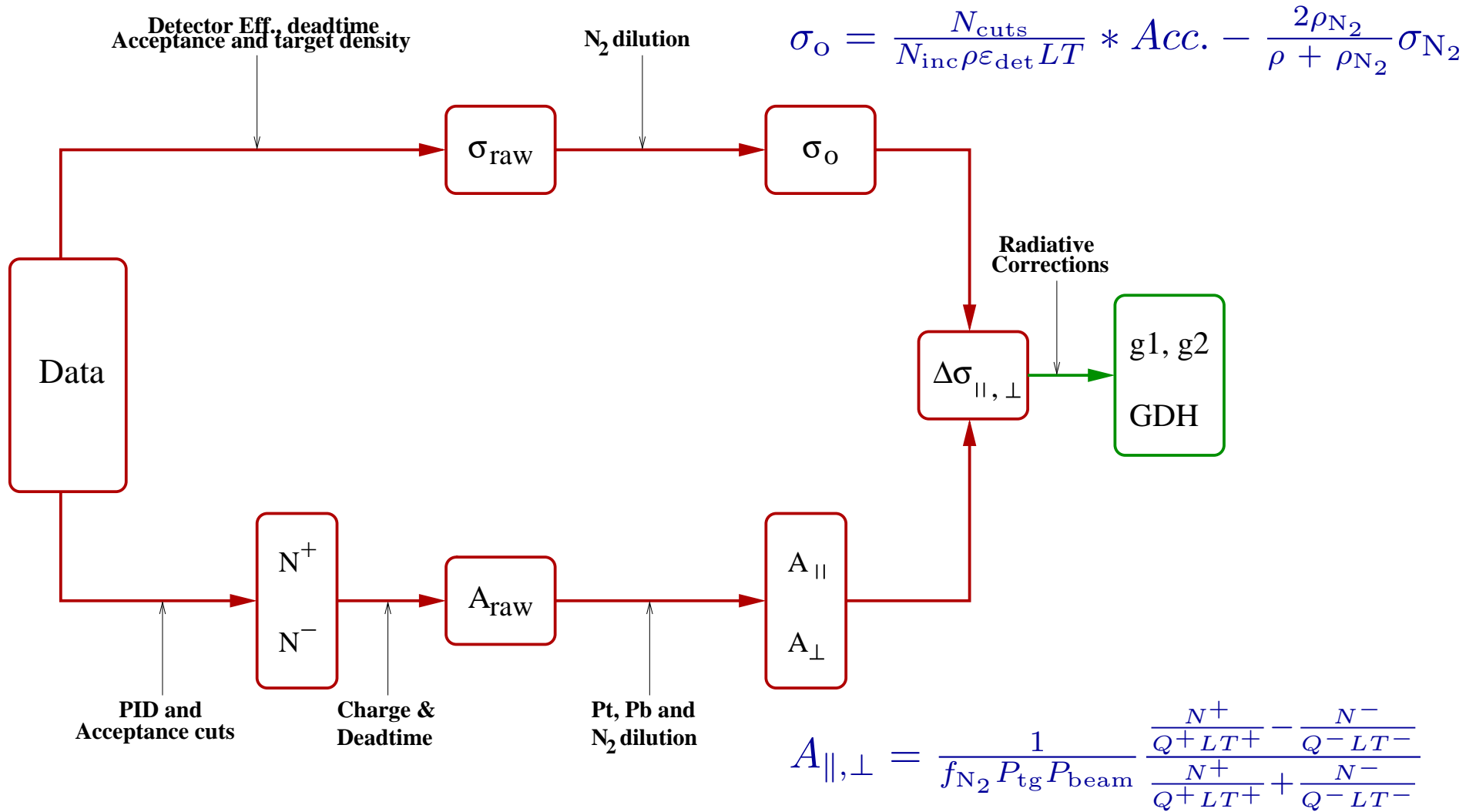
	$M$ [GeV]	Spin	$\kappa$	$I_{\text{GDH}}$ [ $\mu$ b]
Proton	0.938	$\frac{1}{2}$	1.79	-204.8
Neutron	0.940	$\frac{1}{2}$	-1.91	-233.2
Deuteron	1.876	1	-0.14	-0.65
Helium-3	2.809	$\frac{1}{2}$	-8.38	-498.0

$$1 \mu\text{b} = 10^{-34} \text{ m}^2$$

- Proton sum rule was verified to  $\sim 10\%$ , Mainz and Bonn.
- Measurements for the **neutron** are in progress.



# Analysis Procedure



# Chiral Symmetry

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$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} G_{\mu\nu}^\alpha G_{\alpha}^{\mu\nu} + \bar{q} i \gamma^\mu D_\mu q - \bar{q} \mathcal{M} q$$
$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_0 + \mathcal{L}_{sb}$$

- Consider the limit where the light quark masses vanish.
- For massless fermions, chirality (handedness) is identical to a particle's helicity.
- Extra symmetry to the Lagrangian and obtain left and right handed quark fields.

$$q_{L,R} = \frac{1}{2}(1 \mp \gamma_5)q,$$

# *GDH Derivation*

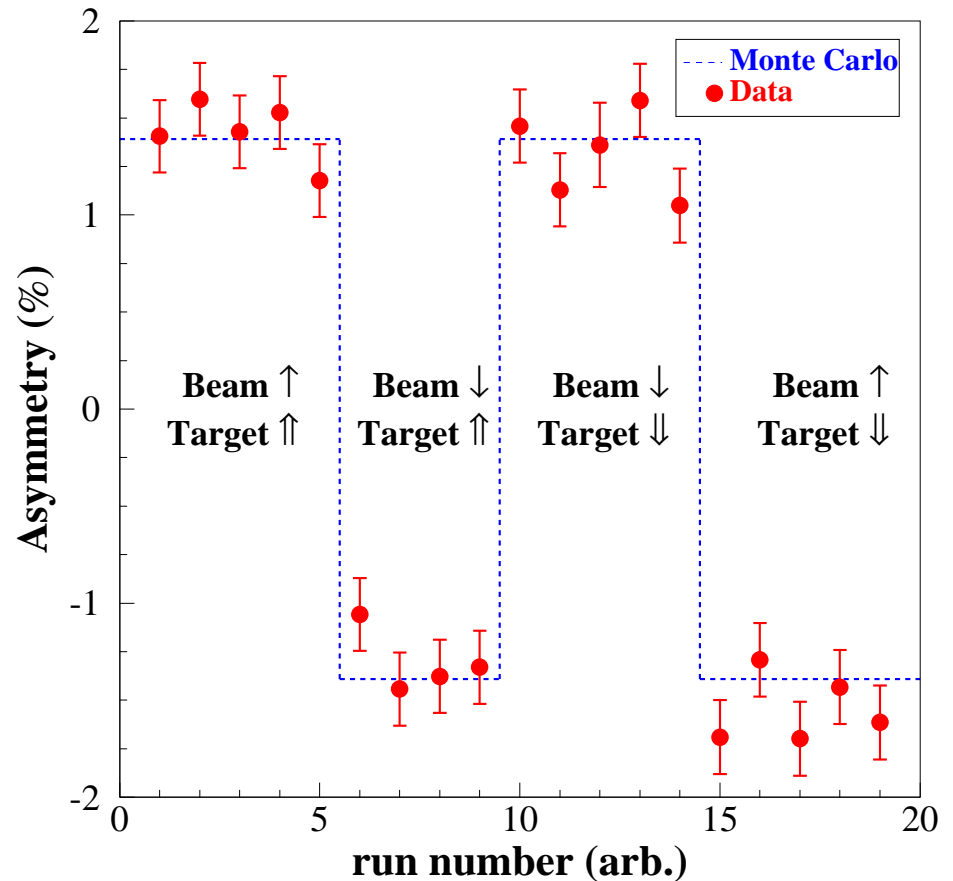
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Based on fundamental physical arguments

- Lorentz and gauge invariance: low energy theorem, Phys. Rev. **96**, 1428 (1954).
- Unitarity of the S-matrix: optical theorem.
- Causality: dispersion relations for forward compton scattering.

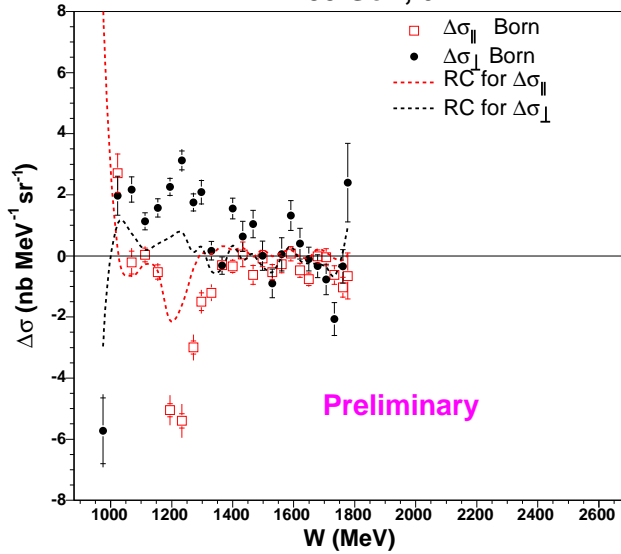
# $^3\text{He}$ Elastic Asymmetry

- Monte Carlo prediction: 1.390%
- Preliminary data analysis:  
 $(1.403 \pm 0.044)\%$  (stat. only)  
 $\chi^2/N_{\text{dof}} = 1.08.$
- Four target and beam configurations
- For **seven** out of the twelve beam energies, **elastic data** were acquired.

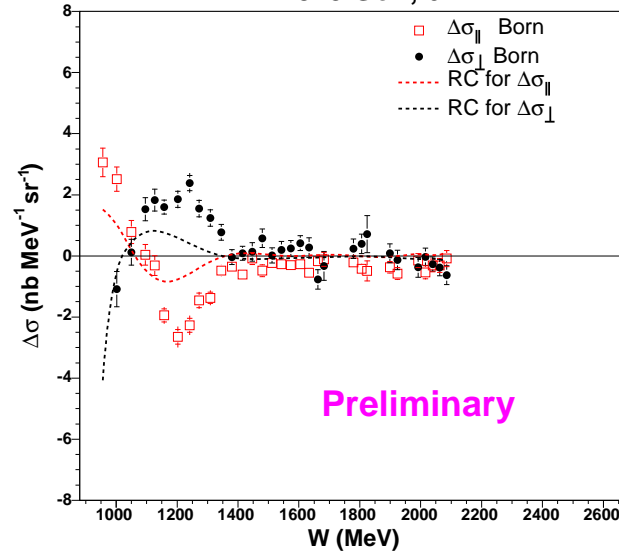


# Cross Section Differences

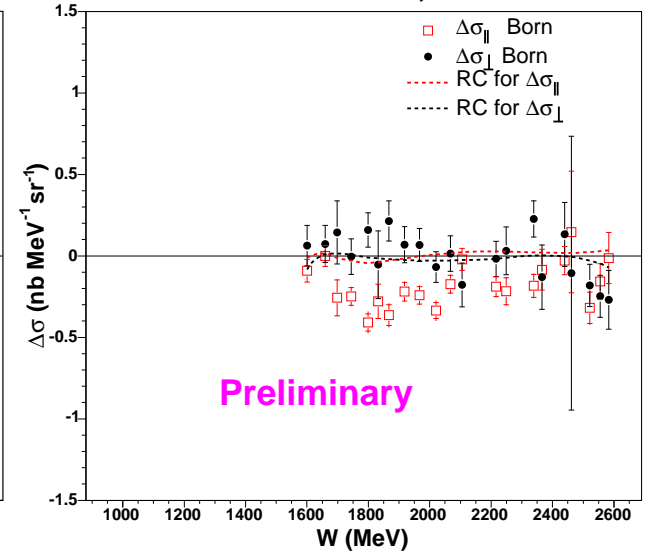
2.135 GeV, 6°



2.845 GeV, 6°

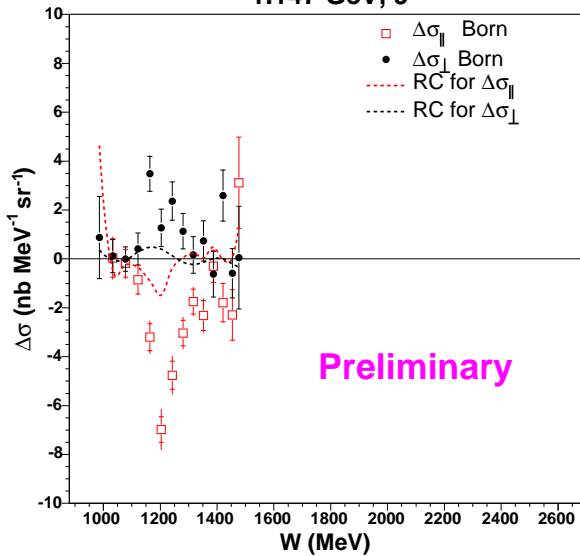


4.209 GeV, 6°

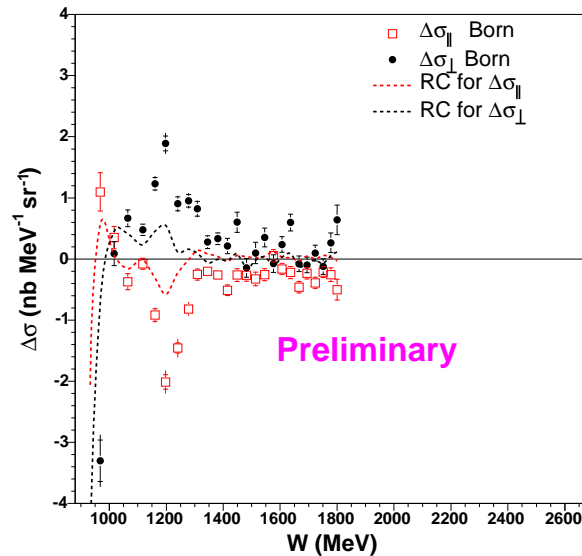


# Cross Section Differences

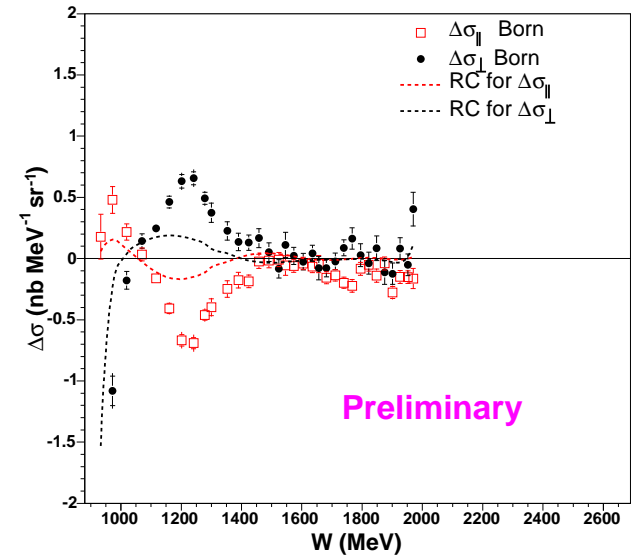
1.147 GeV, 9°



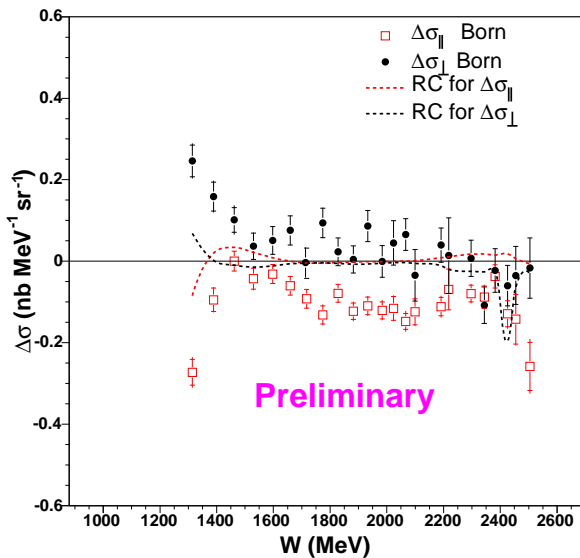
2.234 GeV, 9°



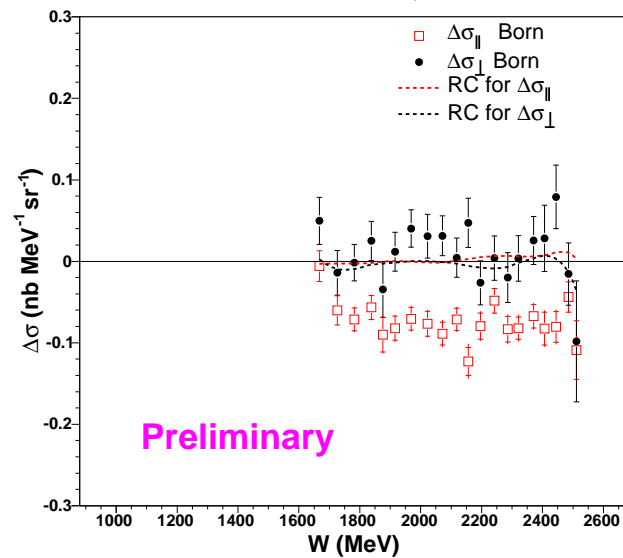
3.319 GeV, 9°



3.775 GeV, 9°

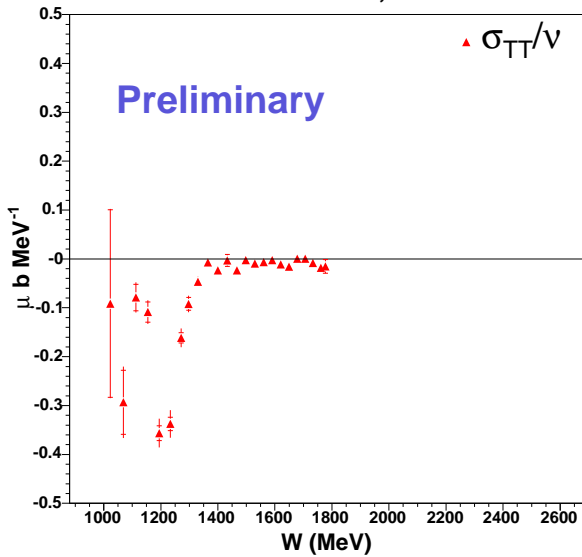


4.404 GeV, 9°

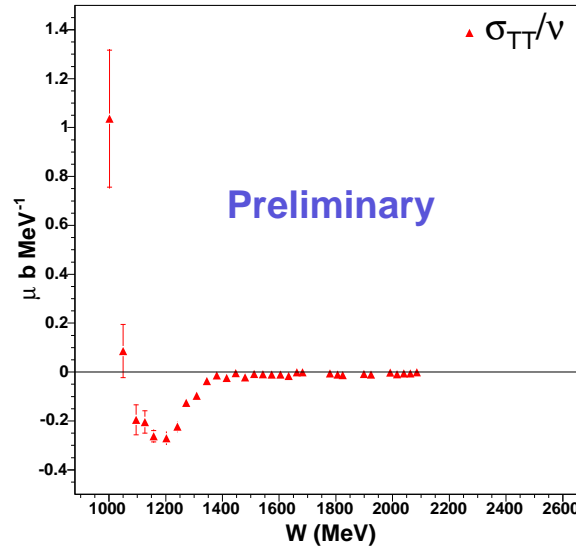


# The GDH Integrand: $\sigma_{TT}$

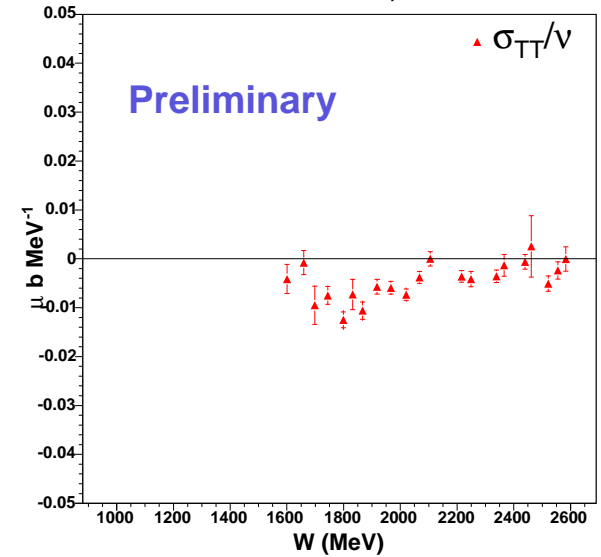
2.135 GeV,  $6^\circ$



2.845 GeV,  $6^\circ$



4.209 GeV,  $6^\circ$

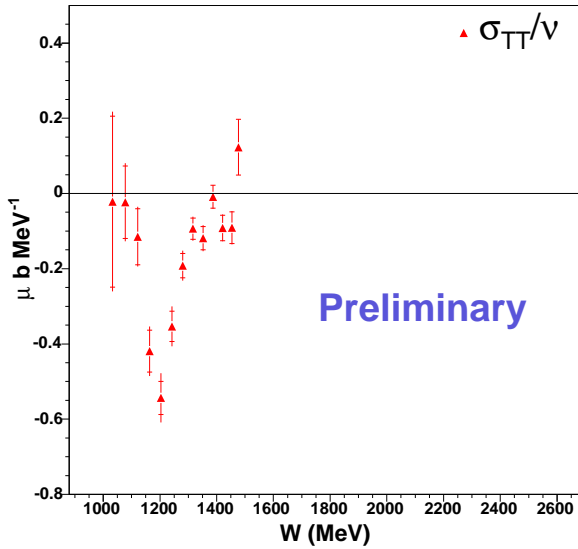


$$2\sigma_{TT} = \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)$$

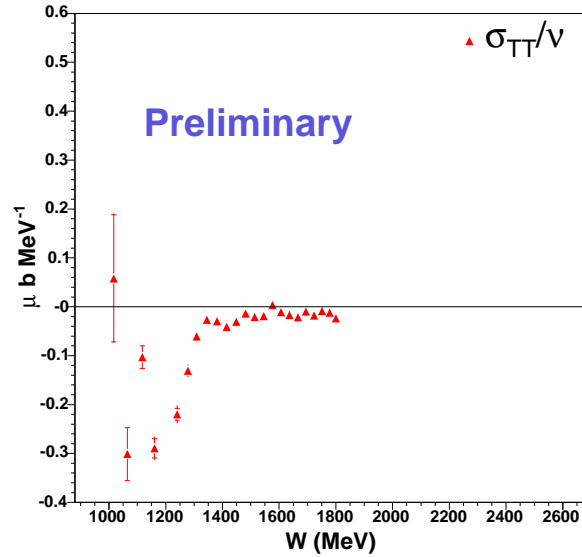
$$\sigma_{TT} = \frac{4\pi^2\alpha}{MK} \left[ g_1(\nu, Q^2) - \left( \frac{Q^2}{\nu^2} \right) g_2(\nu, Q^2) \right]$$

# The GDH Integrand: $\sigma_{TT}$

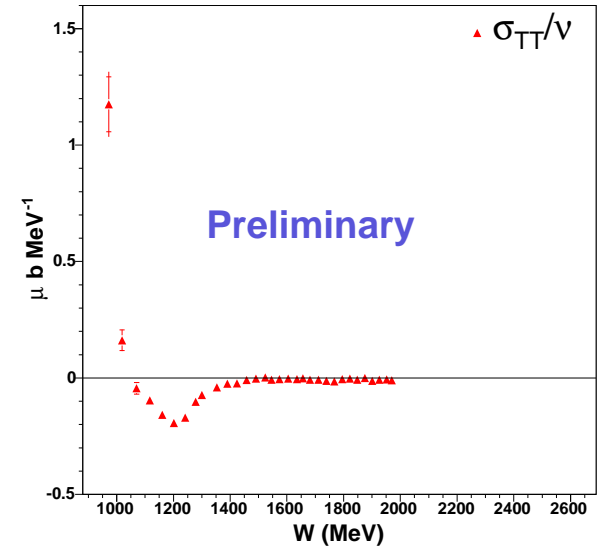
1.147 GeV, 9°



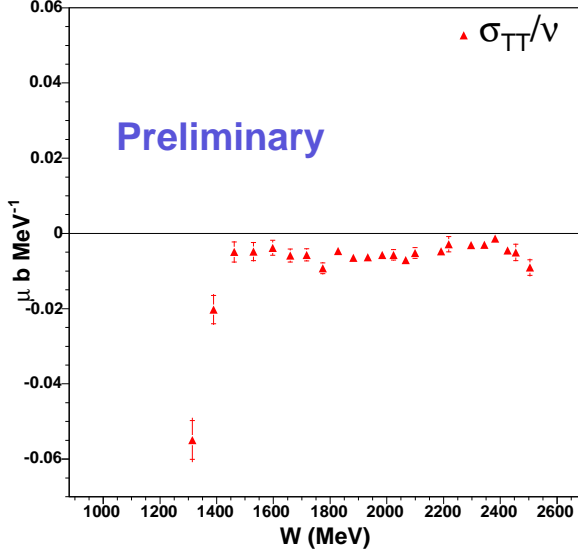
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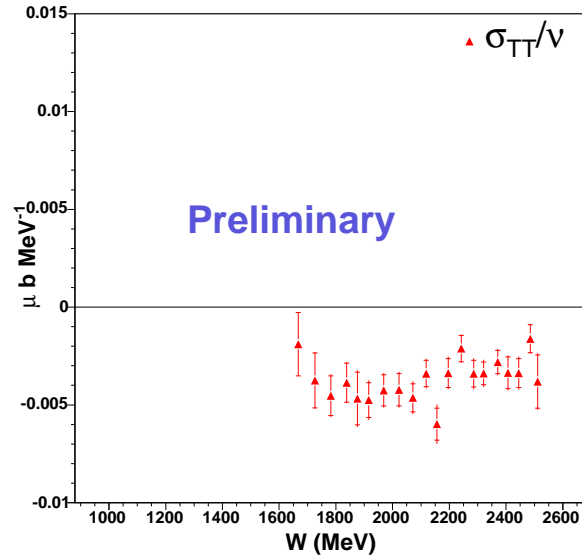
3.319 GeV, 9°



3.775 GeV, 9°



4.404 GeV, 9°



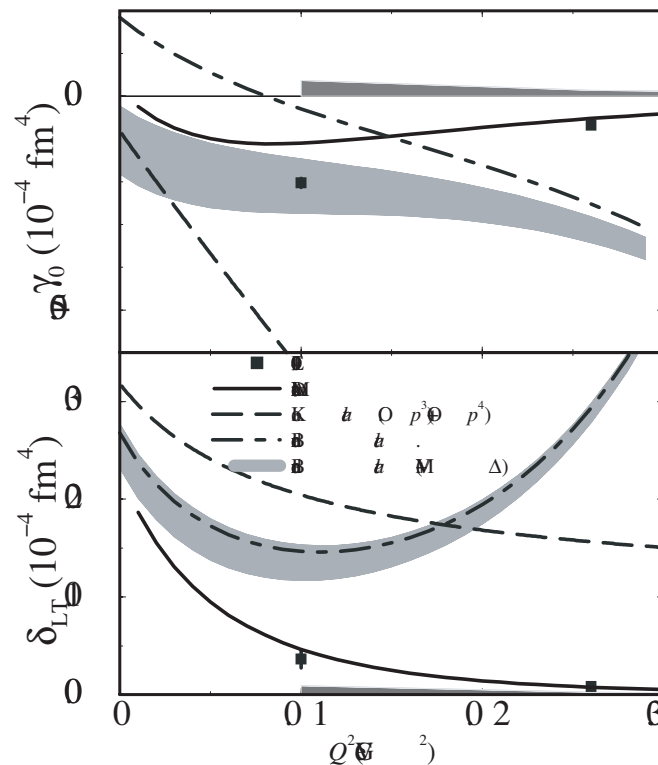
$$I(Q^2) = \int_{\nu_{th}}^{\infty} \frac{2\sigma_{TT}}{\nu} d\nu$$



# Spin Polarizabilities

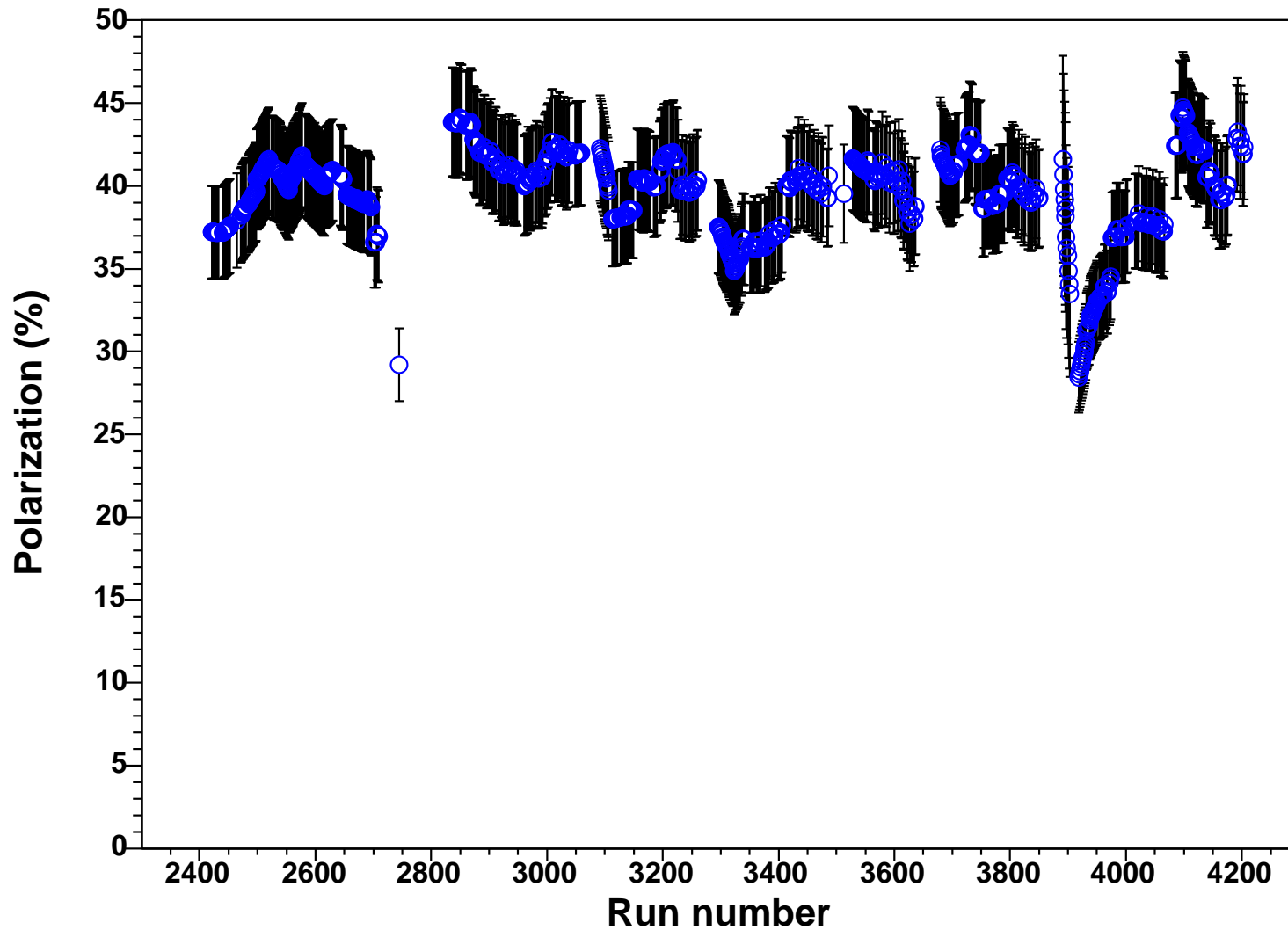
$$\gamma_0 = \frac{4e^2 M^2}{\pi Q^6} \int_0^{x_0} x^2 \left( g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right) dx$$

$$\delta_{LT} = \frac{4e^2 M^2}{\pi Q^6} \int_0^{x_0} x^2 (g_1 + g_2) dx$$

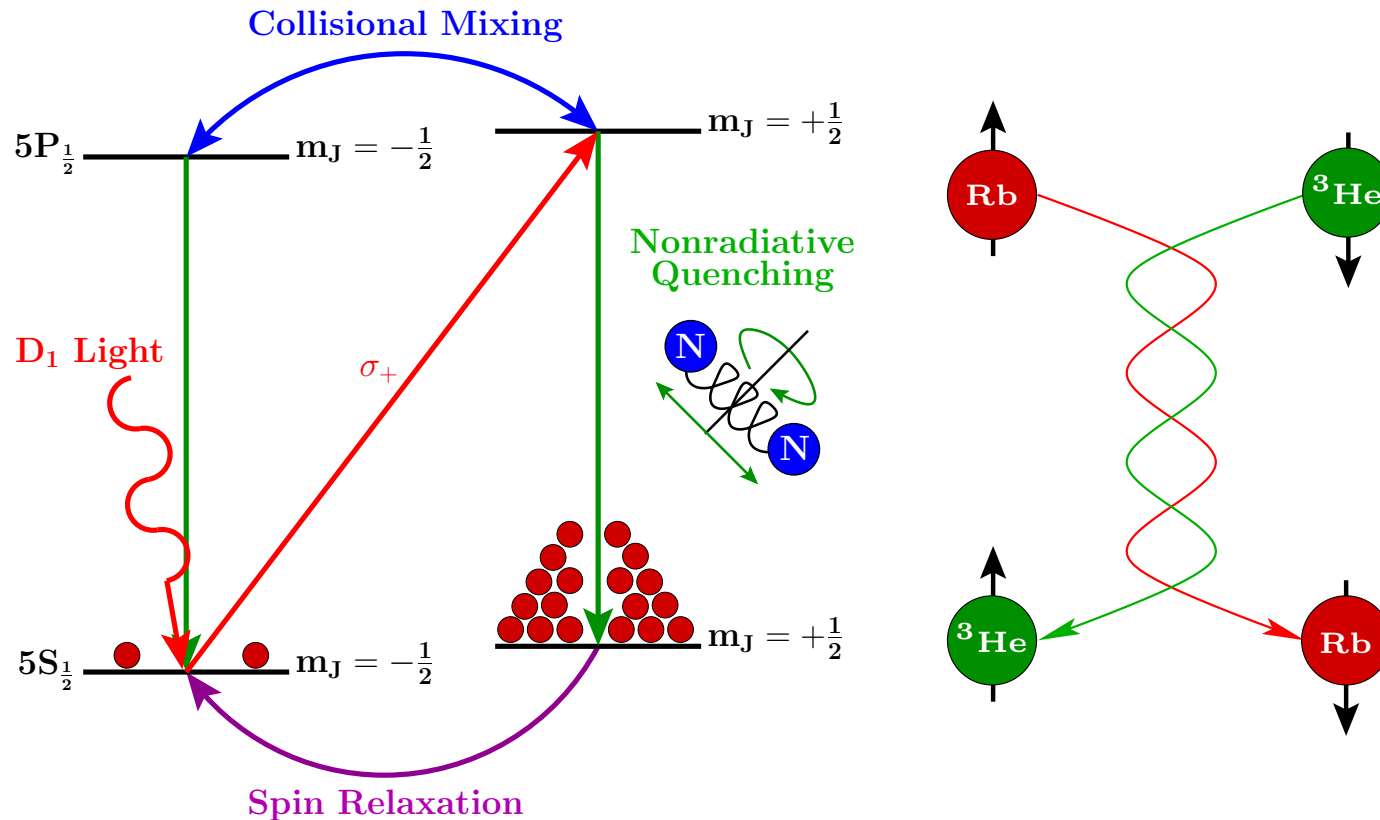


M. Amarian *et al.*, PRL **93**, 152301 (2004)

# Preliminary Target Polarization



# Spin Exchange Optical Pumping



$^3\text{He}$  nucleus is polarized via **spin-exchange** with **optically pumped** Rb atoms.