Nijmegen, August 1999
http://www.nat.vu.nl/~ mulders (Talks)

Structure of Hadrons in Hard Scattering Processes

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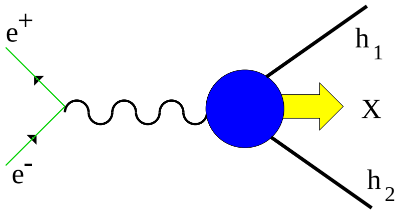
- Structure functions and what do we learn from them
  (Form factors, polarization, inclusive, semi-inclusive, distributions)
- Quark and gluon correlations
  (distribution and fragmentation functions)
- Examples of cross sections
- Specific topics

[1] collaborators: D. Boer, J. Rodrigues, A. Henneman, R. Jakob, M. Boglione, A. Bacchetta, O. Teryaev, E. Leader

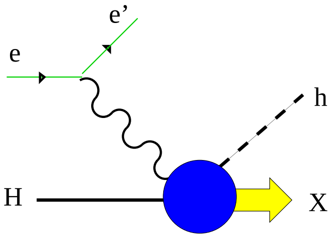
⇒ Excitation spectrum (invariant masses, lifetimes)
Gives information, e.g. via sum rules (indirect!)

⇒ Jets or specific particles (J/ψ)
. . . originate from specific quarks or gluons
(requires high energies, algorithms)

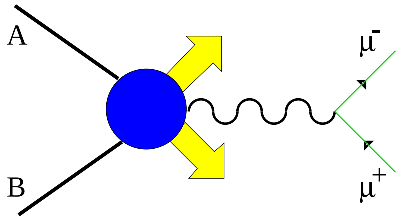
⇒ Electroweak processes (via color-blind (!) γ, Z^0, $W^\pm$)
. . . measure specific 'well-defined' quantities
(requires care, patience, polarization, . . .)



Electron-positron annihilation

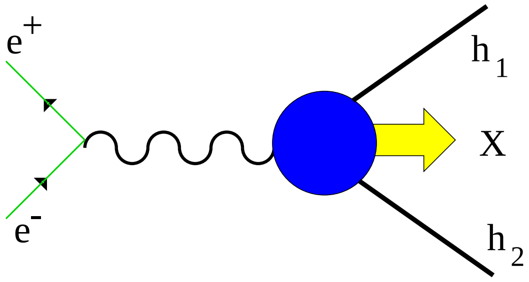


Lepton-hadron scattering



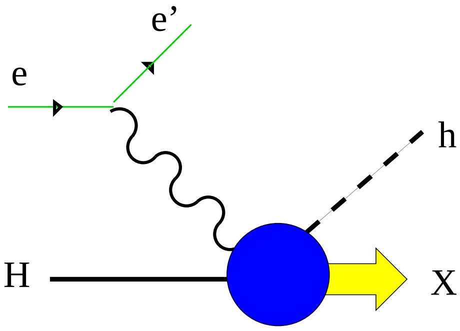
lepton pair production (Drell-Yan)

HARD ELECTROWEAK SCATTERING PROCESSES



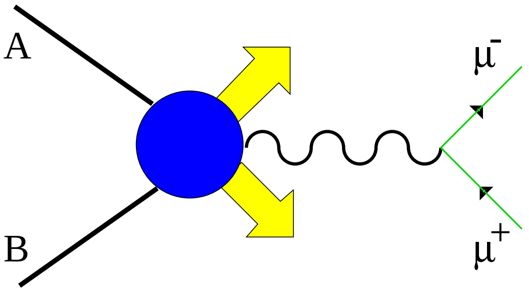
Electron-positron annihilation
inclusive: $e^+e^- \longrightarrow X$
1-particle incl.: $e^+e^- \longrightarrow h\,X$
2-particle incl.: $e^+e^- \longrightarrow h_1h_2\,X$

$$q^2 = (k+k')^2 \equiv Q^2$$



Lepton-hadron scattering
inclusive: $\ell H \longrightarrow \ell'\,X$
1-particle incl.: $\ell H \longrightarrow \ell' h\,X$

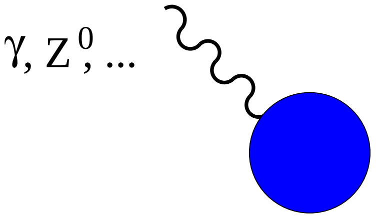
$$q^2 = (k-k')^2 \equiv -Q^2$$

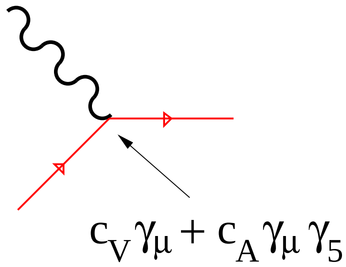


lepton pair production (Drell-Yan)
inclusive: $AB \longrightarrow \mu^+\mu^-\,X$

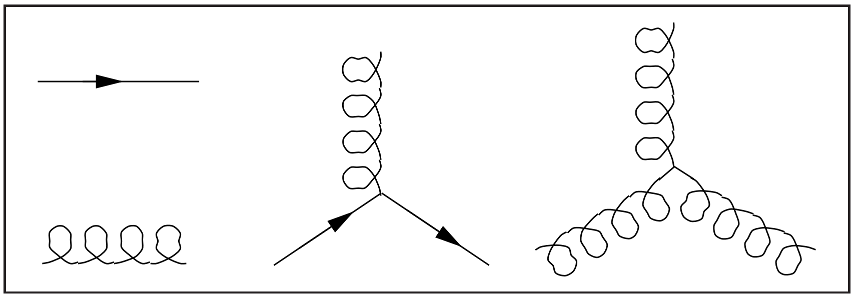
$$q^2 = (k+k')^2 \equiv Q^2$$

Electroweak interactions

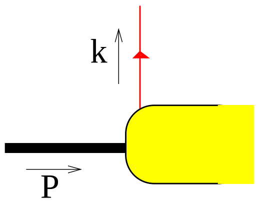




Quantum Chromodynamics

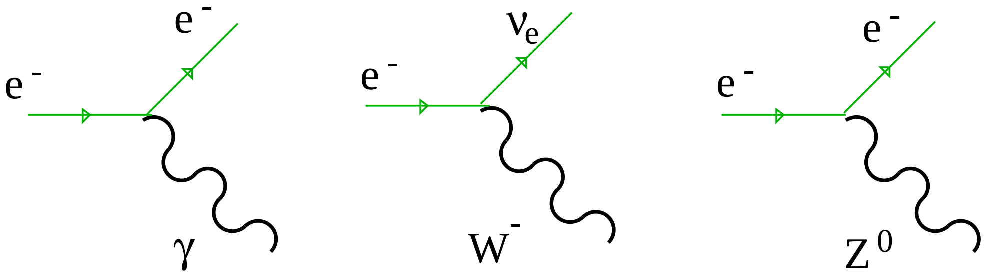


Structure of hadrons



Uncalculable for soft momenta
($k \cdot P \;\sim\; k^2 \;\sim\; P^2$)
$\Rightarrow$ Parametrization

Leptons interact with matter via the exchange of virtual particles:

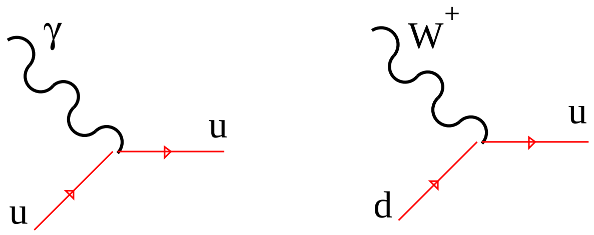


- probe = virtual (spacelike): $q^2 = -Q^2 \leq 0$

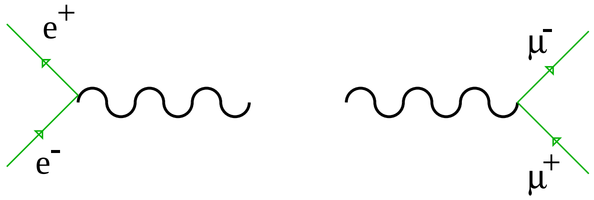
Proof:

$$\left.\begin{aligned} k &= (E, \boldsymbol{q}/2) \\ k' &= (E, -\boldsymbol{q}/2) \\ q = k - k' &= (0, \boldsymbol{q}) \end{aligned}\right\} \text{In Brick-wall frame}$$

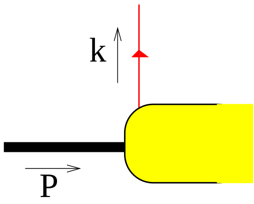
- electroweak probing of ... quarks

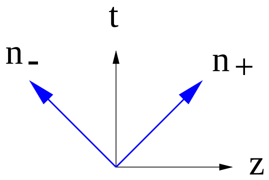


- Similarly timelike probing: $q^2 = Q^2 \geq 0$
  annihilation or creation of pairs



In a hard scattering process hadrons are about lightlike!





$$n_+^2 = n_-^2 = 0$$
$$n_+ \cdot n_- = 1$$

A hard scale (e.g. exchanged photon) provides naturally the lightlike directions such that P^+ can be considered large

$$P^\mu \equiv P^+\, n_+^\mu + \frac{M^2}{2P^+}\, n_-^\mu$$

$\uparrow$ collinear $\downarrow$ $\uparrow$ suppressed $\downarrow$

$$k^\mu \equiv xP^+\, n_+^\mu + k_T^\mu + \mathcal{O}\left(\frac{1}{P^+}\right)$$

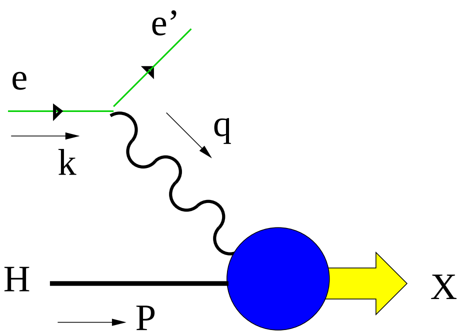
$\uparrow$ beyond collinearity

Similarly for spin:

$$S^\mu \equiv \lambda \frac{P^+}{M}\, n_+^\mu + S_T^\mu + \mathcal{O}\left(\frac{1}{P^+}\right)$$

$\ell H \longrightarrow \ell' X$

Scale : $-q^2 \equiv Q^2$ (Hard : $Q^2 \to \infty$)



$$x_B = \frac{Q^2}{2P \cdot q}$$
$$y = \frac{P \cdot q}{P \cdot k}$$

Hadronic tensor

$$\begin{aligned} 2M\, W_{\mu\nu}^{(\ell H)}(q; PS) \\ &= \frac{1}{(2\pi)} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q + P - P_X) \\ &\quad \times \langle PS | J_\mu(0) | P_X \rangle \langle P_X | J_\nu(0) | PS \rangle \end{aligned}$$

Inclusive scattering (unpolarized lepton and hadron, γ-exchange)[2]

$$2MW_S^{\mu\nu}(q,P) = \underbrace{\left(-g^{\mu\nu} + \hat{q}^\mu\hat{q}^\nu - \hat{t}^\mu\hat{t}^\nu\right)}_{-g_\perp^{\mu\nu}} F_1 + \hat{t}^\mu\hat{t}^\nu \underbrace{\left(\frac{F_2}{2x_{_B}} - F_1\right)}_{F_L}$$

Cross section

$$\frac{d\sigma_{OO}}{dx_{_B}\,dy} = \frac{4\pi\,\alpha^2\,x_{_B}\,s}{Q^4}\left\{\left(1 - y + \frac{1}{2}\,y^2\right) F_T + (1-y)\,F_L\right\}$$

Elastic limit and Parton model

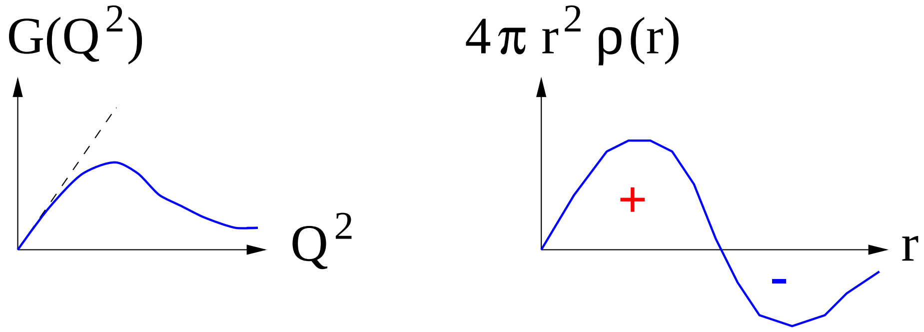
$$\begin{aligned} F_T(x_{_B},Q) &\overset{elastic}{=} G_M^2(Q^2)\,\delta(1-x_{_B}) \\ &\overset{DIS}{=} \frac{1}{2}\sum_a e_a^2\,f_1^a(x_{_B}) \\ F_L(x_{_B},Q) &\overset{elastic}{=} \frac{4M^2}{Q^2}\,G_E^2(Q^2)\,\delta(1-x_{_B}) \\ &\overset{DIS}{=} 0 \end{aligned}$$

[2]

$$\begin{aligned} \hat{q}^\mu &= \frac{q^\mu}{Q} \\ \hat{t}^\mu &= \frac{\tilde{P}^\mu}{\sqrt{\tilde{P}^2}} = \frac{1}{\sqrt{\tilde{P}^2}}\left(P^\mu - \frac{P\cdot q}{q^2}\,q^\mu\right) \end{aligned}$$

Form factors: charge and current distribution

Form factor of the neutron



Explanation 1: $\langle r^2 \rangle_u \ < \ \langle r^2 \rangle_d$

(unlikely)

Explanation 2: $n = n_{\text{bare}} + \underbrace{\ldots p_{\text{bare}}\pi^- + \ldots n_{\text{bare}}\pi^0}_{\text{isospin in the sea!}}$

$$\pi^- = d\bar{u} \ \Rightarrow \ n_{\bar{u}/n} > n_{\bar{d}/n} \ \Rightarrow \ n_{\bar{d}} > n_{\bar{u}}$$

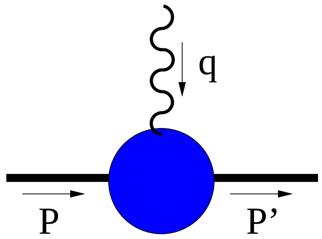
Structure functions: quark (and gluon) distributions

Gottfried sum rule

$$\begin{aligned} S_G &= \int dx\, \frac{F_2^p(x) - F_2^n(x)}{x} = \frac{1}{3}\,(n_u + n_{\bar{u}} - n_d - n_{\bar{d}}) \\ &= \frac{1}{3}\underbrace{(n_u - n_{\bar{u}} - n_d + n_{\bar{d}})}_{1} + \frac{2}{3}\,(n_{\bar{u}} - n_{\bar{d}}) < \frac{1}{3} \end{aligned}$$

ELASTIC SCATTERING $\Rightarrow$ FORM FACTORS

X = target hadron: $x_B = 1$



$$J_\mu^{(\gamma)} = \sum_q Q\, V_\mu^q$$

$$J_\mu^{(Z)} = \sum_q \left(I_W^3 - 2Q\,\sin^2\theta_W\right)\, V_\mu^q - I_W^3\, A_\mu^q$$

Form factors in vector and axial vector currents:

$$\langle P'|V_\mu^q(x)|P\rangle = e^{\,i\,q\cdot x}\,\bar{U}(P')\Big[F_1^q(Q^2)\,\gamma_\mu + F_2^q(Q^2)\,\frac{i\sigma_{\mu\nu}q^\nu}{2M}\Big]U(P)$$

$$\downarrow$$

$$V_\mu^u(x) = \overline{\psi}^{\,u}(x)\gamma_\mu\psi^u(x) \qquad F_1^u(0) = n_u, \ldots$$

$$\vdots \qquad F_2^u(0) = \kappa_u, \ldots$$

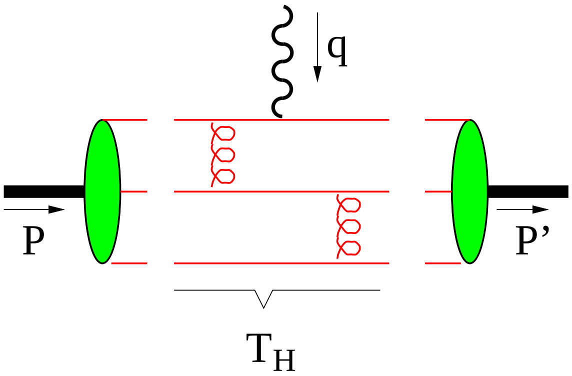
$$\langle P'|A_\mu^q(x)|P\rangle = e^{\,i\,q\cdot x}\,\bar{U}(P')\Big[G_A^q(Q^2)\,\gamma_\mu\gamma_5 + G_P^q(Q^2)\,\gamma_5 q_\mu\Big]U(P)$$

$$\downarrow$$

$$A_\mu^u(x) = \overline{\psi}^{\,u}(x)\gamma_\mu\gamma_5\psi^u(x) \qquad G_A^u(0) = g_A^u, \ldots$$

$$\vdots \qquad (g_A^u - g_A^d = 1.26)$$

Factorization into a hard scattering amplitude T_H and distribution amplitudes



dominant contribution at (very) large Q^2

S. Brodsky
G. Farrar
P. Kroll
A. Radyushkin

Inclusive scattering (polarized leptons and hadrons, γ-exchange)

$$2MW_A^{\mu\nu}(q,P,S) = \underbrace{-i\,\lambda\,\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma}{P\cdot q}}_{-i\,\lambda\,\epsilon_\perp^{\mu\nu}}\,g_1 + i\,\frac{2Mx_B}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,g_T$$

Cross section

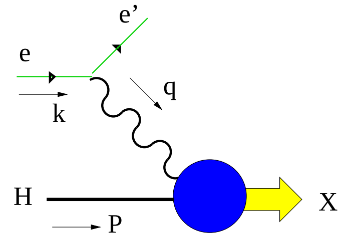
$$\frac{d\sigma_{LL}}{dx_B\,dy} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\left\{\lambda\,\left(1-\frac{y}{2}\right)g_1 \right.$$
$$\left. -|S_\perp|\,\cos\,\phi_S^\ell\,\frac{2Mx_B}{Q}\sqrt{1-y}\;g_T\right\}$$

Parton model

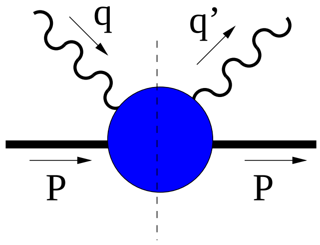
$$g_1(x_B,Q) = \frac{1}{2}\sum_a e_a^2\,g_1^a(x_B)$$

$$g_T(x_B,Q) = (g_1+g_2)(x_B,Q) = \sum_a e_a^2\,g_T^a(x_B)$$

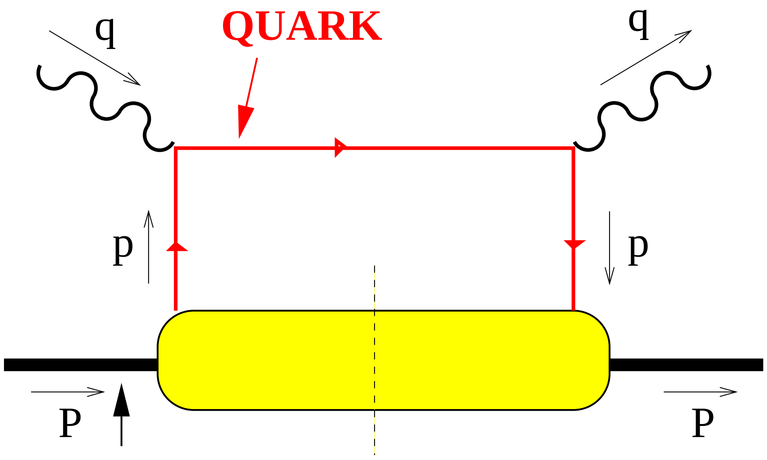
INCLUSIVE LEPTON-NUCLEON SCATTERING



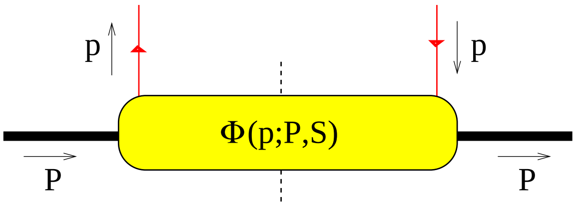
$$\sigma \;\propto\; \mathcal{I}m\, T_{\mu\nu} \;=$$



Parton picture (at large Q^2)



$+$ higher twist stuff $\sim (1/Q)^{t-2}$
$+$ perturbative corrections $\sim \alpha_s^n(Q^2)\, \ln^m Q^2$
$+ \ldots$



Hermiticity:
$\Phi^\dagger(k,P,S) = \gamma_0\,\Phi(k,P,S)\,\gamma_0$

Parity:
$\Phi(k,P,S) = \gamma_0\,\Phi(\bar{k},\bar{P},-\bar{S})\,\gamma_0$
$[\bar{k} = (k^0, -\boldsymbol{k})]$

$$\begin{aligned}\Phi(k,P,S) \;=\;& M\,A_1 + A_2\,\not{P} + A_3\,\not{k} + i\,A_4\,\frac{[\not{P},\,\not{k}]}{2M}\\ &+i\,A_5\,(k\cdot S)\gamma_5 + M\,A_6\,\not{S}\gamma_5 + A_7\,\frac{k\cdot S}{M}\,\not{P}\gamma_5\\ &+A_8\,\frac{k\cdot S}{M}\,\not{k}\gamma_5 + A_9\,\frac{[\not{P},\,\not{S}]\gamma_5}{2} + A_{10}\,\frac{[\not{k},\,\not{S}]\gamma_5}{2}\\ &+A_{11}\,\frac{k\cdot S}{M}\,\frac{[\not{P},\,\not{k}]\gamma_5}{2M} + A_{12}\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma}{M}\end{aligned}$$

with real $A_i = A_i(k^2, k\cdot P)$

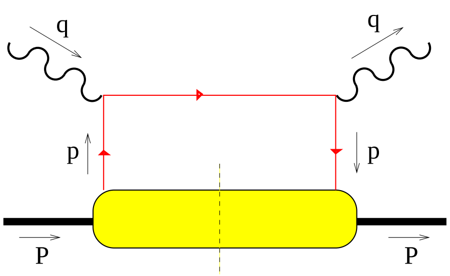
Time reversal invariance

$\Phi^*(k,P,S) = \gamma_5 C\,\Phi(\bar{k},\bar{P},\bar{S})\,C^\dagger\gamma_5 \quad \Rightarrow \quad A_4 = A_5 = A_{12} = 0$

This general form is a nice starting point for modelling

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \frac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ \end{array}\right.$$



part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

Note: the lightlike vectors may be rescaled into dimensionful vectors $\hat{p} = (Q/x_B\sqrt{2})n_+$ and $\hat{n} = (x_B\sqrt{2}/Q)n_-$

$$P = \hat{p} + \frac{M^2}{2}\,\hat{n}$$

$$q = -x_B\,\hat{p} + \frac{Q^2}{2x_B}\,\hat{n}$$

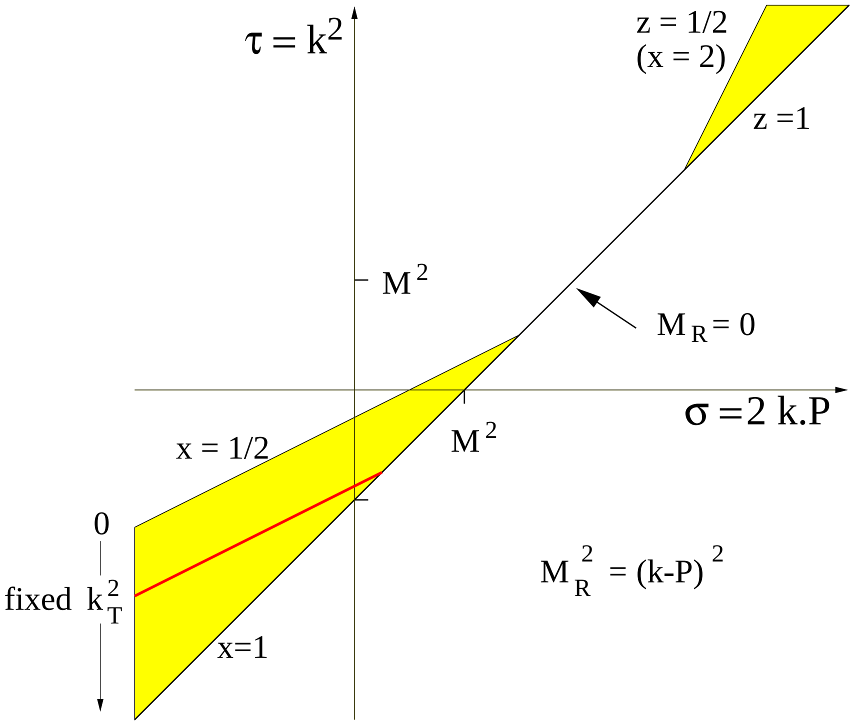
LORENTZ STRUCTURE OF SOFT PARTS

$$\Phi^{[\gamma^+]}(x,\boldsymbol{k}_T) = \frac{1}{2}\int dk^-\; Tr[\gamma^+\,\Phi(k,P,S)]\Big|_{k^+=xP^+,\;\boldsymbol{k}_T}$$

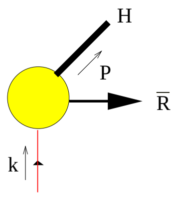
$$= \int d(2k\cdot P)\;\frac{Tr[\gamma^+\,(\not{P}\,A_2 + \not{k}\,A_3)]}{4P^+}\Big|_{k^+=xP^+,\;\boldsymbol{k}_T}$$

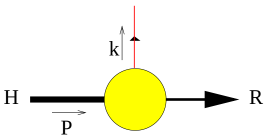
$$= \int d\sigma d\tau\;\; [A_2(\sigma,\tau) + x\,A_3(\sigma,\tau)]\;\underbrace{\delta(\tau - x\sigma + \boldsymbol{k}_T^2 + x^2\,M^2)}_{constraint}$$

$$\equiv f_1(x,\boldsymbol{k}_T)$$



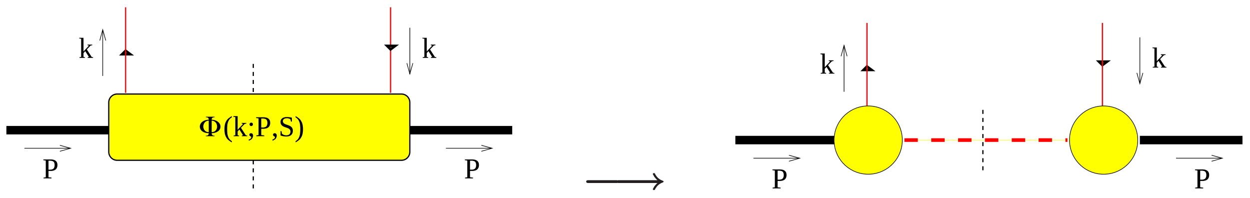
$$k^- = \frac{1}{z}\,P^-$$



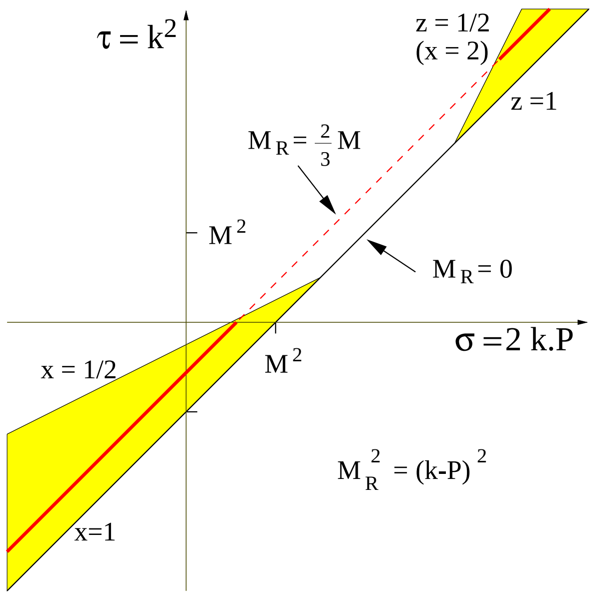


$$k^+ = x\,P^+$$

SPECTATOR MODEL FOR SOFT PART



Spectral decomposition $\Rightarrow$ spectator (diquark)

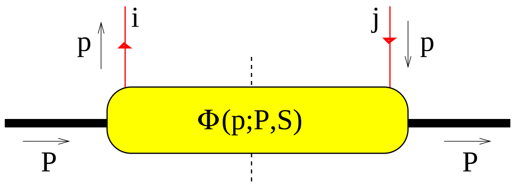


$$A_i(\tau,\sigma)$$
$$= \tilde{A}_i(k^2)\,\delta\left((P-k)^2 - M_R^2\right)$$
$$= \tilde{A}_i(\tau)\,\delta\left(\tau - \sigma + M^2 - M_R^2\right)$$

$$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T) = \left.\frac{Tr(\Gamma\tilde{\Phi})}{4(1-x)P^+}\right|_{k^2(x,k_T^2)} \qquad k^2 = \frac{x(1-x)M^2 - xM_R^2 - \boldsymbol{p}_T^2}{1-x}$$

SOFT PART IN DIS

Hard inclusive (polarized) deep inelastic lepton-hadron scattering allows isolation of the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$



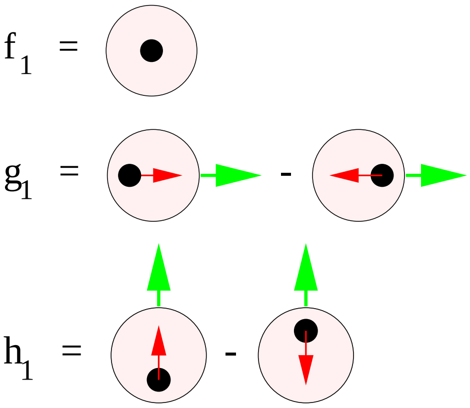
$$\begin{aligned}\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S)\Big|_{p^+ = x\,P^+} \\ &= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+ = \xi_T = 0}\end{aligned}$$

It is parametrized in terms of distribution functions

$$\boxed{\begin{aligned}\Phi(x) &= \frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + \lambda\, g_1(x)\,\gamma_5\,\not{n}_+ + h_1(x)\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2}\right\} \\ &\quad + \mathcal{O}\left(\frac{M}{P^+}\right)\end{aligned}}$$

The (leading) distribution functions f_1, g_1 and h_1 have a natural interpretation as densities in momentum space

$$
\begin{aligned}
f_1 &\sim \langle\overline{\psi}(0)\,\gamma^+\psi(\xi)\rangle \sim \langle\psi_+^\dagger(0)\,\psi_+(\xi)\rangle \\
&\sim \langle b^\dagger(p^+)\,b(p^+) - d(p^+)\,d^\dagger(p^+)\rangle
\end{aligned}
$$



chirality distribution

transverse spin distribution
chirally odd!
(off-diagonal in chiral basis)

For each of the distributions there are many aspects to be discussed (they are the subject under discussion at workshops like DISyy, SPINyy and NUCLEONyy, etc.)

- Interpretation as (lightcone) momentum distributions
- Flavor dependence
  $f_1^u(x) = u(x), g_1^u(x) = \Delta u(x), \ldots$
- positivity, e.g. $f_1^a(x) \geq 0$
  bounds, e.g. $|g_1^a(x)| \leq f_1^a(x)$
- Symmetry relations and antiquark distributions
  $\bar{f}_1(x) = -f_1(-x),\ \bar{g}_1(x) = g_1(-x)$
- Sum rules

$$\int_{-1}^{1} dx\ f_1^a(x) = \int_0^1 dx\ (f_1^a(x) - \bar{f}_1^a(x)) = n_a^{\text{val}}$$

$$\int_{-1}^{1} dx\ (g_1^u(x) - g_1^d(x)) = g_A \quad \text{(Bjorken sumrule)}$$

PROBLEM 1: GAUGE INVARIANCE

QCD is a (non-abelian) gauge theory

$$\begin{aligned} \psi(x) &\longrightarrow e^{i\,g\Lambda(x)}\,\psi(x) \\ A_\mu(x) &\longrightarrow A_\mu(x) + \frac{1}{g}\,\partial_\mu \Lambda(x) \end{aligned}$$

Ensure local gauge invariance by

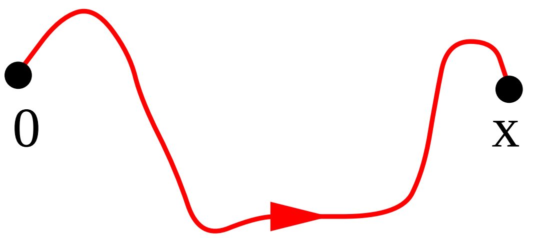
$$i\,\partial_\mu \quad\longrightarrow\quad i\,D_\mu \equiv i\,\partial_\mu + g\,A_\mu$$

or for nonlocal operators:

$$\psi^\dagger(0)\,\psi(x) \;=\; \sum_{n=0}^{\infty} (-i)^n\,\frac{x_{\mu_1}\ldots x_{\mu_n}}{n!}\,\psi^\dagger(0)\,i\partial_{\mu_1}\ldots i\partial_{\mu_n}\psi(0)$$

$$\downarrow$$

$$\psi^\dagger(0)\,U(0,x)\,\psi(x) \;=\; \sum_{n=0}^{\infty} (-i)^n\,\frac{x_{\mu_1}\ldots x_{\mu_n}}{n!}\,\psi^\dagger(0)\,iD_{\mu_1}\ldots iD_{\mu_n}\psi(0)$$



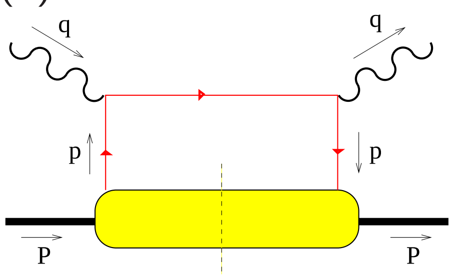
$$U(0,x) = \mathcal{P}\,e^{-ig\int ds^\mu\,A_\mu(s)}$$

THE FULL CALCULATION OF $W_{\mu\nu}$

Field theoretical approach

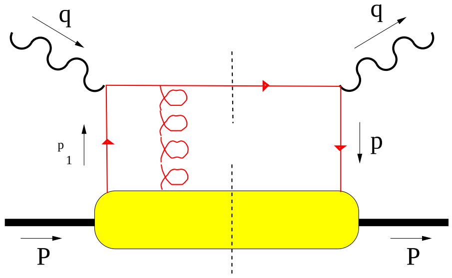
- Introduce soft parts that allow inclusion of hadrons in the field theoretical framework of QCD
- If $|q^2| \to \infty$ only a limited number of diagrams survives, each of them with characteristic behavior

(1)



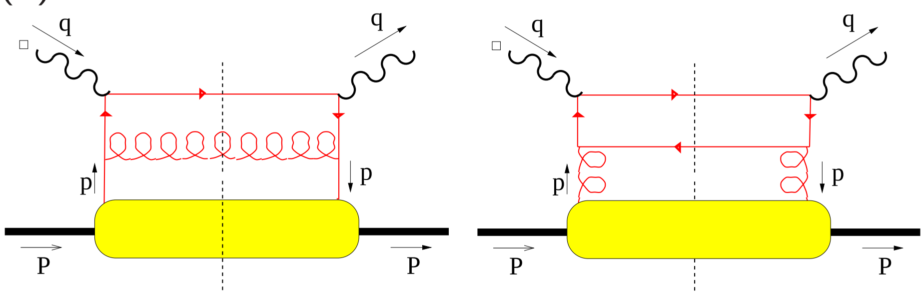
Parton model result

(2)



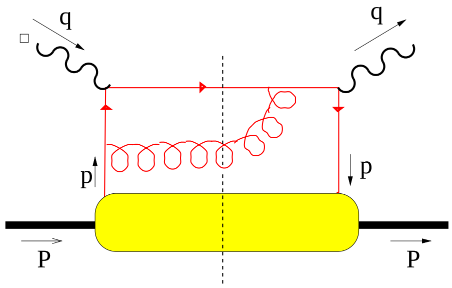
- 'Physical' A_T gluons
  [e.m. gauge invariance at $\mathcal{O}(1/Q)$]
- Longitudinal A^+ gluons
  [create link, color gauge invariance]

(3)



$\alpha_s \log(Q^2/\mu^2)$ corrections
[DGLAP equations]

(5)

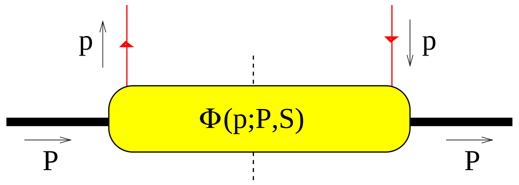


(Process dependent) α_s corrections

$$F_L = \frac{F_2}{x} - 2F_1 \sim \alpha_s(Q^2) \times \ldots$$

D. Soper; J.C. Collins; R.L. Jaffe; ...

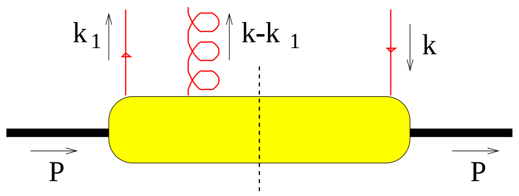
(1) Quark-quark correlator



$$p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2$$

$$\Phi_{ij}(p, P, S) = \frac{1}{(2\pi)^4} \int d^4x\, e^{ip\cdot x}\, \langle P, S|\overline{\psi}_j(0)\psi_i(x)|P, S\rangle$$

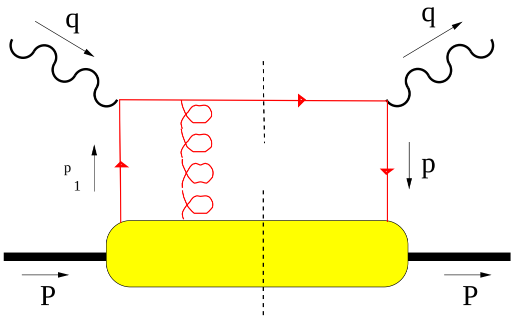
(2) Quark-quark-gluon correlator



$$k^2 \sim k \cdot P \sim k_1^2 \sim k_1 \cdot P \sim k \cdot k_1$$
$$\sim P^2 = M^2 \ll Q^2$$

$$\Phi^{\alpha}_{A\,ij}(k, k_1; P, S) = \frac{1}{(2\pi)^4} \int d^4x\, d^4y\; e^{i\,k_1\cdot(x-y)+ik\cdot y}$$
$$\times \langle P, S|\overline{\psi}_j(0)\, gA_T^{\alpha}(y)\psi_i(x)|P, S\rangle$$

PROBLEM 2: LONGITUDINAL GLUONS



$\Longrightarrow$ In DIS one also probes quark-quark-gluon correlations

$$\overline{\psi}(0)A^\alpha(\eta)\,\psi(\xi)$$

- Only (longitudinal) A^+ gluons cause problems

$$\Phi_A^+ \;\propto\; \langle P,S|\overline{\psi}(0)\left[g\int_0^{\xi^-} d\eta^- \;\underbrace{A^+(\eta)}_{\uparrow}\right]\psi(\xi)|P,S\rangle$$

spoils 'twist' reasoning

(not suppressed)

- SOLUTION TO BOTH PROBLEMS:
  Include longitudinal gluons in the definition of distributions which leads to a modified but color gauge invariant form

$$\Phi(x) = \int \frac{d\xi^-}{4\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{P}e^{-ig\int d\eta^\mu A_\mu(\eta)}\psi(\xi)|P,S\rangle\Big|_{LC}$$

- Practical hint: work in lightcone gauge ($A^+ = 0$): No link and no gluon diagrams at leading order!

EXTENSION TO $1/Q$

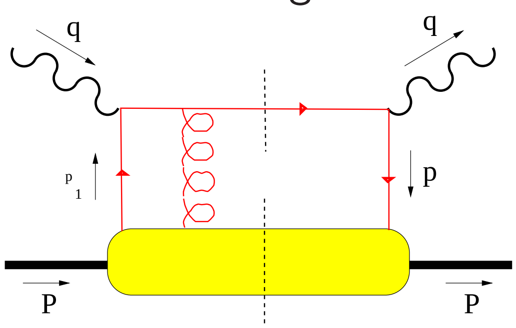
Jaffe, Ji, ...

Extension to order $1/Q$ requires:

- Inclusion of M/P^+ 'handbag'contributions in $\Phi(x)$:

$$\boxed{\begin{aligned}\Phi(x) &= \frac{1}{2}\left\{f_1\,\not{n}_+ + \lambda\,g_1\gamma_5\,\not{n}_+ + h_1\,\frac{[\,\not{S}_T,\,\not{n}_+]\gamma_5}{2}\right\}\\ &\quad+\frac{M}{2P^+}\left\{e + g_T\,\gamma_5\,\not{S}_T + \lambda\,h_L\,\frac{[\,\not{n}_+,\,\not{n}_-]\gamma_5}{2}\right\}\end{aligned}}$$

- Inclusion of diagrams with (transverse) gluons in soft part:



$$\overline{\psi}(0)A_T^\alpha(\eta)\,\psi(\xi)$$

- Use of equations of motion relates gluonic matrix elements to above quark-quark correlation function
- Partonic graph (although no partonic interpretation!) + gluon graph ensure electromagnetic gauge invariance at $\mathcal{O}(1/Q)^1$

Twist-3 functions in polarized DIS

Reaction: $\vec{\ell} + H^{\uparrow} \to \ell + X$

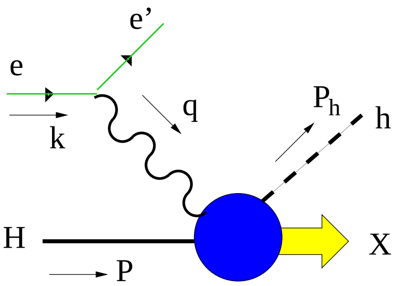
Cross sections:

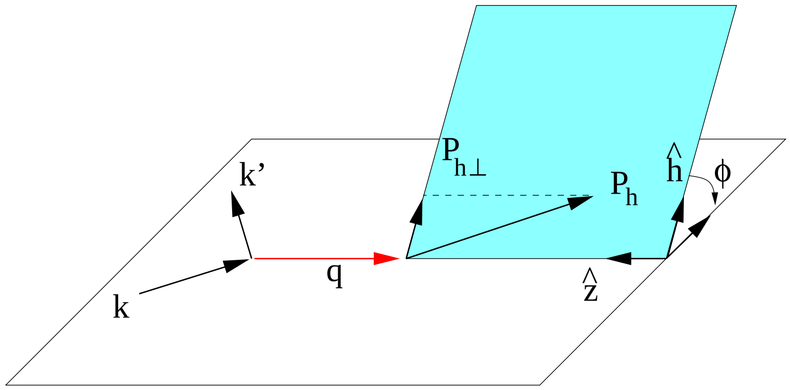
$$\frac{d\sigma_{LT}}{dx_{B}\, dy\, d\phi^{\ell}} = -\frac{4\,\alpha^2\, s}{Q^4}\, \lambda_e\, |\boldsymbol{S}_{T}|\, \cos(\phi_S^{\ell})\, y\sqrt{1-y}\, \frac{M x_B^2}{Q} \underbrace{\sum_{a,\bar{a}} e_a^2\, g_T^a(x_{B})}_{g_T(x_B,Q^2)}$$

- The compensation of the transverse index of the target spin requires a transverse index in the matrix element, which effectively means a transverse gluon index, $\langle \overline{\psi} A_T^{\alpha} \psi \rangle$, leading to suppression factor M/Q (higher twist).
- In 1-particle inclusive leptoproduction S_T^{α} can be correlated with the transverse momentum of the outgoing hadron
  $\Rightarrow$ azimuthal asymmetries at leading order in $1/Q$

$$\ell H \longrightarrow \ell' h X$$

$$-q^2 \equiv Q^2 \to \infty$$





$$x_B = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Hadronic tensor

$$2M\mathcal{W}^{(\ell H)}_{\mu\nu}(q;PS;P_hS_h)$$
$$= \frac{1}{(2\pi)^4}\int\frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X-P_h)$$
$$\times\langle PS|J_\mu(0)|P_X;P_hS_h\rangle\langle P_X;P_hS_h|J_\nu(0)|PS\rangle$$

Semi-inclusive scattering (unpolarized lepton and hadron) [3]

$$
\begin{aligned}
2M\mathcal{W}_S^{\mu\nu}(q,P,P_h) \;&=\; -g_\perp^{\mu\nu}\,\mathcal{H}_T + \hat{t}^\mu\hat{t}^\nu\,\mathcal{H}_L \\
&\quad + \hat{t}^{\,\{\mu}\hat{h}^{\nu\}}\,\mathcal{H}_{LT} + \left(2\,\hat{h}^\mu\hat{h}^\nu + g_\perp^{\mu\nu}\right)\mathcal{H}_{TT}
\end{aligned}
$$

Cross section

$$
\begin{aligned}
\frac{d\sigma_O}{dx_{_B}\,dy\,dz_h d^2q_T} \;=\; \frac{4\pi\,\alpha^2\,s}{Q^4}\,x_{_B}\,z_h \Bigg\{ &\left(1-y+\frac{1}{2}\,y^2\right)\mathcal{H}_T + (1-y)\,\mathcal{H}_L \\
&-(2-y)\sqrt{1-y}\,\cos\phi_h^\ell\;\mathcal{H}_{LT} \\
&+(1-y)\,\cos 2\phi_h^\ell\;\mathcal{H}_{TT}\Bigg\}
\end{aligned}
$$

3

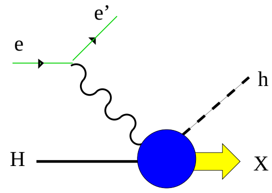
$$
\begin{aligned}
&\hat{q}^\mu = \frac{q^\mu}{Q}, \qquad \hat{t}^\mu = \frac{q^\mu + 2x_{_B}\,P^\mu}{Q}, \\
&g_\perp^{\mu\nu} = g^{\mu\nu} - \hat{q}^\mu\hat{q}^\nu + \hat{t}^\mu\hat{t}^\nu \\
&q_T^\mu = q^\mu + x_{_B}\,P^\mu - \frac{P_h^\mu}{z_h} = -\frac{P_{h\perp}^\mu}{z_h} \equiv -Q_T\,\hat{h}^\mu
\end{aligned}
$$

Semi-inclusive scattering (polarized lepton, but unpolarized hadron)

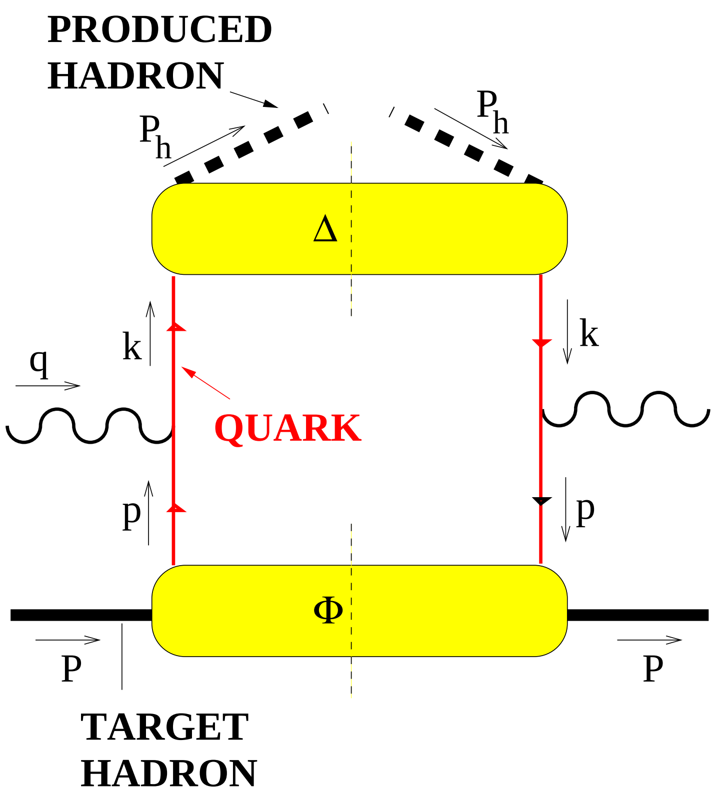
$$2M\mathcal{W}_A^{\mu\nu}(q,P,P_h) = -i\hat{t}^{\,[\mu}\hat{h}^{\nu]}\,\mathcal{H}'_{LT}$$

Cross section

$$\frac{d\sigma_L}{dx_{_B}\,dy\,dz_h d^2q_T} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\,z_h\,\sqrt{1-y}\,\sin\phi_h^\ell\;\mathcal{H}'_{LT}$$



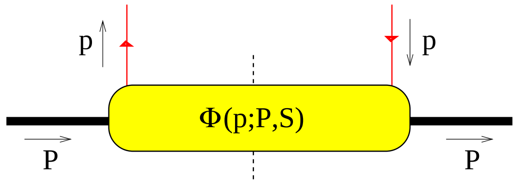
Parton picture (at large Q^2)



+ target fragmentation
+ higher twist stuff $\sim (1/Q)^{t-2}$
+ perturbative corrections $\sim \alpha_s^n(Q^2)\ \ln^m Q^2$
+ ...

D. Soper; J.C. Collins; R.L. Jaffe; . . .

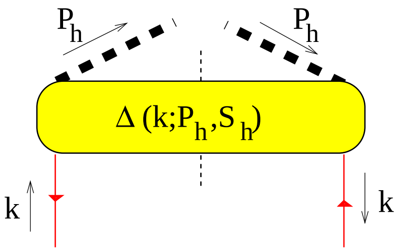
(1) distribution part



$$p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2$$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4} \int d^4x \; e^{ip\cdot x} \; \langle P,S|\overline{\psi}_j(0)\psi_i(x)|P,S\rangle$$

(2) fragmentation part



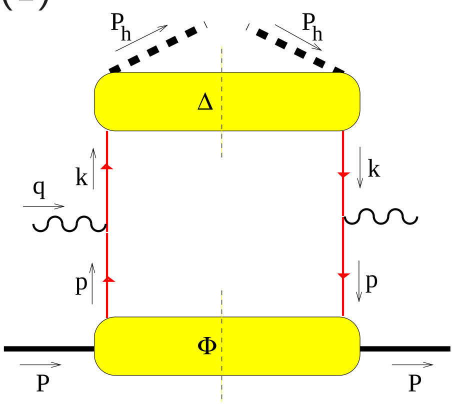
$$k^2 \sim k \cdot P_h \sim P_h^2 = M_h^2 \ll Q^2$$

$$\Delta_{ij}(k,P_h,S_h) = \sum_X \frac{1}{(2\pi)^4} \int d^4x \; e^{ik\cdot x}$$
$$\times \langle 0|\psi_i(x)|P_h,S_h;X\rangle\langle P_h,S_h;X|\overline{\psi}_j(0)|0\rangle$$

Field theoretical approach

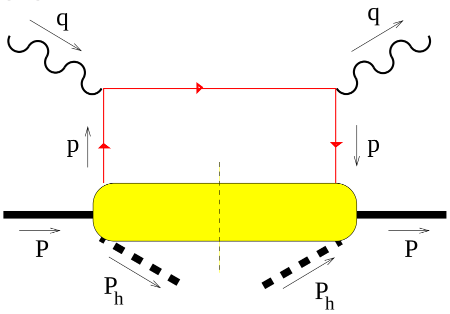
- Introduce soft parts that allow inclusion of hadrons in the field theoretical framework of QCD
- If $|q^2| \to \infty$ only a limited number of diagrams survives, each of them with characteristic behavior

(1)



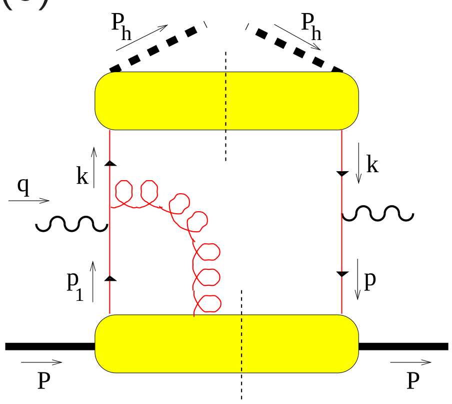
Parton model result
[current fragmentation]

(2)



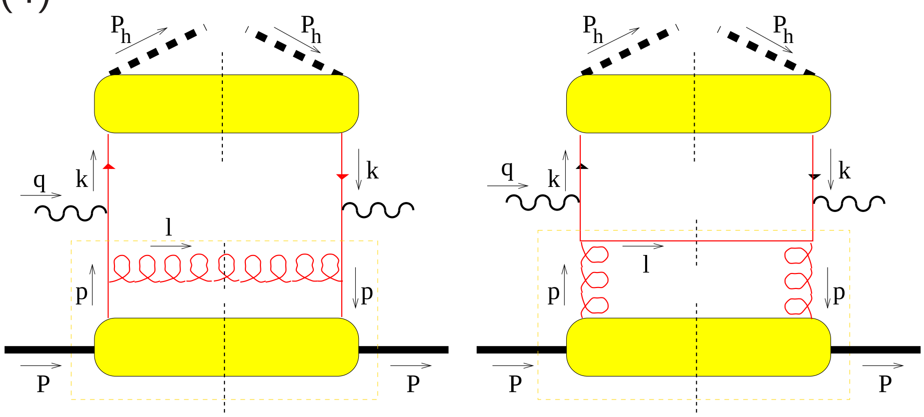
$P \cdot P_h \sim M^2$
[target fragmentation, fracture functions]

(3)



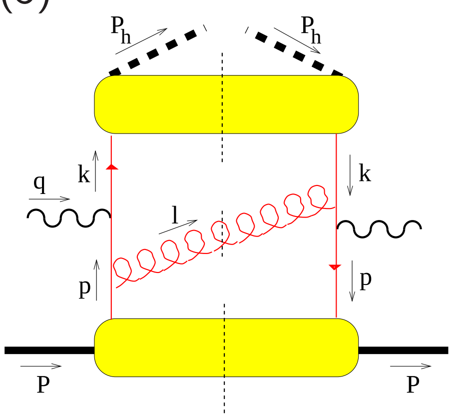
- 'Physical' A_T gluons
  [e.m. gauge invariance at $\mathcal{O}(1/Q)$]
- Longitudinal A^+ gluons
  [create link, color gauge invariance]

(4)



$\alpha_s \log(Q^2/\mu^2)$
corrections
[DGLAP equations]

(5)

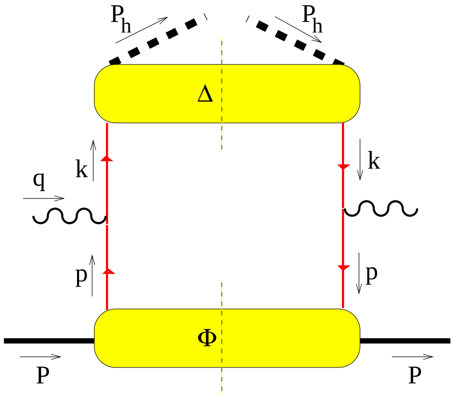


(Process dependent) α_s corrections
Compare inclusive DIS:

$$F_L = \frac{F_2}{x} - 2F_1 \sim \alpha_s(Q^2) \times \ldots$$

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

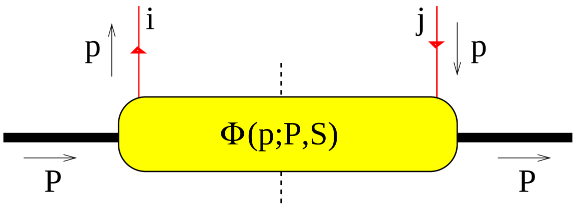
$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \dfrac{Q^2}{x_B} \\ 2\,P_h\cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \dfrac{z_h\,Q}{\sqrt{2}}\,n_- + \dfrac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ q = \dfrac{Q}{\sqrt{2}}\,n_- - \dfrac{Q}{\sqrt{2}}\,n_+ + q_T \\ P = \dfrac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \dfrac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$



	$-$	$+$	
$q \to h$	$\sim Q$	$\sim 1/Q$	$\to \int dk^+ \ldots$
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

Three external momenta $(P_A,\, P_B,\, q)$

$\longrightarrow$ transverse direction relevant



Hard 1-particle inclusive scattering allows isolation of $\Phi(x, p_T)$

$$\begin{aligned}\Phi(x,\boldsymbol{p}_T) &= \int dp^- \, \Phi(p;P,S)\Big|_{p^+=xP^+,\,p_T} \\ &= \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}\end{aligned}$$

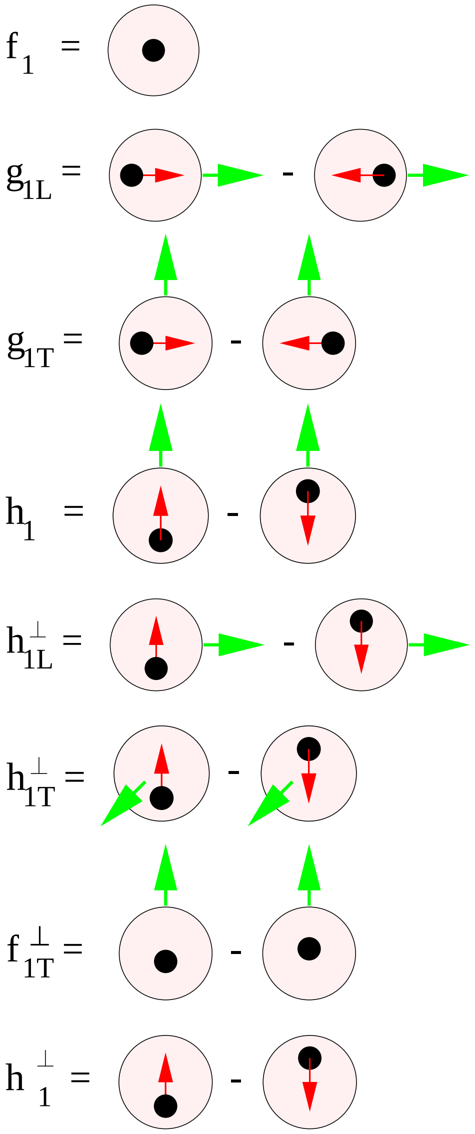
$$\begin{aligned}\Phi(x,\boldsymbol{p}_T) = \frac{1}{2}\Big\{ & f_1\, \not{n}_+ + \lambda\, g_{1L}\gamma_5\, \not{n}_+ + g_{1T}\,\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M}\gamma_5\,\not{n}_+ \\ & + h_{1T}\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2} + \lambda\, h_{1L}^\perp\,\frac{[\not{p}_T,\not{n}_+]\gamma_5}{2} \\ & + h_{1T}^\perp\,\frac{(\boldsymbol{p}_T\cdot\boldsymbol{S}_T)}{M}\,\frac{[\not{p}_T,\not{n}_+]\gamma_5}{2}\Big\} \\ & + \frac{1}{2}\Big\{ f_{1T}^\perp\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} + h_1^\perp(x,p_T^2)\,\frac{i\,[\not{p}_T,\not{n}_+]}{2M}\Big\}\end{aligned}$$

T-odd

$f_1 = f_1(x,\boldsymbol{p}_T^2)$, etc.

Ralston & Soper, Tangerman & M

All these functions can be interpreted as momentum densities:

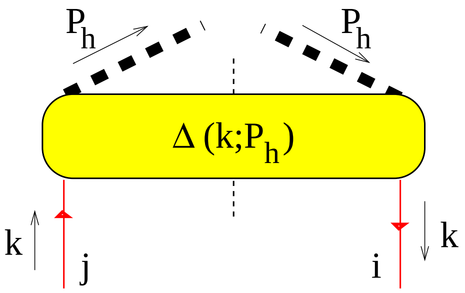


Longitudinally polarized quarks in a transversely polarized target

Transversely polarized quarks in a longitudinally polarized target

Unpolarized quarks in a transversely polarized target (T-odd)

Transversely polarized quarks in an unpolarized target (T-odd)



$$P_h = P_h^- \, n_- + \frac{M_h^2}{2P_h^-} \, n_+$$

$$k = \frac{1}{z} \, P_h^- \, n_- + k_T + \mathcal{O}\left(\frac{1}{P_h^-}\right)$$

Hard processes need the quantity (lightfront correlation)

$$\Delta(z, \boldsymbol{k}_T) = \int dk^- \; \Delta(k, P_h, S_h)\Big|_{k^- = P_h^-/z,\, k_T}$$

$$= \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} \, e^{ik\cdot\xi} \, \langle 0|\psi(\xi) a_h^\dagger a_h \overline{\psi}(0)|0\rangle\Big|_{\xi^- = 0}$$

Using parity and hermiticity one finds for an unpolarized hadron

$$\boxed{\Delta(z, k_T) = z D_1(z, k_T') \, \not{n}_- + z H_1^\perp(z, k_T') \, \frac{i[\not{k}_T, \not{n}_-]}{2M_h} + \mathcal{O}\left(\frac{M_h}{P_h^-}\right)}$$

- $k_T' = -z k_T$ is transverse momentum of hadron wrt quark.
- $H_1^\perp$ is T-odd (natural for fragmentation functions)

FROM THE LIGHTFRONT TO THE LIGHTCONE

$\boldsymbol{p}_T$-averaging: relevant for inclusive ℓH or $\boldsymbol{q}_T$-integrated DY

$$\Phi(x) = \int dp^- \, d^2\boldsymbol{p}_T \; \Phi(p,P,S)$$
$$= \int \frac{d\xi^-}{2\pi} \; e^{ip\cdot\xi} \, \langle P,S|\overline{\psi}(0)\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\boxed{\Phi(x) = \frac{1}{2}\left\{f_1 \, \not{n}_+ + \lambda \, g_1\gamma_5 \, \not{n}_+ + h_1 \, \frac{[\,\not{S}_T, \, \not{n}_+]\gamma_5}{2}\right\}}$$

$$f_1 = f_1(x) = \int d^2\boldsymbol{p}_T \;\; f_1(x,\boldsymbol{p}_T)$$

$\boldsymbol{p}_T$-weighting: relevant for asymmetries in ℓH or DY

$$\Phi_\partial^\alpha(x) = \int dp^- \, d^2\boldsymbol{p}_T \; p_T^\alpha \, \Phi(p,P,S)$$
$$= \int \frac{d\xi^-}{2\pi} \; e^{ip\cdot\xi} \, \langle P,S|\overline{\psi}(0)\mathcal{U}\, i\partial^\alpha \, \mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

$$\boxed{\begin{aligned}\frac{1}{M}\,\Phi_\partial^\alpha(x) = \frac{1}{2}\Big\{ & g_{1T}^{(1)} \, S_T^\alpha \, \gamma_5 \, \not{n}_+ + \lambda \, h_{1L}^{\perp(1)} \, \frac{[\,\not{n}_+, \gamma^\alpha]\gamma_5}{2} \\ & + f_{1T}^{\perp(1)} \, \epsilon_T^{\alpha\beta} S_{T\beta} \, \not{n}_+ + h_1^{\perp(1)} \, \frac{i\,[\,\not{n}_+, \gamma^\alpha]}{2}\Big\}\end{aligned}}$$

T-odd

$$g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T \, \frac{\boldsymbol{p}_T^2}{2M^2} \, g_{1T}(x,\boldsymbol{p}_T)$$

Including subleading order for the $\boldsymbol{p}_T$-averaged functions

$$\begin{aligned}\Phi(x) \;&=\; \frac{1}{2}\left\{f_1\,\not{n}_+ + \lambda\,g_1\gamma_5\,\not{n}_+ + h_1\,\frac{[\,\not{S}_T,\,\not{n}_+]\gamma_5}{2}\right\}\\ &\quad+\frac{M}{2P^+}\left\{e + g_T\,\gamma_5\,\not{S}_T + \lambda\,h_L\,\frac{[\not{n}_+,\,\not{n}_-]\gamma_5}{2}\right\}\\ &\quad+\frac{M}{2P^+}\left\{f_T\,\epsilon_T^{\rho\sigma}S_{T\rho}\gamma_\sigma - i\,\lambda\,e_L\,\gamma_5 + h\,\frac{i\,[\,\not{n}_+,\,\not{n}_-]}{2}\right\}\end{aligned}$$

T-odd

- $M/P^+ \;\propto\; M/Q \longrightarrow$ subleading
- The (twist-3) functions do not have a density interpretation
  $e = (m/Mx)\,f_1 + \tilde{e}$, $g_T = g_{1T}^{(1)}/x + (m/Mx)\,h_1 + \tilde{g}_T$,
  $h_L = -2\,h_{1L}^{\perp(1)}/x + (m/Mx)\,g_1 + \tilde{h}_L$
- From the general amplitude expansion for Φ (covariance), constrained by Hermiticity, P, and T one obtains:

$$\underbrace{g_T - g_1}_{g_2} = \frac{d}{dx}\,g_{1T}^{(1)} \qquad\qquad \underbrace{h_L - h_1}_{\frac{1}{2}h_2} = -\frac{d}{dx}\,h_{1L}^{\perp(1)}$$

$$f_T = -\frac{d}{dx}\,f_{1T}^{\perp(1)} \qquad\qquad h = -\frac{d}{dx}\,h_1^{\perp(1)}$$

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$$\Phi_D^\alpha(x) = \frac{M}{2}\left\{\left(x g_T - \frac{m}{M}\,h_1\right) S_T^\alpha\,\gamma_5\,\not{n}_+ + \lambda\left(x h_L - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2}\right\}$$

$$\Phi_\partial^\alpha(x) = \frac{M}{2}\left\{g_{1T}^{(1)}\,S_T^\alpha\,\gamma_5\,\not{n}_+ + \lambda\,h_{1L}^{\perp(1)}\,\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2}\right\}$$

$$\Phi_A^\alpha(x) = \frac{M}{2}\left\{x\tilde{g}_T\,S_T^\alpha\,\gamma_5\,\not{n}_+ + \lambda\,x\tilde{h}_L\,\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2}\right\}$$

$$x\tilde{g}_T = x g_T - g_{1T}^{(1)} - \frac{m}{M}\,h_1$$

$$x\tilde{h}_L = x h_L - h_{1L}^{\perp(1)} - \frac{m}{M}\,g_1$$

NOTE:

In parton model interaction-dependent parts vanish: $\Phi_A^\alpha = 0$
Combined with Lorentz invariance

$$\Rightarrow \quad g_T(x) = \int_x^1 dy\,\frac{g_1(y)}{y} + \frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 dy\,\frac{h_1(y)}{y^2}\right]$$

S. Wandzura, F. Wilczek

INTERACTION DEPENDENT PIECES (T-ODD)

$$\Phi_D^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{x f_T\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho + x\, h\, \frac{i[\gamma^\alpha, \not n_+]}{4}\right\}$$

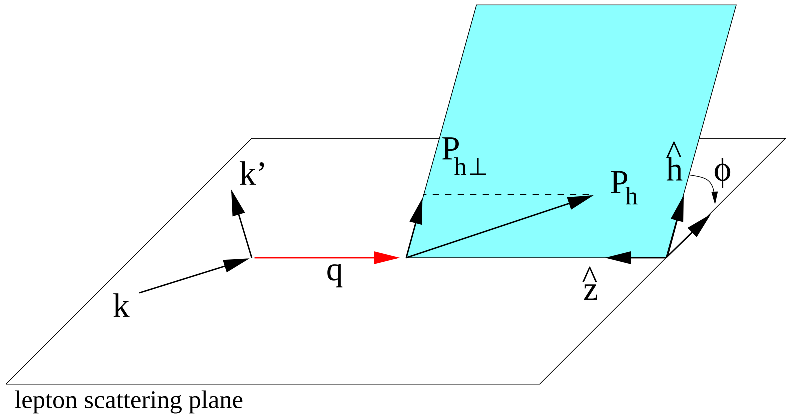
$$\Phi_\partial^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{-f_{1T}^{\perp(1)}\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho - h_1^{\perp(1)}\, \frac{i[\gamma^\alpha, \not n_+]}{2}\right\}$$

$$\Phi_A^\alpha(x)\Big|_{T-odd} = \frac{M}{2}\left\{x \tilde{f}_T\, \epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho + x\, \tilde{h}\, \frac{i[\gamma^\alpha, \not n_+]}{4}\right\}$$

$$x\tilde{f}_T = x f_T + f_{1T}^{(1)}$$
$$x\tilde{h} = x h + 2h_1^{\perp(1)}$$

In parton model interaction-dependent parts vanish: $\Phi_A^\alpha = 0$
Combined with Lorentz invariance

$$\Rightarrow \quad f_T(x) = h(x) = 0$$



$$\ell H \longrightarrow \ell' h X$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot k}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q}$$

$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Cross sections (leading in $1/Q$)

$$\frac{d\sigma_{OO}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,\left(1+(1-y)^2\right)\,\sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\,D_1^a(z_h)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy\,dz_h} = \frac{2\pi\alpha^2\,s}{Q^4}\,x_B\,y(2-y)\,\lambda_e\,\lambda\,\sum_{a,\bar{a}} e_a^2 g_1^a(x_B)\,D_1^a(z_h)$$

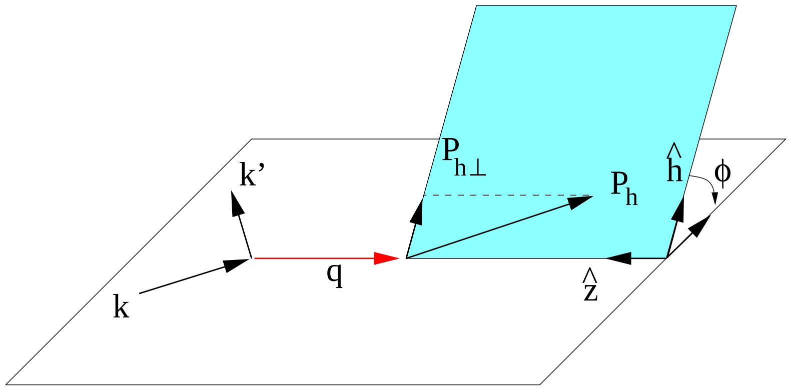
Where to find a specific correlation function?

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$

- cross sections are chirally even
  Examples: $\propto f_1 \otimes D_1$
  $\propto h_1 \otimes H_1$
  $\propto e \otimes H_1^\perp$
  $\propto m\, f_1 \otimes H_1^\perp$

- # of spin vectors is even in case of a T-even combination
  Examples: $\propto \lambda_e\, \lambda\, g_1 \otimes D_1$
  $\propto \lambda_e\, |\boldsymbol{S}_T|\, g_{1T}^{(1)} \otimes D_1$
  $\propto |\boldsymbol{S}_T|\, |\boldsymbol{S}_{hT}|\, h_1 \otimes H_1$
  $\propto h_1^\perp \otimes H_1^\perp$

- # of spin vectors is odd in case of a T-odd combination (single spin asymmetries)
  Examples: $\propto \lambda_e\, e \otimes H_1^\perp$
  $\propto |\boldsymbol{S}_T|\, h_1 \otimes H_1^\perp$
  $\propto |\boldsymbol{S}_T|\, f_{1T}^\perp \otimes D_1$



$$\ell H \longrightarrow \ell' h X$$

$$x_{_B} = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}$$

$$Q_{_T} = \frac{|P_{h\perp}|}{z_h}$$

Azimuthally weighted cross sections

$$\langle W \rangle_{\underbrace{P_e P_H P_h}_{\substack{\uparrow \\ (O,L,T)}}} \equiv \int d\phi^\ell \, d^2\boldsymbol{q}_{_T} \underbrace{W}_{\substack{\uparrow \\ W(Q_{_T}, \phi_h^\ell, \phi_S^\ell, \phi_{S_h}^\ell)}} \frac{d\sigma_{P_e P_H}}{dx_{_B}\, dy\, dz_h\, d\phi^\ell\, d^2\boldsymbol{q}_{_T}}$$

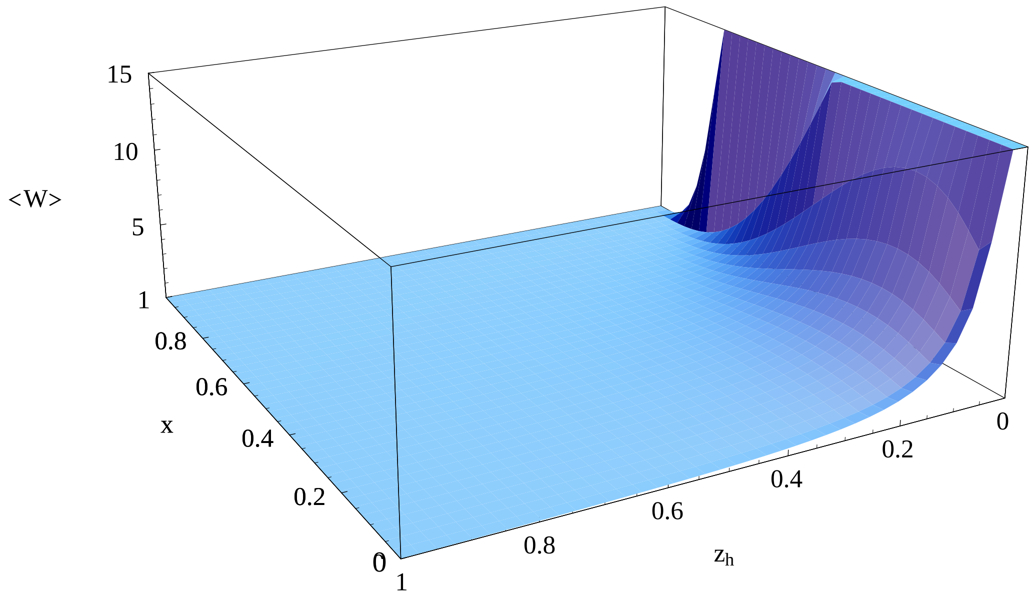
e.g.

$$\langle 1 \rangle_{OOO} = \frac{2\pi\alpha^2 s}{Q^4} x_{_B} \left(1 + (1-y)^2\right) \sum_{a,\bar{a}} e_a^2 f_1^a(x_{_B}) \, D_1^a(z_h)$$

π^+ **production on a proton:** $ep \longrightarrow e\pi^+ X$

$$\langle 1 \rangle_{OOO} \quad \propto \sum_{a,\bar{a}} e_a^2 f_1^a(x_B)\, D_1^a(z_h)$$

For valence quarks one finds



Azimuthal asymmetries for unpolarized targets (higher twist)

$$\left\langle \frac{Q_T}{M}\cos(\phi_h^\ell)\right\rangle_{OOO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,(2-y)\sqrt{1-y}\sum_{a,\bar{a}} e_a^2\left\{\frac{2M}{Q}\,x_{_B}^2\,f^{\perp(1)a}(x_{_B})\,D_1^a(z_h) + \frac{2M_h}{Q}\,x_{_B}\,f_1^a(x_{_B})\,\frac{\tilde{D}^{\perp(1)a}(z_h)}{z_h}\right\}$$

Note: $\tilde{D}^{\perp a}(z) = D^{\perp a}(z) - zD_1^a(z)$

J. Levelt and PJM, PRD 49 (1994) 96

$$\left\langle \frac{Q_T}{M}\sin(\phi_h^\ell)\right\rangle_{LOO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,y\sqrt{1-y}\,\frac{2M}{Q}\sum_{a,\bar{a}} e_a^2\,x_{_B}^2\,\tilde{e}^a(x_{_B})\,H_1^{\perp(1)a}(z_h)$$

Note: $\tilde{e}^a(x) = e^a(x) - \frac{m_a}{M}\,\frac{f_1^a(x)}{x}$

J. Levelt and PJM, PLB 338 (1994) 357

Azimuthal asymmetries for polarized targets (leading twist)

$$\left\langle \frac{Q_T}{M}\,\cos(\phi_h^\ell - \phi_S^\ell)\right\rangle_{LTO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,|\boldsymbol{S}_T|\,y\left(1-\frac{y}{2}\right)\sum_{a,\bar a} e_a^2\,x_B\,g_{1T}^{(1)a}(x_B)D_1^a(z_h)$$

$$\left\langle \frac{Q_T^2}{4MM_h}\,\sin(2\phi_h^\ell)\right\rangle_{OLO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(1-y)\sum_{a,\bar a} e_a^2\,x_B\,h_{1L}^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h)$$

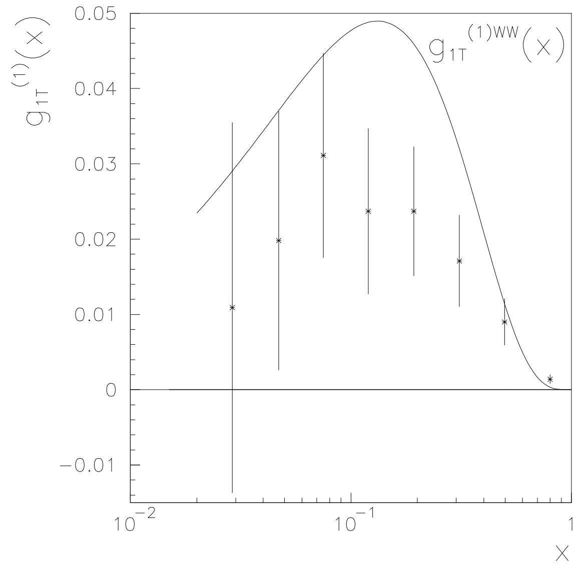
$$\left\langle \frac{Q_T}{M_h}\,\sin(\phi_h^\ell + \phi_S^\ell)\right\rangle_{OTO} = \frac{4\pi\alpha^2\,s}{Q^4}\,|\boldsymbol{S}_T|\,(1-y)\sum_{a,\bar a} e_a^2\,x_B\,h_1^a(x_B)H_1^{\perp(1)a}(z_h)$$

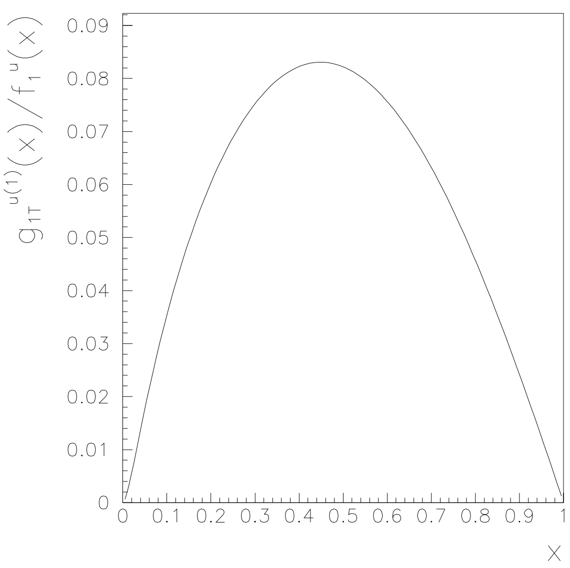
Note: there exists also an (OTO) $\sin(3\phi_h^\ell - \phi_S^\ell)$ asymmetry $\sim h_{1T}^{\perp(2)}H_1^{\perp(1)}$

J.C. Collins, NPB 396 (1993) 161
A. Kotzinian, NPB 441 (1995) 234
R.D. Tangerman and PJM, PLB 352 (1995) 129

Estimate of $g_{1T}^{(1)}$ A. Kotzinian & PJM, PRD 54 (1996) 1229

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$
- line: above relation with $g_2(x) = g_2^{WW}(x)$





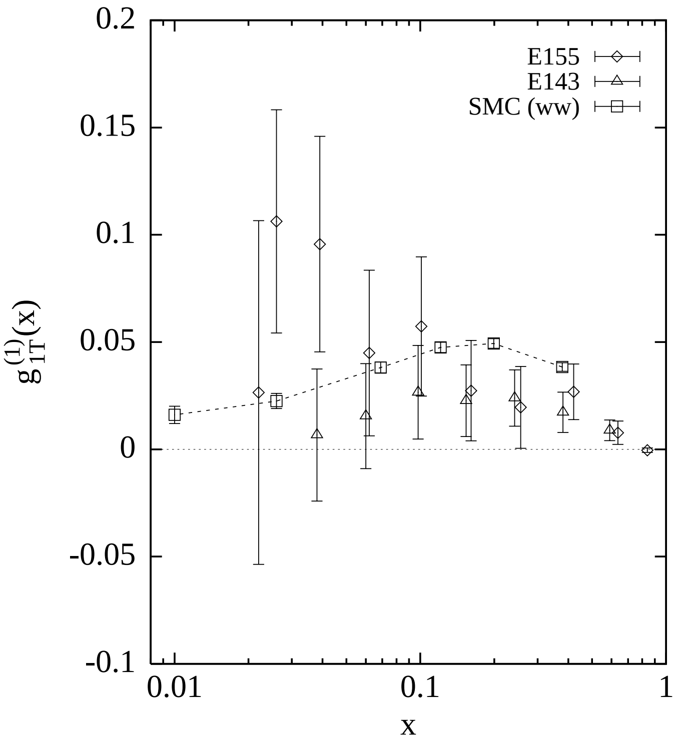
Resulting asymmetry in $\vec{e}\vec{p} \to e\pi^+X$:

$$A_{LT} \propto \frac{g_{1T}^{(1)u}(x)}{f_1^u(x)}$$

Updated stimate of $g_{1T}^{(1)}$

M. Boglione & PJM

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$ (including E155, preliminary)
- line: above relation with $g_2(x) = g_2^{WW}(x)$



Leading functions: $g_1,\ g_{1T}^{(1)}$

Subleading function

$$\left.\begin{aligned} g_T &= \frac{g_{1T}^{(1)}}{x} + \tilde{g}_T \\ &= g_1 + \frac{d}{dx}\, g_{1T}^{(1)} \end{aligned}\right\} \Rightarrow g_T = \int_x^1 dy\, \frac{g_1(y)}{y} + \underbrace{\left(\tilde{g}_T - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y}\right)}_{\bar{g}_T}$$

Approximation 1: $g_{1T}^{(1)}(x) \approx 0$

$$\begin{aligned} g_T(x) &= g_1(x) \\ \bar{g}_T(x) &= g_1(x) - \int_x^1 dy\, \frac{g_1(y)}{y} \end{aligned}$$

Approximation 2: $\bar{g}_T(x) \approx 0$

$$\begin{aligned} g_T(x) &= \int_x^1 dy\, \frac{g_1(y)}{y} \\ g_{1T}^{(1)}(x) &= x \int_x^1 dy\, \frac{g_1(y)}{y} = x\, g_T \end{aligned}$$

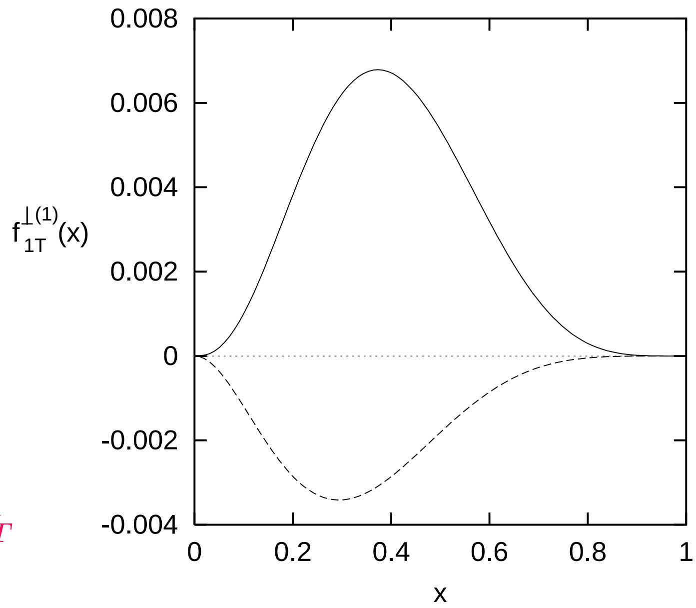
Single spin asymmetry A_N in $pp^{\uparrow} \rightarrow \pi\, X$

Explanation 1: Scattering via an unpolarized quark

$$\sigma \;\propto\; \bar{f}_1 \otimes f_{1T}^{\perp} \otimes D_1$$

M. Anselmino, M. Boglione,
F. Murgia, PLB 362 (1995) 164

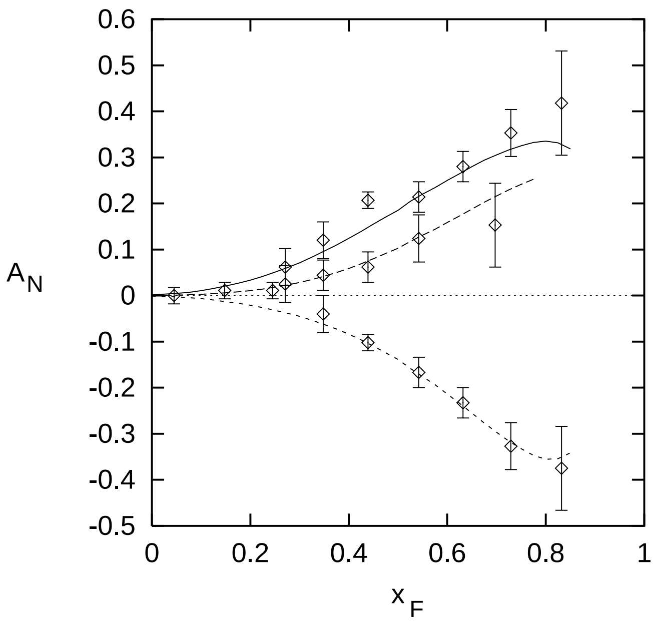
Sivers effect: $f_{1T}^{\perp}$

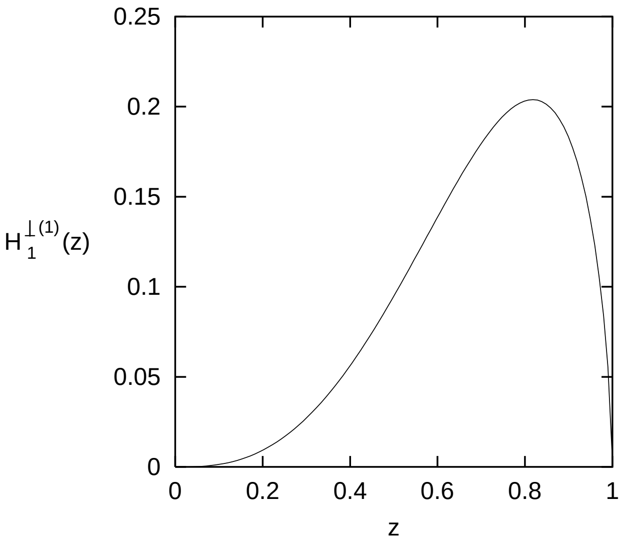


Single spin asymmetry A_N in $pp^\uparrow \to \pi X$

Explanation 2: Scattering via a transversely polarized quark

$$\sigma \propto \bar{f}_1 \otimes h_1 \otimes H_1^\perp$$





M. Anselmino, M Boglione, F. Murgia, hep-ph/9901442

Collins effect: $H_1^\perp$

Single spin asymmetries (leading twist)

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell - \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|\boldsymbol{S}_T|\,\left(1-y+\frac{1}{2}\,y^2\right)\sum_{a,\bar a} e_a^2\,x_B\,f_{1T}^{\perp(1)a}(x_B)D_1^a(z_h)$$

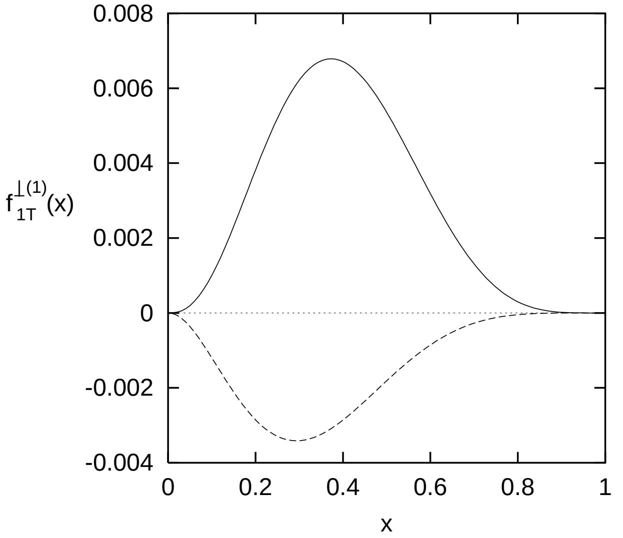
$$\left\langle \frac{Q_T}{M_h}\,\sin(\phi_h^\ell + \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2\,s}{Q^4}\,|\boldsymbol{S}_T|\,2(1-y)\sum_{a,\bar a} e_a^2\,x_B\,h_1^a(x_B)H_1^{\perp(1)a}(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

Use $ep^\uparrow \longrightarrow e\pi^+\,X$ to distinguish different explanations of the asymmetry in $pp^\uparrow \longrightarrow \pi^+\,X$

M. Boglione and PJM, hep-ph/9903354

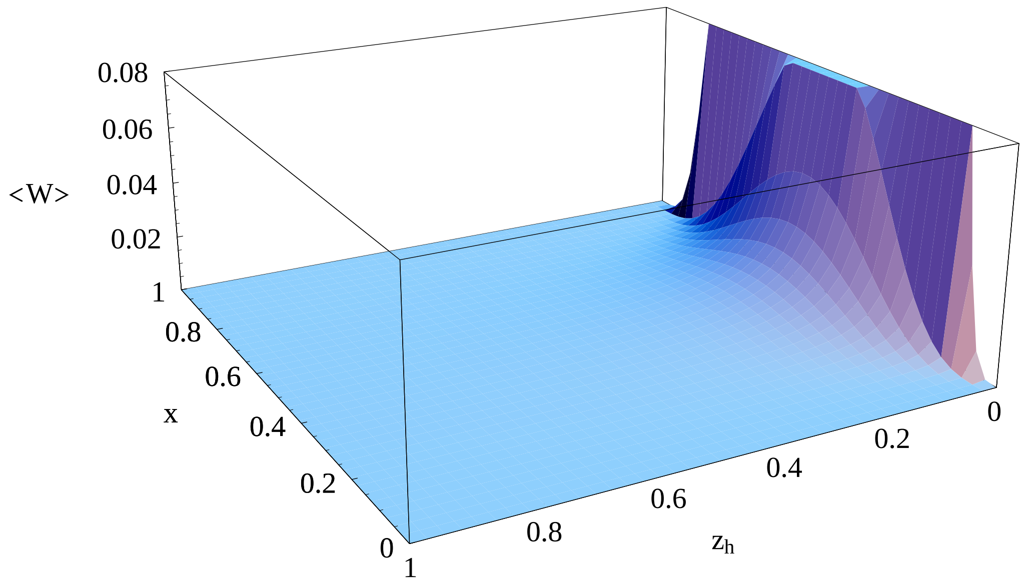
Use $f_{1T}^{\perp}$ from spin asymmetry A_N in $pp^{\uparrow} \rightarrow \pi\, X$



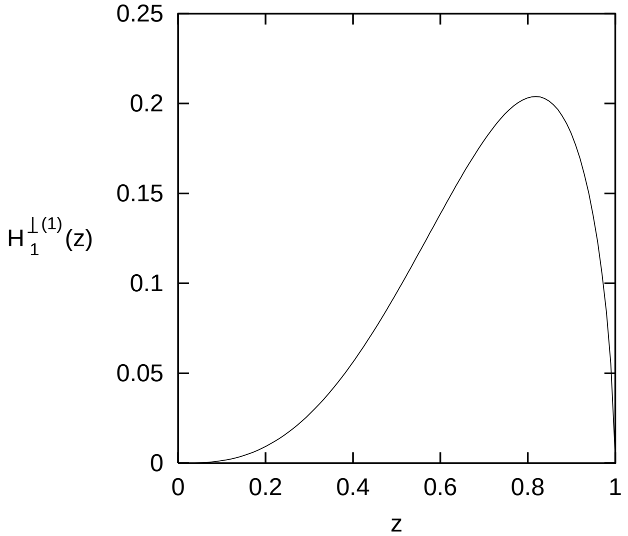
calculate for $ep^{\uparrow} \rightarrow \pi^{+} X$

$$\left\langle \frac{Q_T}{M} \sin(\phi_h^\ell - \phi_S^\ell) \right\rangle_{OTO}$$

$$\propto \sum_{a,\bar{a}} e_a^2\, x_B\, f_{1T}^{\perp(1)a}(x_B) D_1^a(z_h)$$



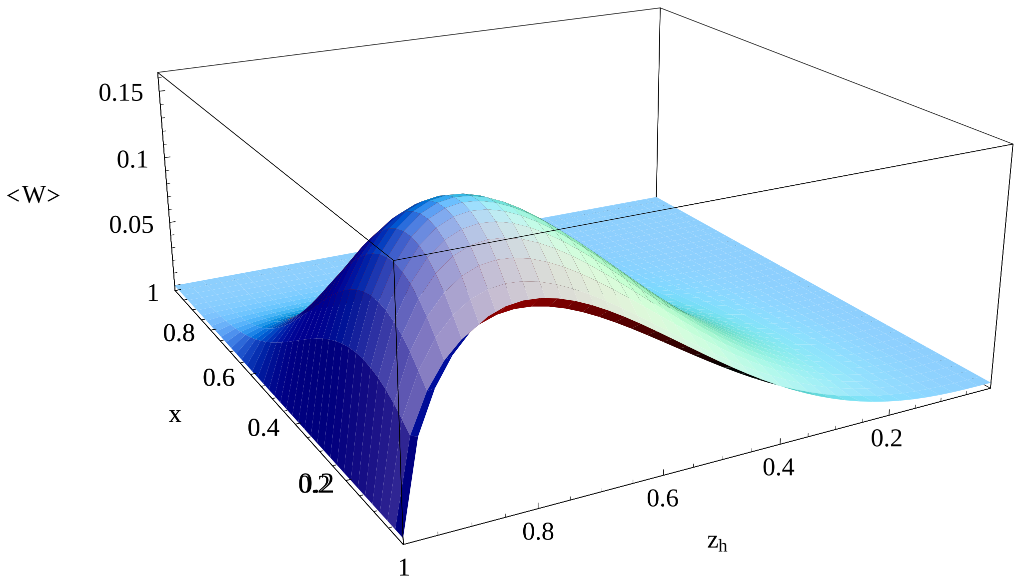
Use $H_1^{\perp}$ from spin asymmetry A_N in $pp^{\uparrow} \to \pi X$



calculate for $ep^{\uparrow} \to \pi^{+} X$

$$\left\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO}$$

$$\propto \sum_{a,\bar{a}} e_a^2 \, x_B \, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$



Single spin asymmetries for longitudinally polarized target
Leading:

$$\left\langle \frac{Q_T^2}{4MM_h}\,\sin(2\phi_h^\ell)\right\rangle_{OLO} = -\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(1-y)\sum_{a,\bar a} e_a^2\,x_B\,h_{1L}^{\perp(1)a}(x_B)H_1^{\perp(1)a}(z_h)$$

Subleading:

$$\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell)\right\rangle_{OLO} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda\,(2-y)\sqrt{1-y}\,\frac{2M_h}{Q} \times \sum_{a,\bar a} e_a^2\Bigg\{x_B\,h_{1L}^{\perp(1)a}(x_B)\,\frac{\tilde H^a(z_h)}{z_h} + x_B\left(2\,h_{1L}^{\perp(1)a} - x_B\,\tilde h_L^a(x_B)\right)\,H_1^{\perp(1)a}(z_h)\Bigg\}$$

Note: $\Rightarrow\ \tilde H^a(z) = H^a(z) + 2z\,H_1^{\perp(1)a}(z) = z\,\frac{d}{dz}\left(z\,H_1^{\perp(1)a}\right)$

$\Rightarrow\ x\,h_L^a(x) - \frac{m}{M}\,g_{1L}^a(x) = -2\,h_{1L}^{\perp(1)a} + x\,\tilde h_L^a(x) = x\,h_1^a - x\,\frac{d}{dx}\,h_{1L}^{\perp(1)a} - \frac{m}{M}\,g_{1L}^a(x)$

$\Rightarrow$ HERMES $\mathcal{A}_{OL}$ results (DIS99): $h_{1L}^{\perp(1)}(x) \approx 0$ (Efremov)

Leading functions: $H_1^{\perp(1)}$

Subleading function

$$\begin{aligned} H(z) &= -2z\, H_1^{\perp(1)}(z) + \tilde{H}(z) \\ &= z^3 \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \end{aligned}$$

Approximation: $H_1^{\perp(1)}(z) \approx 0$

$$\begin{aligned} \frac{H(z)}{z} &= z^2 \frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right) \approx 0 \\ \frac{\tilde{H}(z)}{z} &= \frac{d}{dz}\left(z\, H_1^{\perp(1)}\right) \approx 0 \end{aligned}$$

Note: The approximation $\tilde{H}(z) \approx 0$ leads to inconsistencies

Leading functions: h_1, $h_{1L}^{(1)}$, $h_{1T}^{(2)}$

Subleading function

$$\left.\begin{aligned} h_L &= -2\,\frac{h_{1L}^{\perp(1)}}{x} + \tilde{h}_L \\ &= h_1 - \frac{d}{dx}\,h_{1L}^{\perp(1)} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} h_L &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ &\quad + \underbrace{\left(\tilde{h}_L - 2x\int_x^1 dy\,\frac{\tilde{h}_L(y)}{y^2}\right)}_{\bar{h}_L} \end{aligned}$$

Approximation 1: $h_{1L}^{\perp(1)}(x) \approx 0$

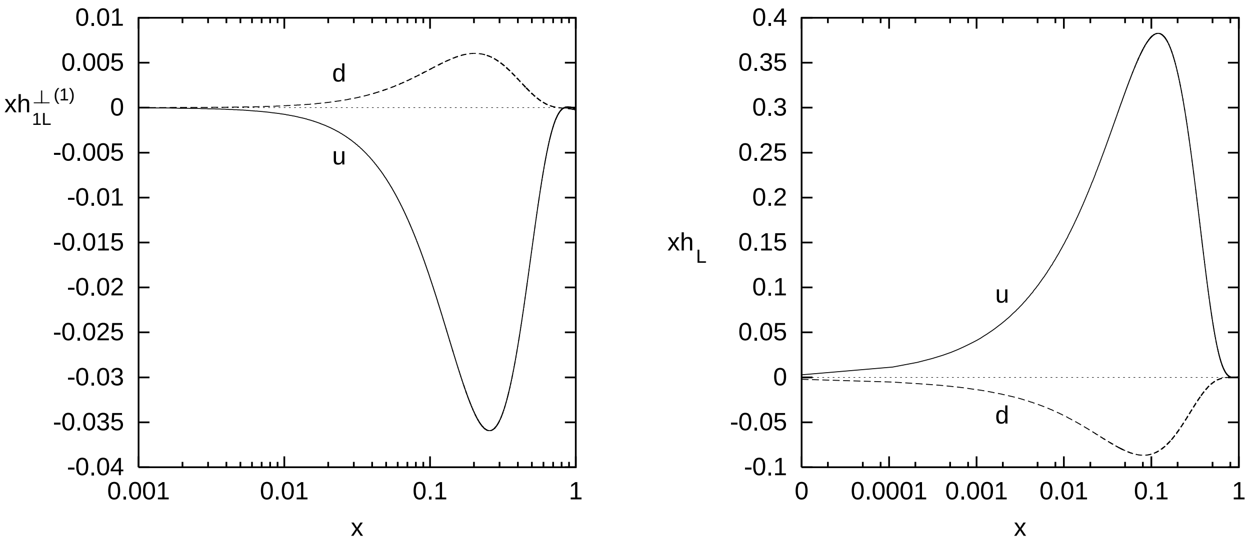
$$\begin{aligned} h_L(x) &= h_1(x) \\ \bar{h}_L(x) &= h_1(x) - 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \end{aligned}$$

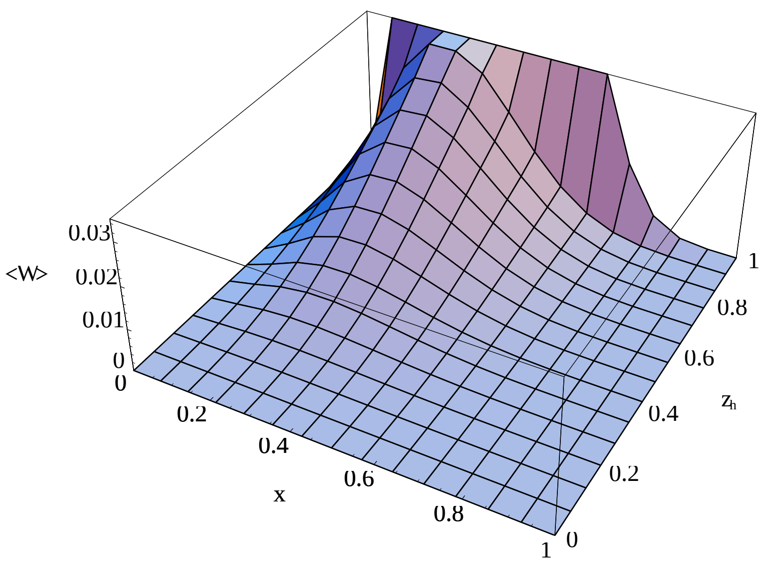
Approximation 2: $\bar{h}_L(x) \approx 0$

$$\begin{aligned} h_L(x) &= 2x\int_x^1 dy\,\frac{h_1(y)}{y^2} \\ h_{1L}^{\perp(1)}(x) &= -x^2\int_x^1 dy\,\frac{h_1(y)}{y^2} = -\frac{1}{2}\,x\,h_L \end{aligned}$$

M. Boglione and M. (PRELIMINARY)

Use $H_1^{\perp}$ from spin asymmetry A_N in $pp^{\uparrow} \to \pi\, X$ and h-distributions in approximation $\bar{h}_L = 0$





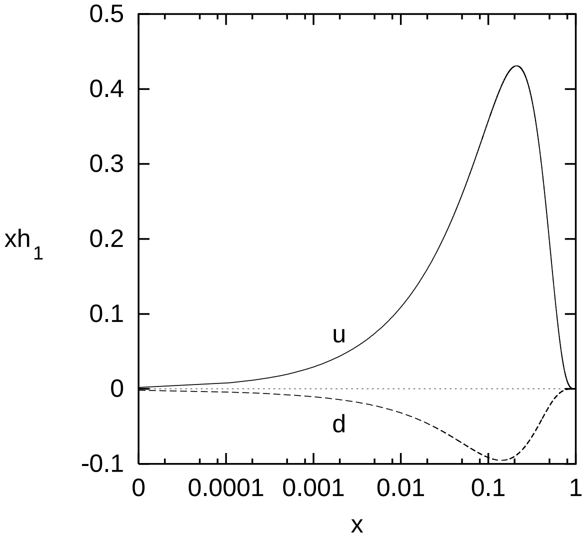
Calculate for $ep^{\uparrow} \to \pi^+ X$

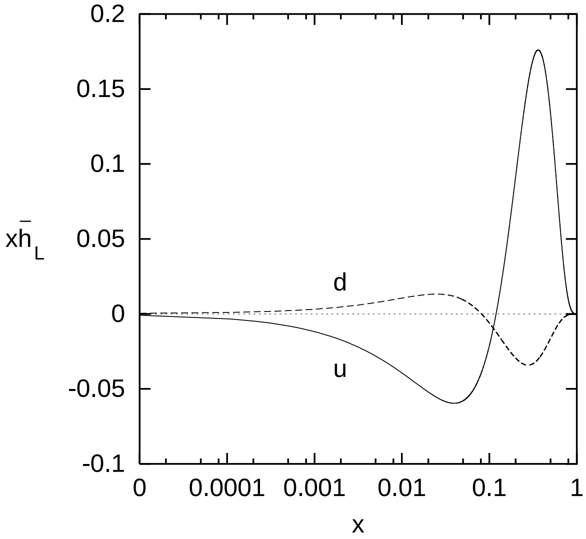
$$\left\langle \frac{Q_T}{M} \sin(\phi_h^{\ell}) \right\rangle_{OLO}$$

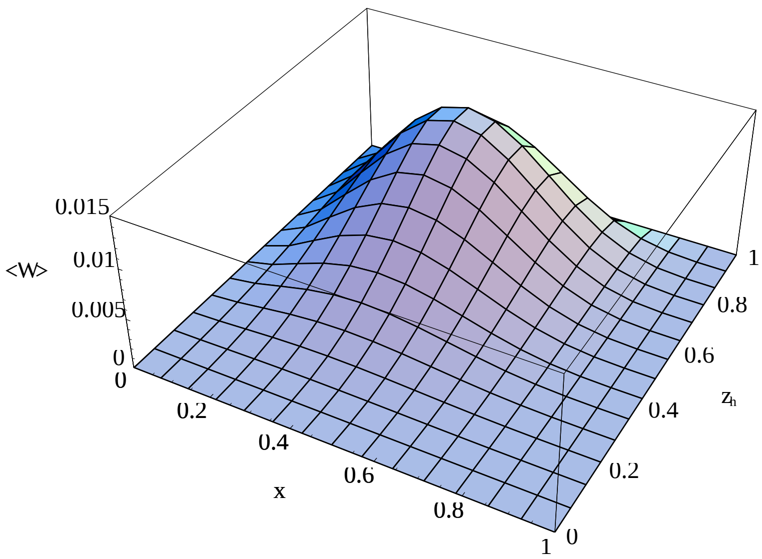
$$\propto -\sum_{a,\bar{a}} e_a^2 \left[x_B\, h_{1L}^{\perp(1)a}(x_B) \frac{\tilde{H}^a(z_h)}{z_h} - x_B^2\, h_L^a(x_B)\, H_1^{\perp(1)a}(z_h) \right]$$

M. Boglione and M. (PRELIMINARY)

Use $H_1^{\perp}$ from spin asymmetry A_N in $pp^{\uparrow} \to \pi\, X$ and h-distributions in approximation $\bar{h}_{1L}^{\perp(1)} = 0$







Calculate for $ep^{\uparrow} \to \pi^+ X$

$$\left\langle \frac{Q_T}{M} \sin(\phi_h^{\ell}) \right\rangle_{OLO}$$

$$\propto -\sum_{a,\bar{a}} e_a^2 \left[x_B\, h_{1L}^{\perp(1)a}(x_B) \frac{\tilde{H}^a(z_h)}{z_h} - x_B^2\, h_L^a(x_B)\, H_1^{\perp(1)a}(z_h) \right]$$

Double T-odd effects

D. Sivers, PRD 41 (1990) 83 and PRD 43 (1991) 261
J. Collins, NPB 396 (1993) 161

Azimuthal asymmetry for unpolarized leptoproduction (leading twist)

$$\left\langle \frac{Q_T^2}{M M_h} \cos(2\phi_h^\ell) \right\rangle_{OOO} = \frac{16\pi\alpha^2\, s}{Q^4}\,(1-y) \sum_{a,\bar{a}} e_a^2\, x_B\, h_1^{\perp(1)a}(x_B)\, H_1^{\perp(1)a}(z_h)$$

D. Boer and PJM, PRD 57 (1998) 5780

Similar azimuthal asymmetry in $e^+e^- \to \pi\pi\, X$

D. Boer, R. Jakob, PJM, NPB 504 (1997) 345
A. Efremov, O. Smirnova, L. Tkatchev, hep-ph/9812522

DISTRIBUTIONS (T-even)					
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	f_1			
	L		g_{1L}	$h_{1L}^\perp$	
	T		g_{1T}	h_1 $h_{1T}^\perp$	
twist 3	U	$f^\perp$			e
	L		$g_L^\perp$	h_L	
	T		g_T $g_T^\perp$	h_T $h_T^\perp$	

(9 independent)

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z, -z\boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1		$H_1^\perp$	
	L		G_{1L}	$H_{1L}^\perp$	
	T	$D_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$	
twist 3	U	$D^\perp$		H	E
	L	$D_L^\perp$	$G_L^\perp$	H_L	E_L
	T	D_T	G_T $G_T^\perp$	H_T $H_T^\perp$	E_T

(12 independent)

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z, -z\boldsymbol{k}_T)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1		$H_1^\perp$	
	L		G_{1L}	$H_{1L}^\perp$	
	T	$D_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$	
twist 3	U	$D^\perp$		H	E
	L	$D_L^\perp$	$G_L^\perp$	H_L	E_L
	T	D_T	G_T $G_T^\perp$	H_T $H_T^\perp$	E_T

$\boldsymbol{k}_T$-integrated results

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1			
	L		G_1		
	T			H_1	
twist 3	U			H	E
	L			H_L	E_L
	T	D_T	G_T		

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SUMMARY OF PROFILE FUNCTIONS

DISTRIBUTION (T-even & T-odd)						
$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T)$			Dirac structure			
			γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2		U	f_1		$h_1^\perp$	
		L		g_{1L}	$h_{1L}^\perp$	
		T	$f_{1T}^\perp$	g_{1T}	$h_1 \quad h_{1T}^\perp$	
twist 3		U	$f^\perp$		h	e
		L	$f_L^\perp$	$g_L^\perp$	h_L	e_L
		T	f_T	$g_T \quad g_T^\perp$	$h_T \quad h_T^\perp$	e_T

$\boldsymbol{k}_T$-integrated results

DISTRIBUTION (T-even & T-odd)						
$\Phi^{[\Gamma]}(x)$			Dirac structure			
			γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2		U	f_1			
		L		g_1		
		T			h_1	
twist 3		U			h	e
		L			h_L	e_L
		T	f_T	g_T		

$\boldsymbol{k}_T$-integrated results

DISTRIBUTION (T-even & T-odd)					
$\Phi^{[\Gamma]}(x)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	f_1			
	L		g_1		
	T			h_1	
twist 3	U			h	e
	L			h_L	e_L
	T	f_T	g_T		

FRAGMENTATION (T-even & T-odd)					
$\Delta^{[\Gamma]}(z)$		Dirac structure γ	$\gamma\,\gamma_5$	$[\gamma,\gamma]\gamma_5$	$1/\gamma_5$
twist 2	U	D_1			
	L		G_1		
	T			H_1	
twist 3	U			H	E
	L			H_L	E_L
	T	D_T	G_T		