QCD and the hadron structure beyond the probability distributions

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T-16 Nuclear Theory Seminar
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based on work with J. -W. Qiu, Vogelsang, and Yuan

Outline

- Introduction: pQCD

- Go beyond probability distributions:

  - Spin-dependent effect: three parton correlation
    - Tri-gluon correlation in ep and pp collision (LO)
    - Evolution of twist-3 correlations (beyond LO)
    - Physical meaning of twist-3 correlations
  - Nuclear size dependent effect: four parton correlation
    - nuclear transverse momentum broadening of vector bosons

- Summary

Quantum Chromodynamics (QCD)

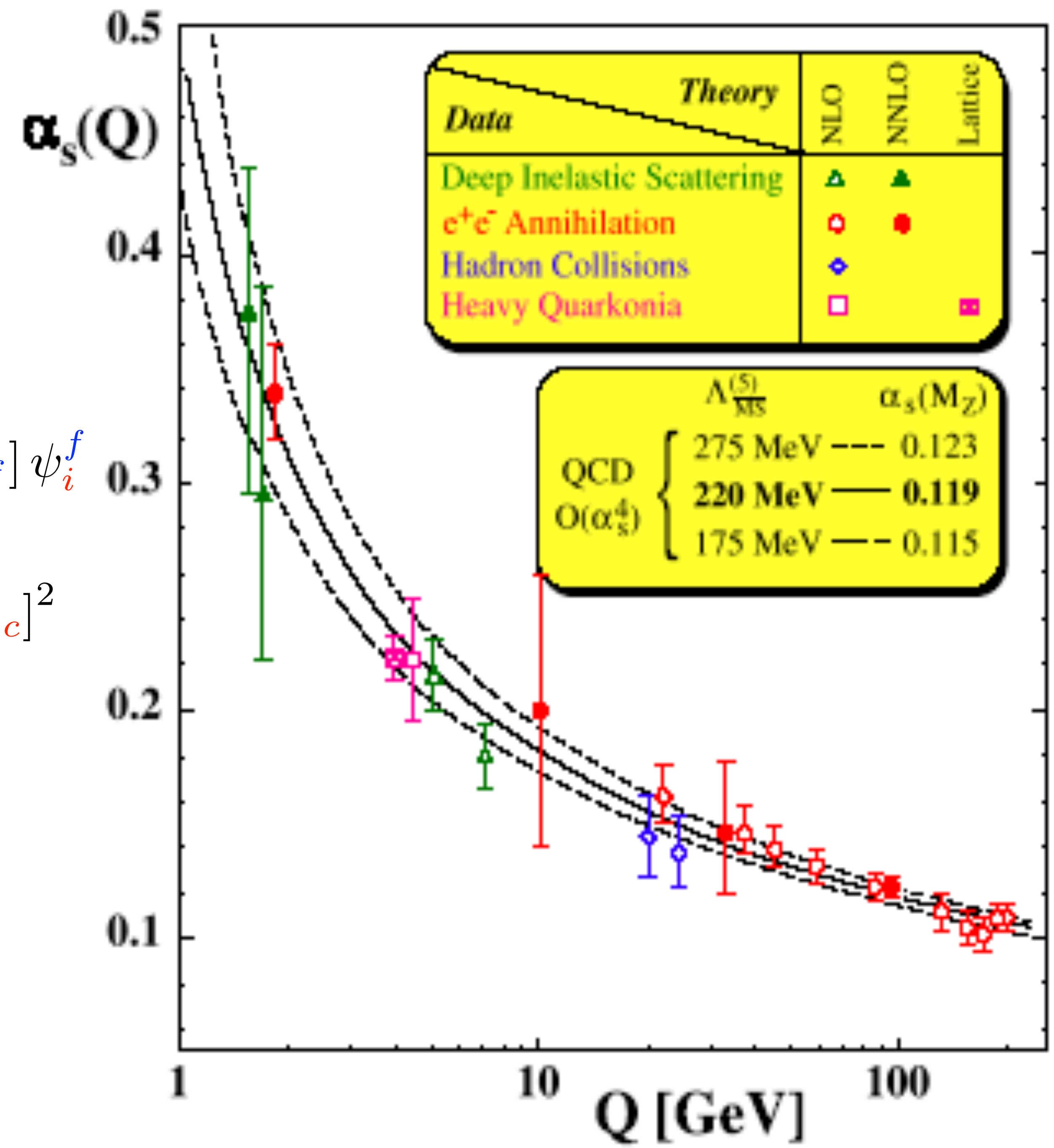
H. Fritzsch, M. Gell-Mann, H.Lentwyler, 1973

- QCD - underlying theory of strong interaction

- Lagrangian density:

ψ_i^f: quark field $\quad A_{\mu,a}$: gluon field

$$L_{QCD}(\psi, A) = \sum_f \bar{\psi}_i^f \left[\gamma^\mu \left(i\partial_\mu - g A_{\mu,a}(t_a)_{ij}\right) - m_f\right] \psi_i^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c}\right]^2$$



- Asymptotic freedom:

  2004 Nobel Prize in Physics

  - perturbative QCD (pQCD)

Factorization Theorems in pQCD

- Can pQCD calculate cross sections involving hadrons?

Scale of hadron wave function: **1/R ~ 1/ fm ~** Λ_{QCD} ~ 200MeV

Scale of hard partonic collision: **Q > 2 GeV >>** Λ_{QCD}

⇒ **pQCD works at** $\alpha_s(Q)$, **but not at** $\alpha_s(\Lambda_{QCD})$

- A way out – Factorization Theorems

For a review, see A. H. Mueller (Ed) perturbative QCD

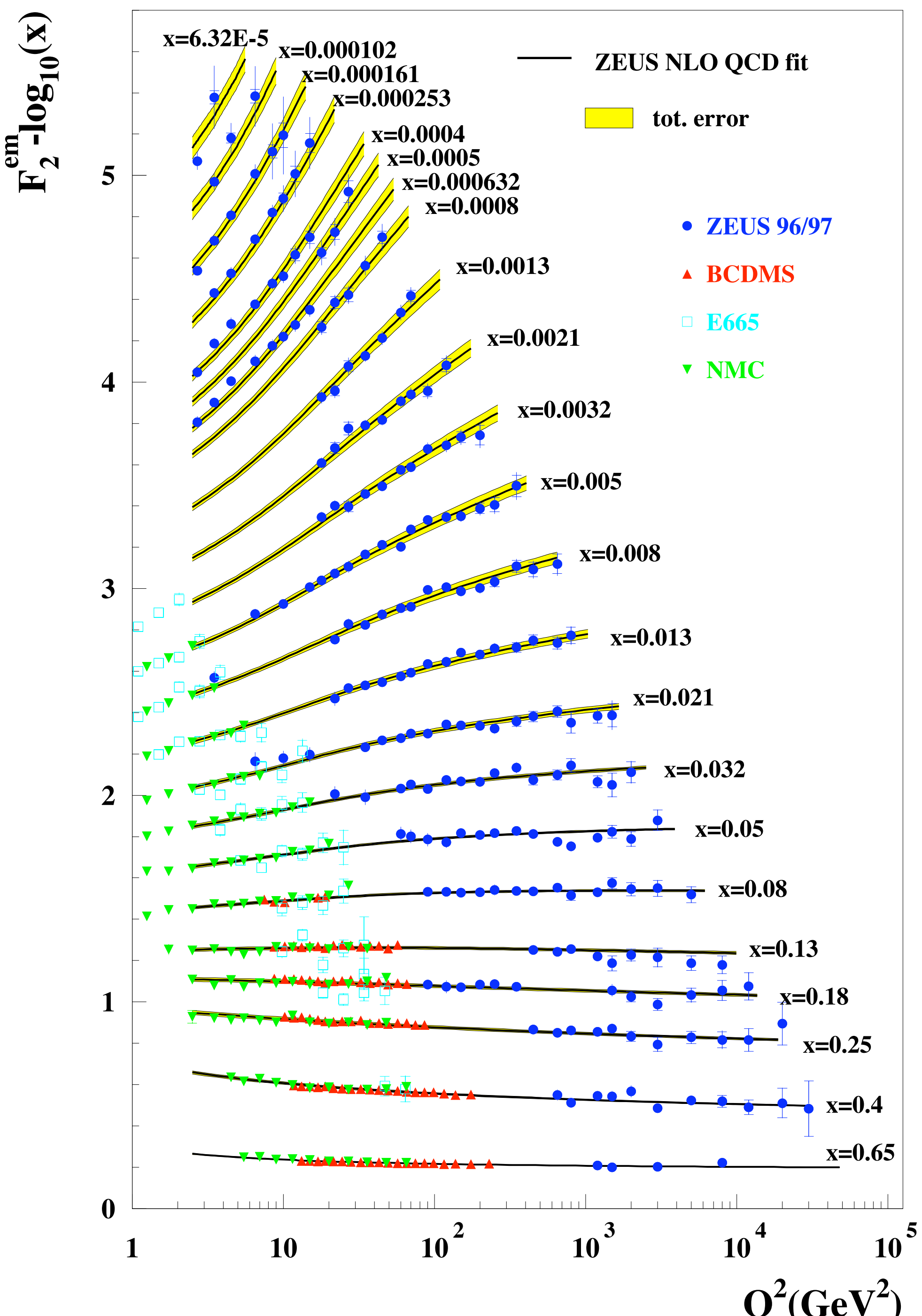
Quantum interference between perturbative and nonperturbative scales can be negelected (power suppressed ~ $O(\Lambda_{QCD}/Q)$)

$$\sigma_{Hadron}(Q) = \underbrace{\phi_{Parton/hadron}(\Lambda_{QCD})}_{\text{universal (measured)}} \otimes \underbrace{\hat{\sigma}_{Parton}(Q)}_{\text{calculable}} + \mathcal{O}(\Lambda_{QCD}/Q)$$

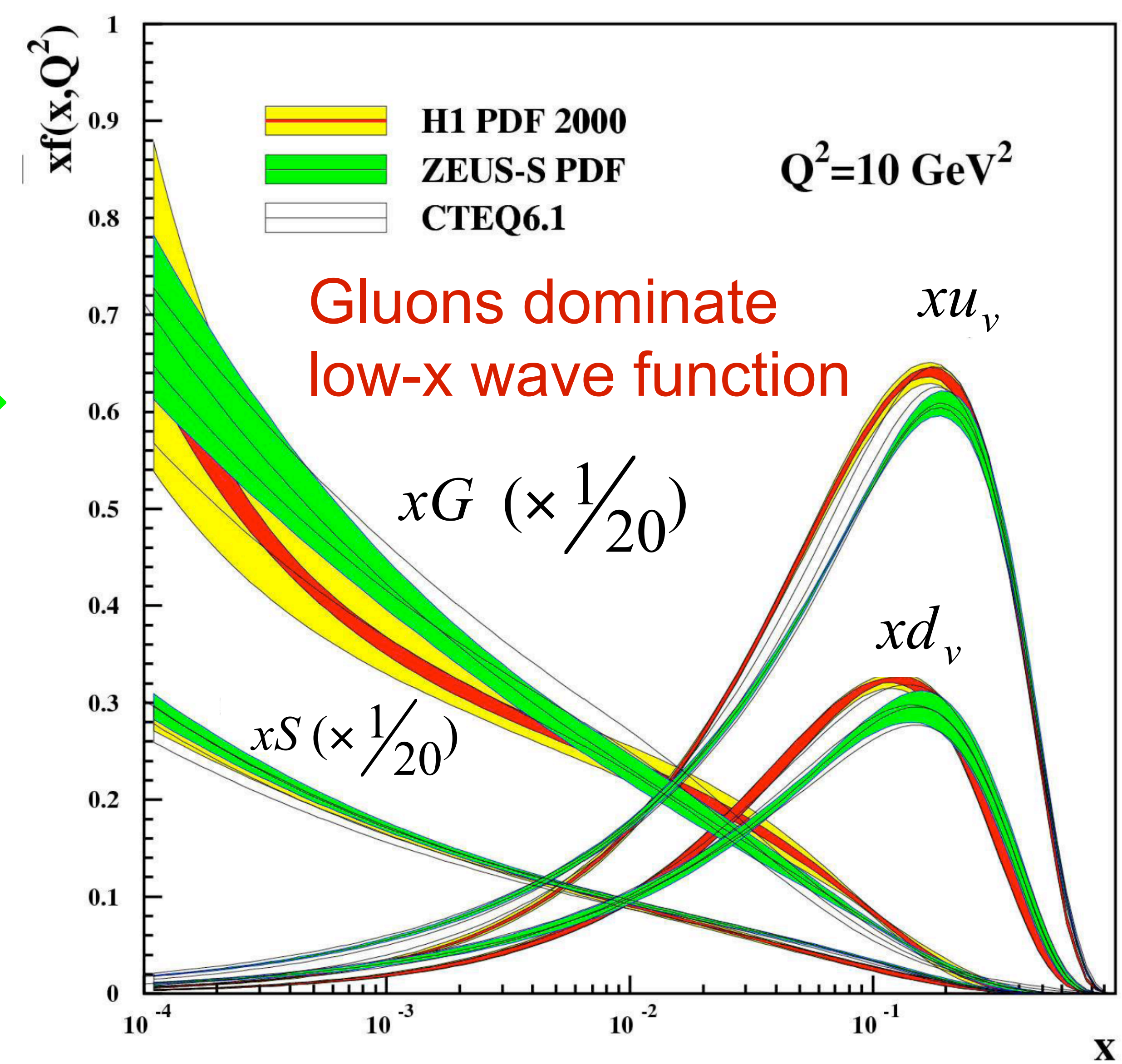
Universality of PDFs: comparison with DIS data

$$\sigma_{Hadron}(Q) = \underbrace{\phi_{Parton/hadron}}_{\text{universal (measured)}}(\Lambda_{QCD}) \otimes \underbrace{\hat{\sigma}_{Parton}}_{\text{calculable}}(Q) + \mathcal{O}(\Lambda_{QCD}/Q)$$

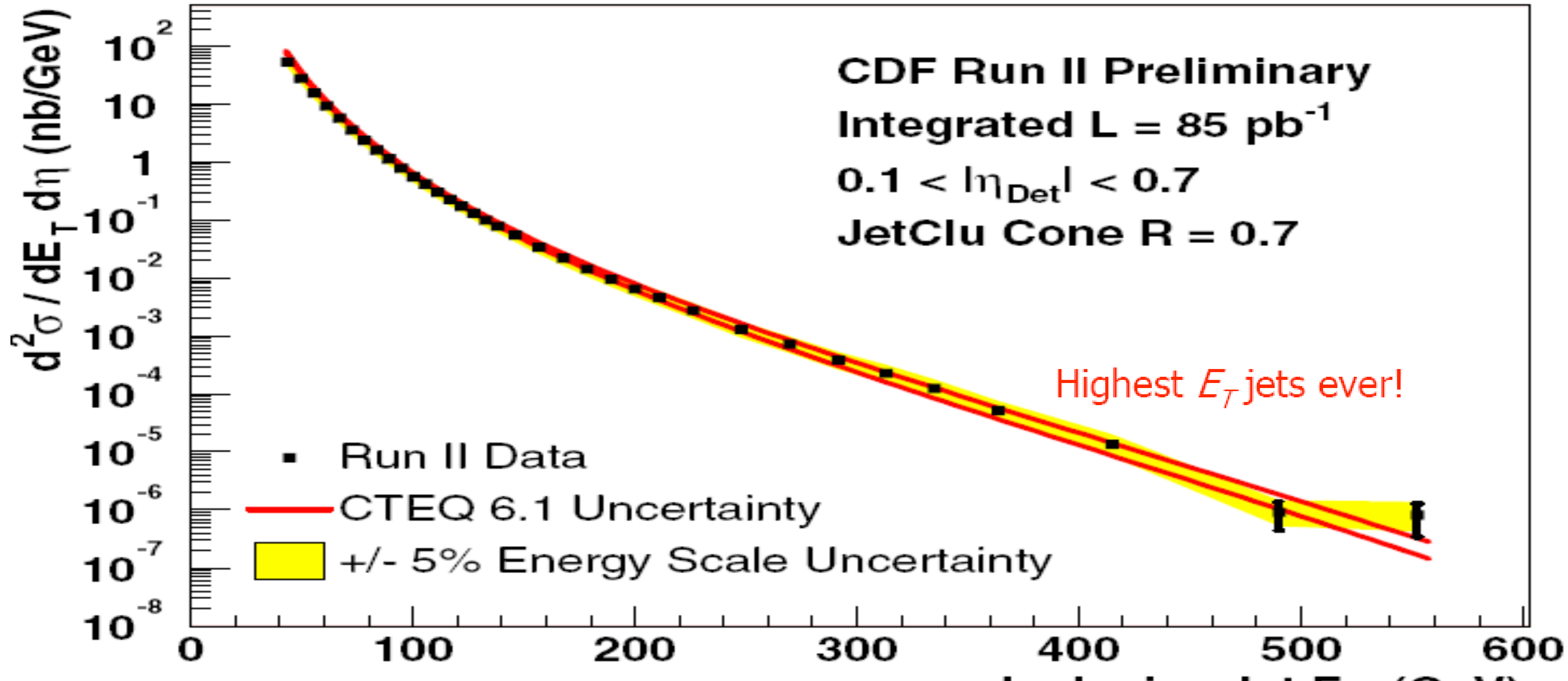
universal (measured) calculable



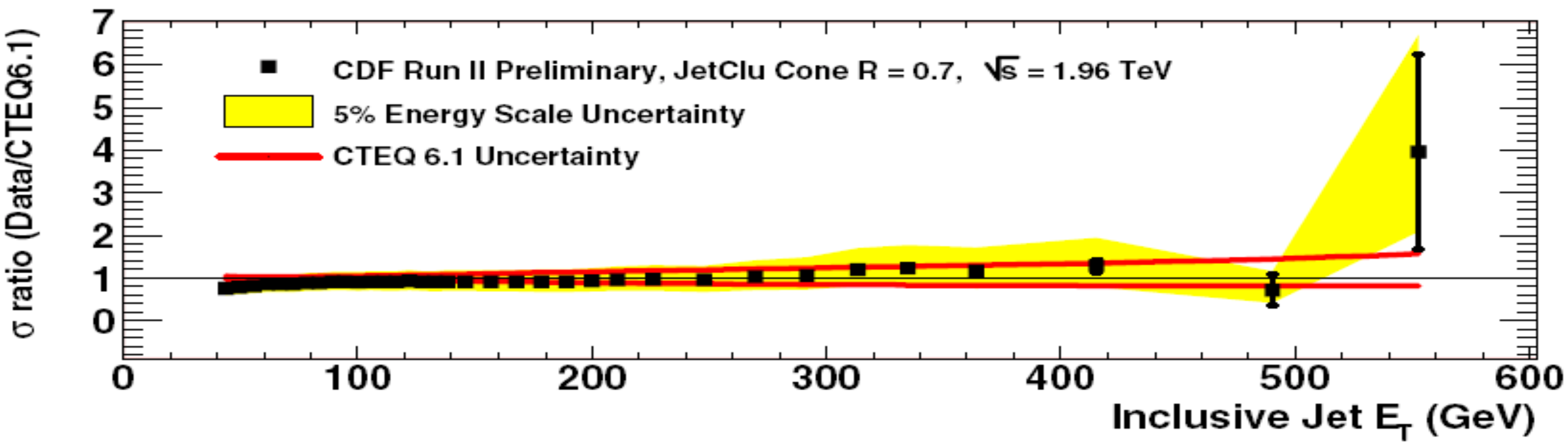
Parton Distribution Functions



Prediction vs CDF Run II data: inclusive jet cross section



- 8 orders of magnitude!



Good agreement (within uncertainties)

Success of pQCD factorization

- Experiments measure cross sections:

$$\sigma_{exp}(Q) \approx \boxed{H_0 \otimes f_2 \otimes f_2} + \frac{1}{Q} H_1 \otimes f_2 \otimes f_3 + \frac{1}{Q^2} H_2 \otimes f_2 \otimes f_4 + \mathcal{O}\left(\frac{1}{Q^3}\right)$$

- test short-distance dynamics at Q > 2GeV (or ≤ 1/10 fm): H_0

- also probe hadron structure via PDFs (probability distributions): f_2

- Question:

$$\sigma_{exp}(Q) \approx H_0 \otimes f_2 \otimes f_2 + \boxed{\frac{1}{Q} H_1 \otimes f_2 \otimes f_3 + \frac{1}{Q^2} H_2 \otimes f_2 \otimes f_4 + \mathcal{O}\left(\frac{1}{Q^3}\right)}$$

What about QCD dynamics for 1/10 fm ~ 1fm? $(H_1, H_2, \cdots)$

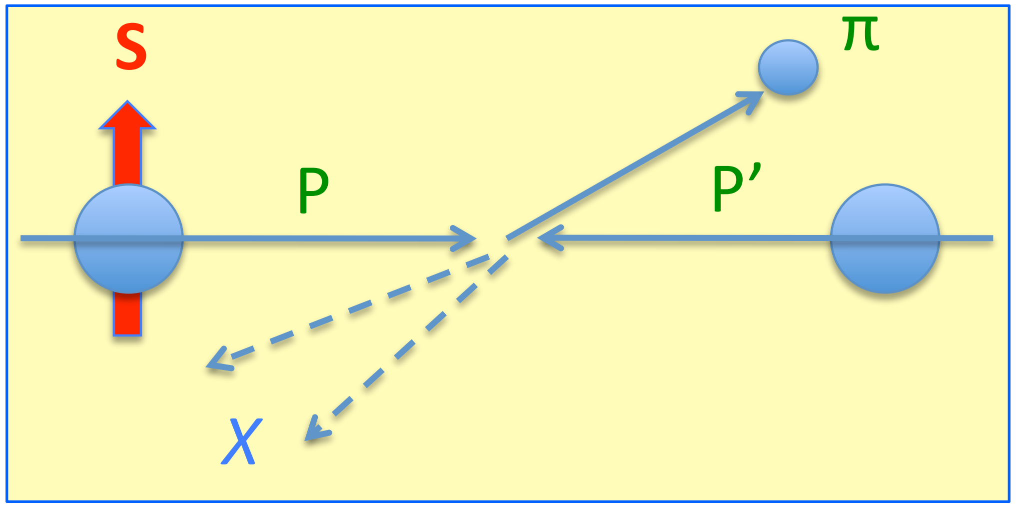
How to go beyond the probability distributions? $(f_3, f_4, \cdots)$

How to go beyond normal PDFs

- Take the difference (spin-dependent cross section)

$p^\uparrow p \to \pi X$

SSA: $A_N = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$



$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + \frac{1}{Q} H_1 \otimes f_2 \otimes f_3 + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

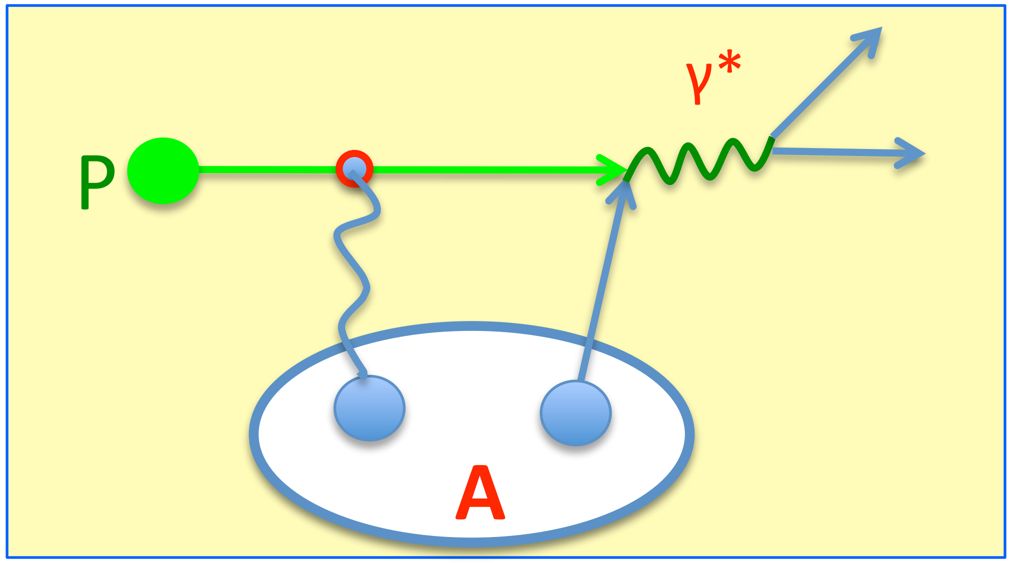
$$\Delta\sigma(Q, s_T) \approx \frac{1}{Q} H_1 \otimes f_2 \otimes f_3$$

➡ three-parton correlations

- Measure the nuclear dependence (A-dependence) in spin-averaged experiment

$$\sigma(Q, A) \approx H_0 \otimes f_2 \otimes f_2 + \frac{1}{Q^2} H_2 \otimes f_2 \otimes f_4 + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

$$\sigma(A) \propto \frac{1}{Q^2} H_2 \otimes f_2 \otimes f_4$$ (beyond normal PDFs)



➡ four-parton correlations

The core of my talk is about
three and four parton correlations and associated QCD dynamics

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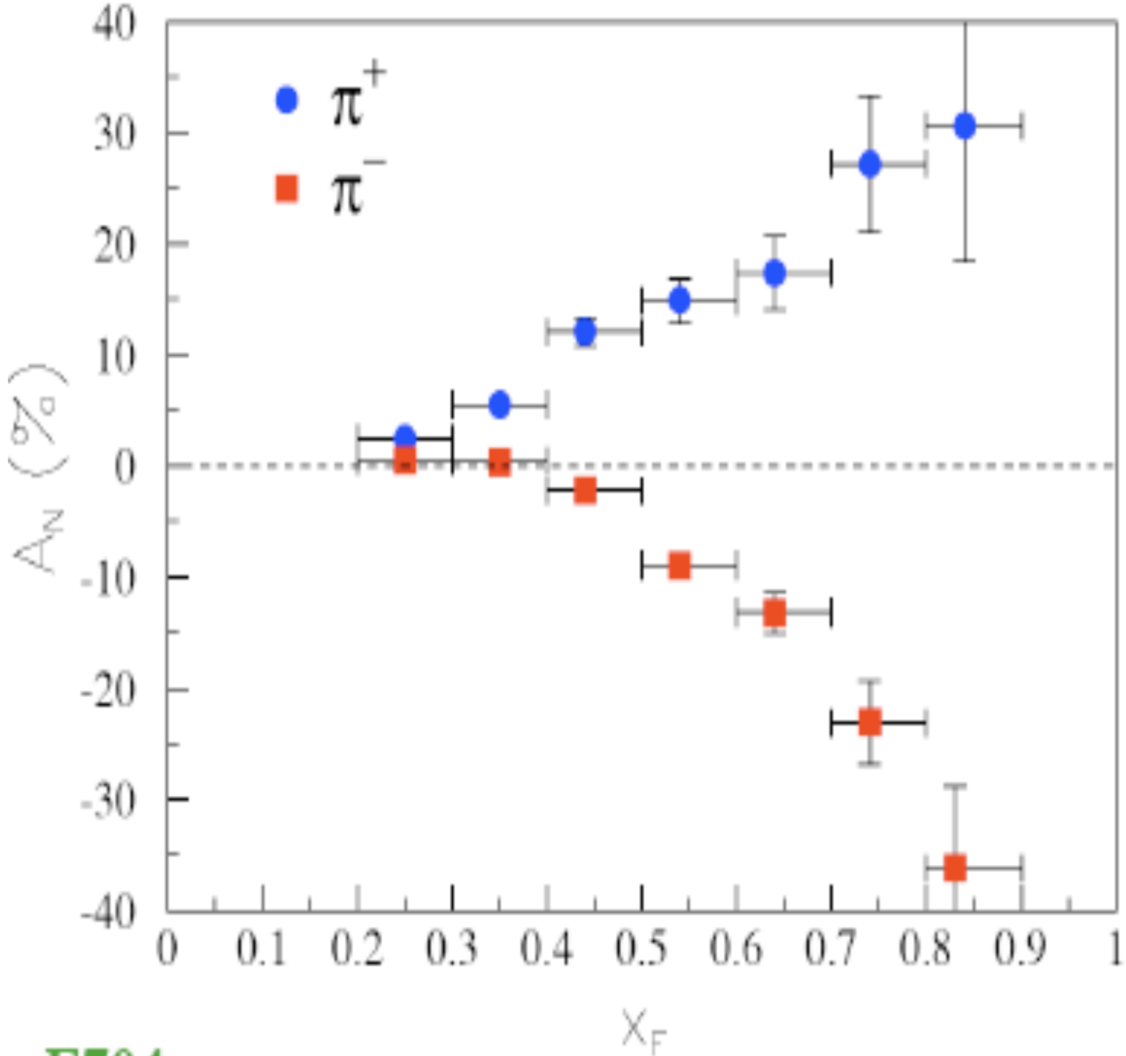
Experimental status of Single Spin Asymmetry (SSA)

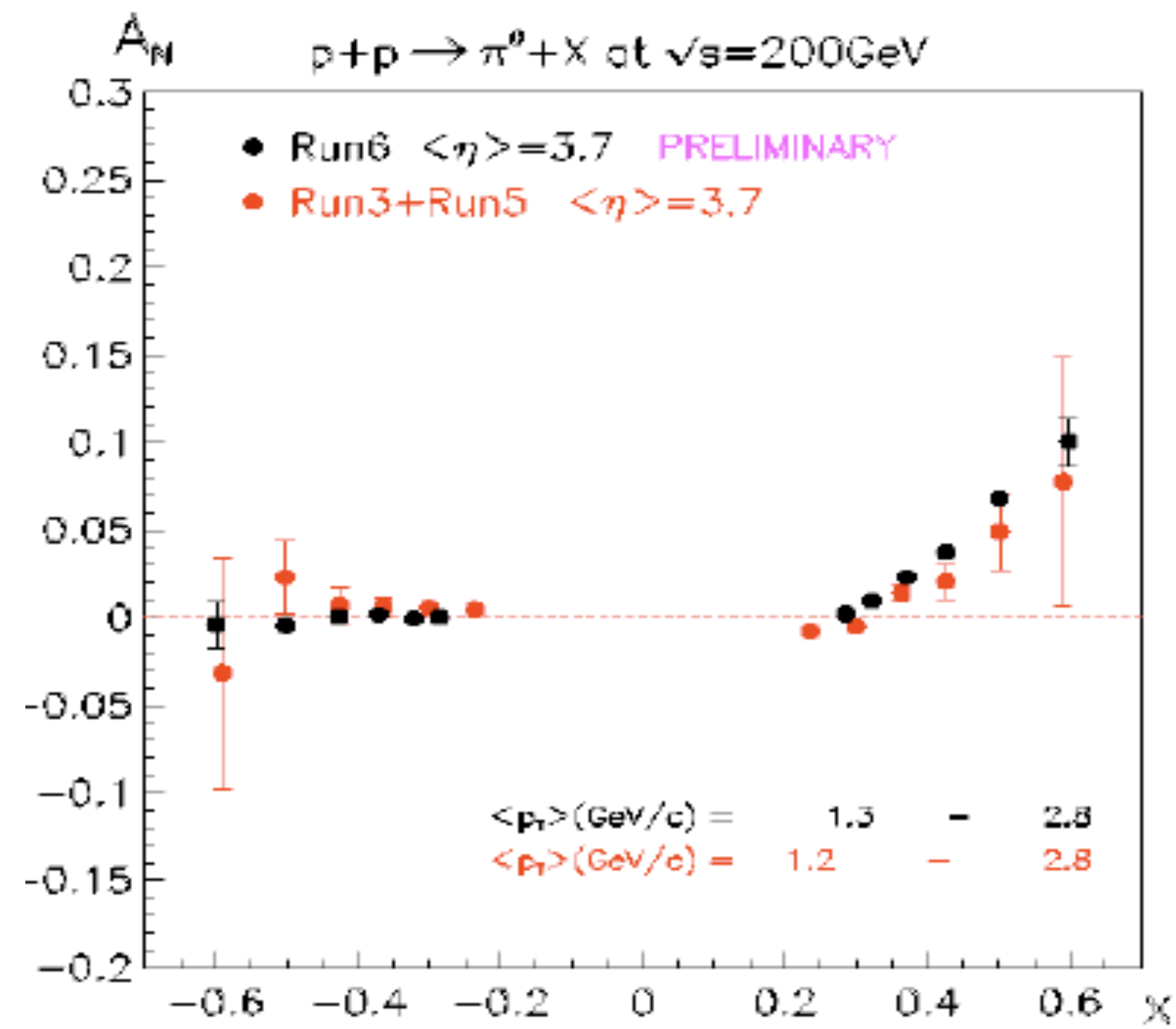
- Fermilab E704, STAR, PHENIX, BRAHMS, COMPASS, HERMES

$$p^{\uparrow}p \to \pi X$$

SSA: $A_N = \dfrac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$



E704



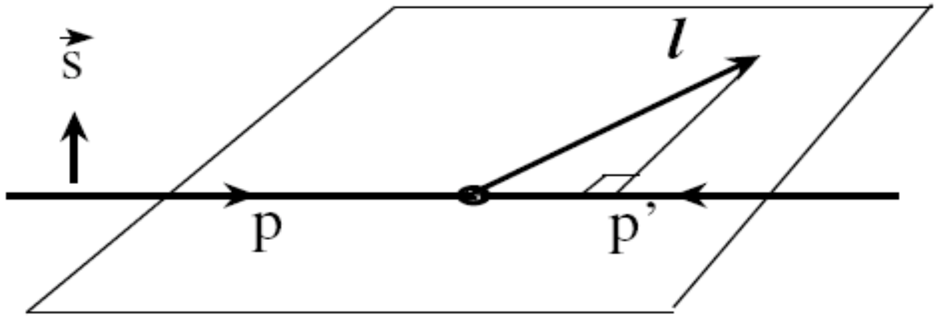
STAR (BRAHMS, too)

SSAs are observed in various experiments at different $\sqrt{s}$

SSA corresponds to a T-odd triplet product

- SSA measures the correlation between the hadron spin and the production plane, which corresponds to $\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$

$$p^{\uparrow} p \rightarrow \pi(\ell) X$$



- Such a product is odd under time reversal (T-odd), and thus they can arise in a time-reversal invariant theory (eg, QCD) only when there is a phase between different spin amplitudes.

$$\Rightarrow A_N \propto i\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$

- the phase "i" is required by time-reversal invariance

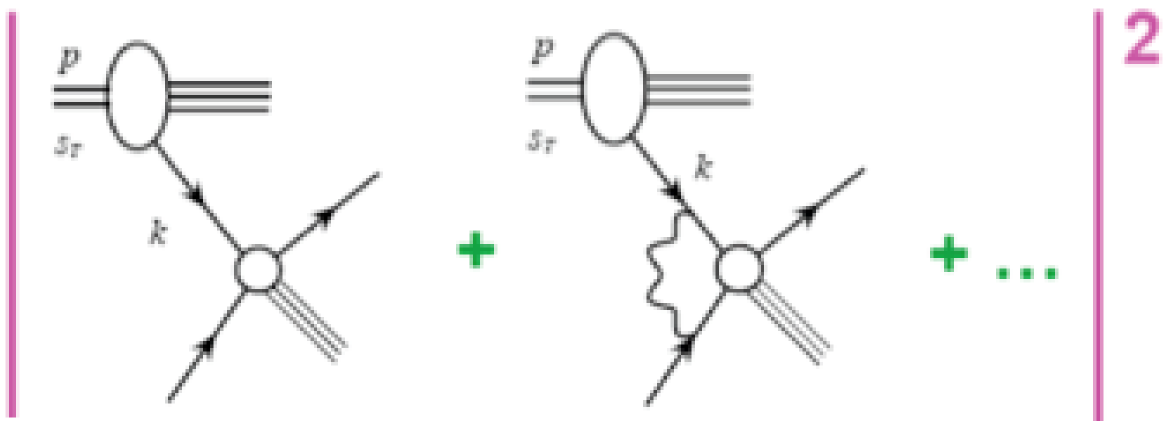
- covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$

Nonvanishing A_N requires a phase, a helicity flip

Non-vanishing SSA due to transverse motion

- If partons are purely collinear:

phase from loop   helicity is conserved for massless partons

$\sigma(s_T) \sim$ 

$$A_N \sim \alpha_s \frac{m_q}{\sqrt{s}} \to 0$$

Kane, Pumplin, Repko, 1978

- $A_N \neq 0$: result of parton's transverse motion or correlations!
- Two approaches to generate A_N:

✓ TMD approach: Transverse Momentum Dependent distributions probe parton's intrinsic transverse momentum (Sivers function, ...)

✓ Collinear factorization approach: twist-3 multi-parton correlation

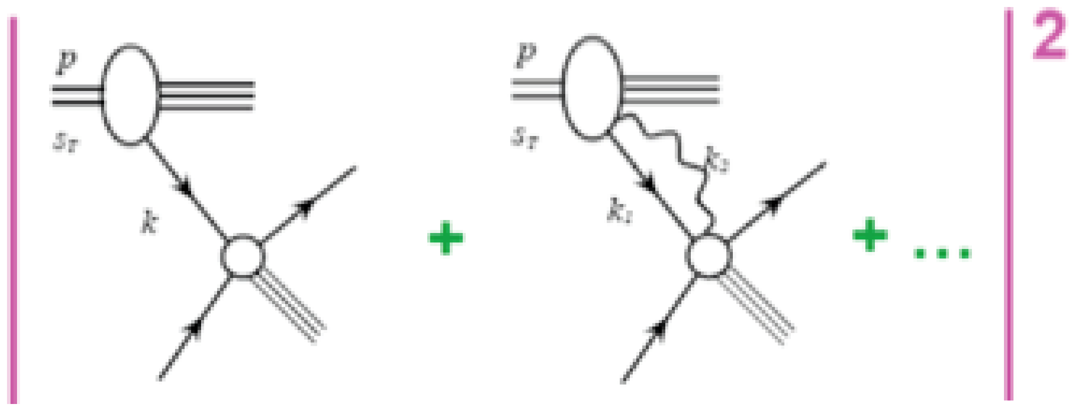
- They apply in different kinematic domain
- Collinear factorization approach is more relevant for single scale hard process

SSA in collinear factorization approach

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

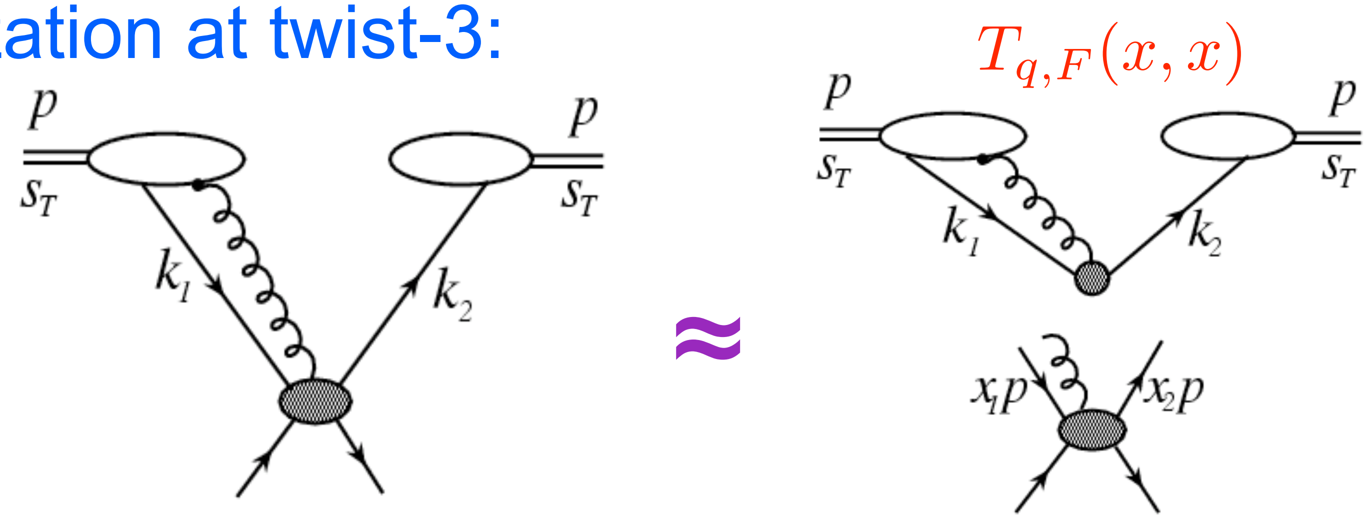
- When all observed scales >> Λ_{QCD}, collinear factorization should work:

$\sigma(s_T) \sim$



- phase: from hard scattering amplitudes (unpinched pole)
- spin flip: from interference between a quark state and a quark-gluon composite state

- Factorization at twist-3:



$$A_N \sim \frac{\langle k_\perp \rangle}{P_\perp}$$

- Twist-3 quark-gluon correlation function $T_{q,F}(x,x)$:

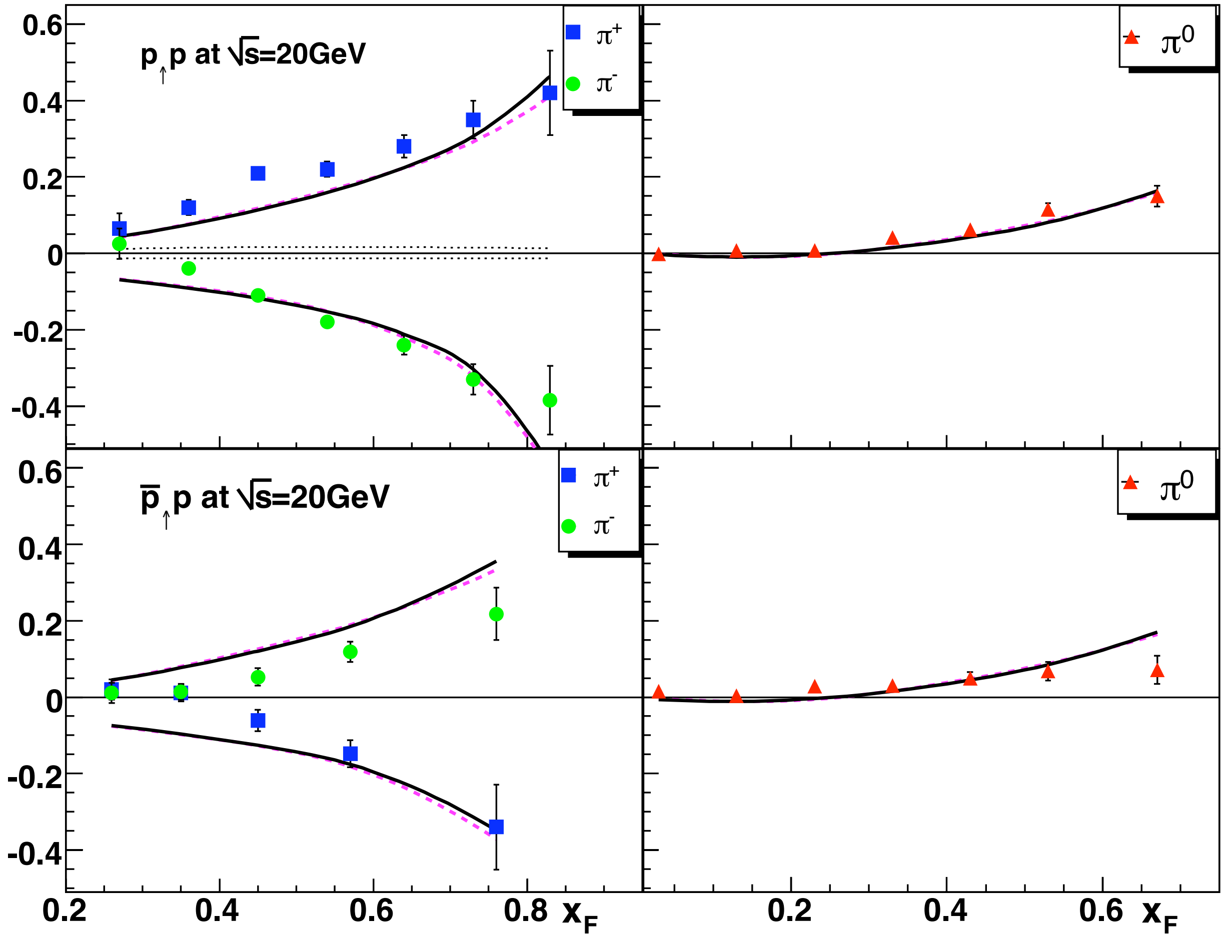
$$T_{q,F}(x,x) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ixP^+y_1^-} \langle P, s_T | \bar{\psi}_q(0)\gamma^+ \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma^{\ +}(y_2^-)\right] \psi_q(y_1^-) | P, s_T \rangle$$

Three field operators do not have the probability interpretation of normal parton distributions

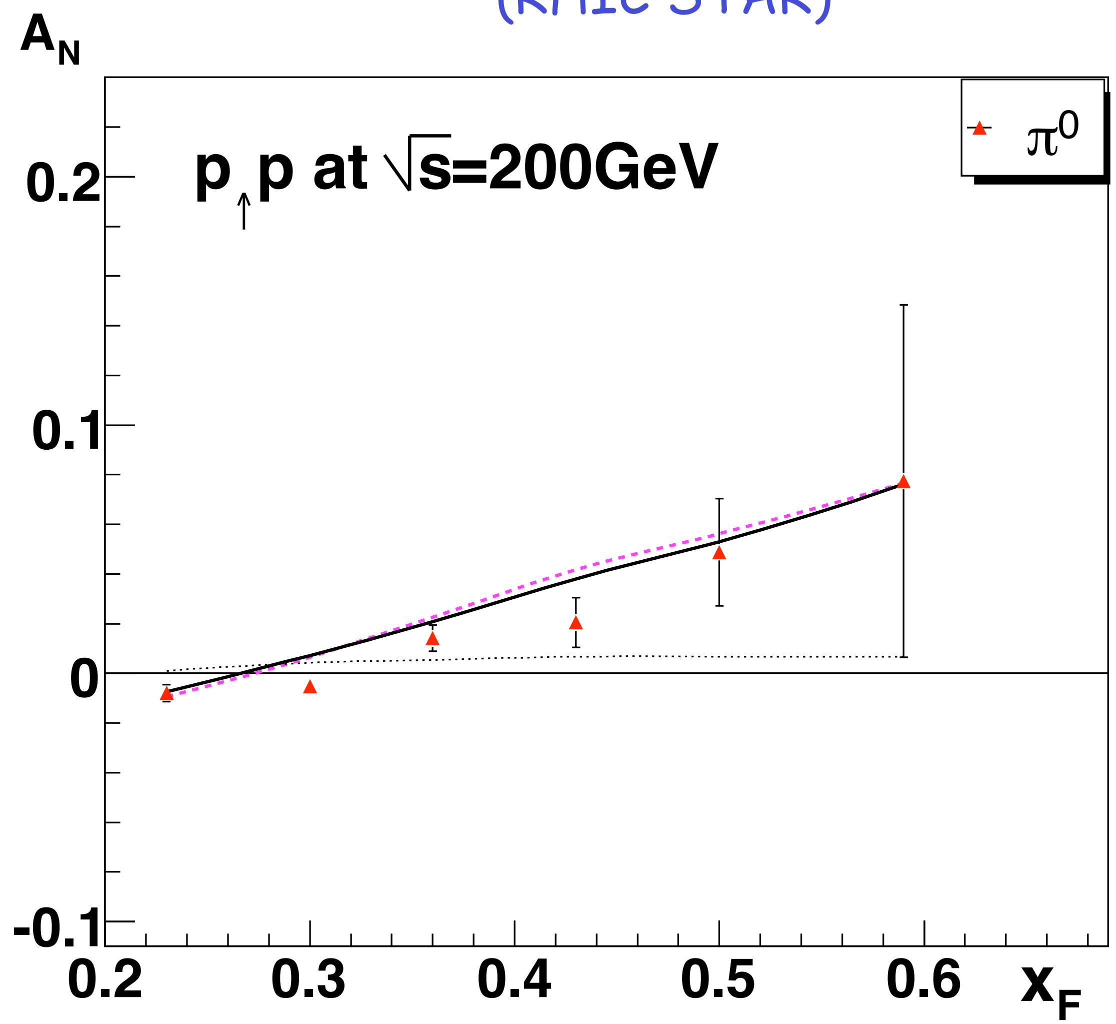
SSAs from twist-3 quark-gluon correlation $T_{q,F}(x,x)$

Kouvaris,Qiu,Vogelsang,Yuan, 2006

(FermiLab E704)



(RHIC STAR)



$T_{q,F}(x,x)$ only in forward region

Initial success of the twist-3 formalism

There are more than just the quark-gluon correlation.
What about the others?

Twist-3 trigluon correlation functions

- Diagonal tri-gluon correlations:

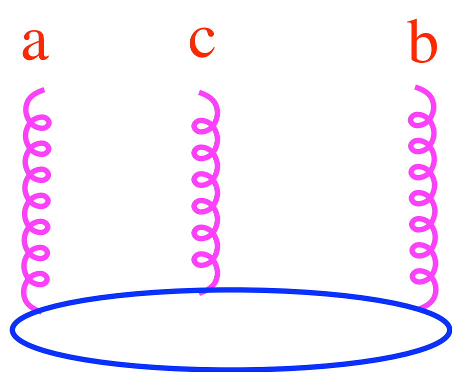
Ji, 1992; Kang, Qiu, 2008
Kang, Qiu, Vogelsang, Yuan, 2008

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \times \frac{1}{xP^+} \langle P, s_T | F^+{}_\alpha(0) \left[\epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] F^{\alpha +}(y_1^-) | P, s_T \rangle$$

- Two tri-gluon correlation functions - different color factors

$$T_G^{(f)}(x,x) \propto i f^{abc} F^a F^c F^b = F^a F^c \left[\mathcal{T}^c\right]^{ab} F^b$$

$$T_G^{(d)}(x,x) \propto d^{abc} F^a F^c F^b = F^a F^c \left[\mathcal{D}^c\right]^{ab} F^b$$



Fermionic correlation: $T_F(x,x) \propto \bar{\psi}_i F^c \left[T^c\right]_{ij} \psi_j$

- D-meson production in Semi Inclusive Deep Inelastic Scattering (SIDIS):
  - Clean probe for twist-3 tri-gluon correlation functions

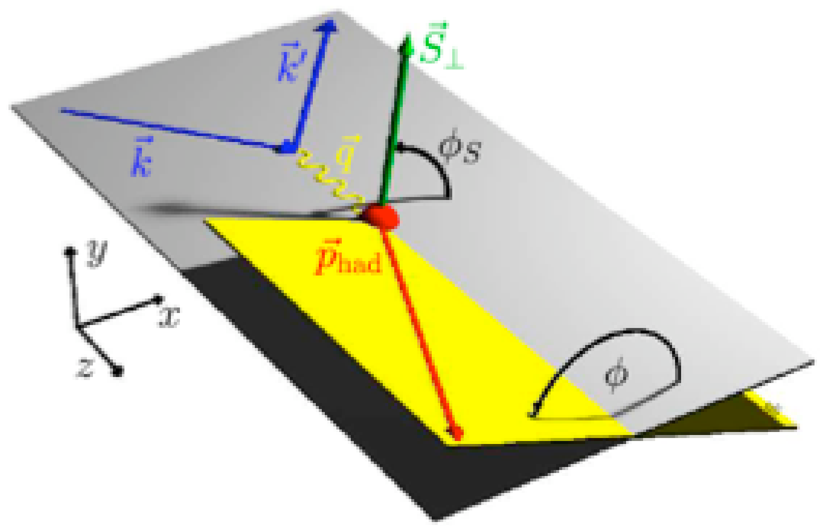
D-meson production in SIDIS: $ep^{\uparrow} \to e + D + X$

- **Frame for SIDIS:**

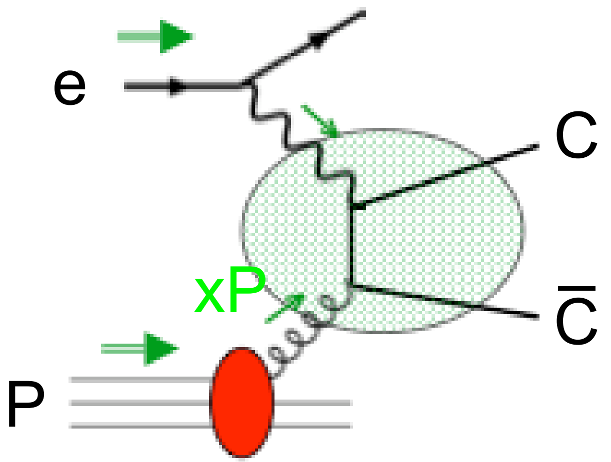
Kang, Qiu, PRD78, 034005 (2008)

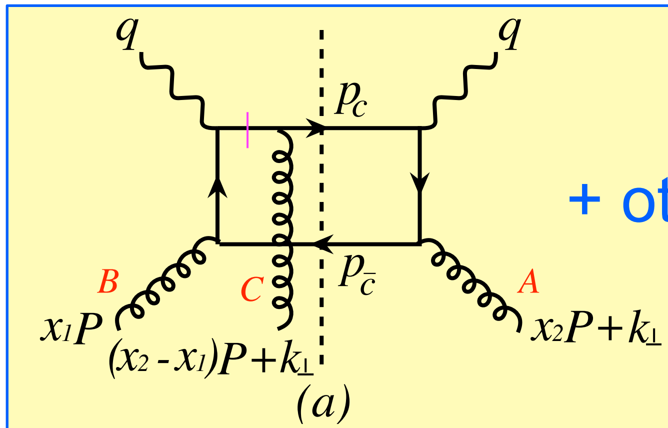
$$e(k) + p^{\uparrow}(P) \to e(k') + D(P_h) + X$$

$$q = k - k' \qquad z_h = \frac{P \cdot P_h}{P \cdot q} = E_h/\nu$$



- **Dominated by the contribution from trigluon correlations**





+ other digrams

- **Single transverse-spin asymmetry:**

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

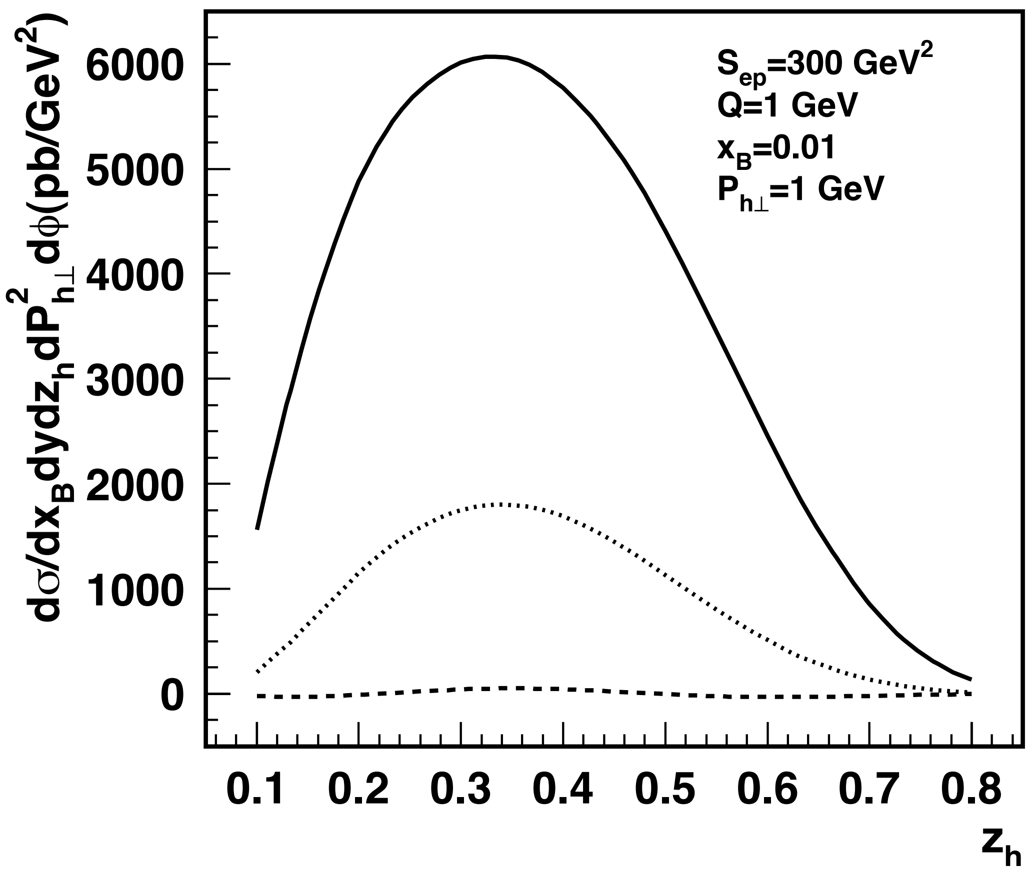
Production rate of D-meson in SIDIS

- Production rate (spin averaged):

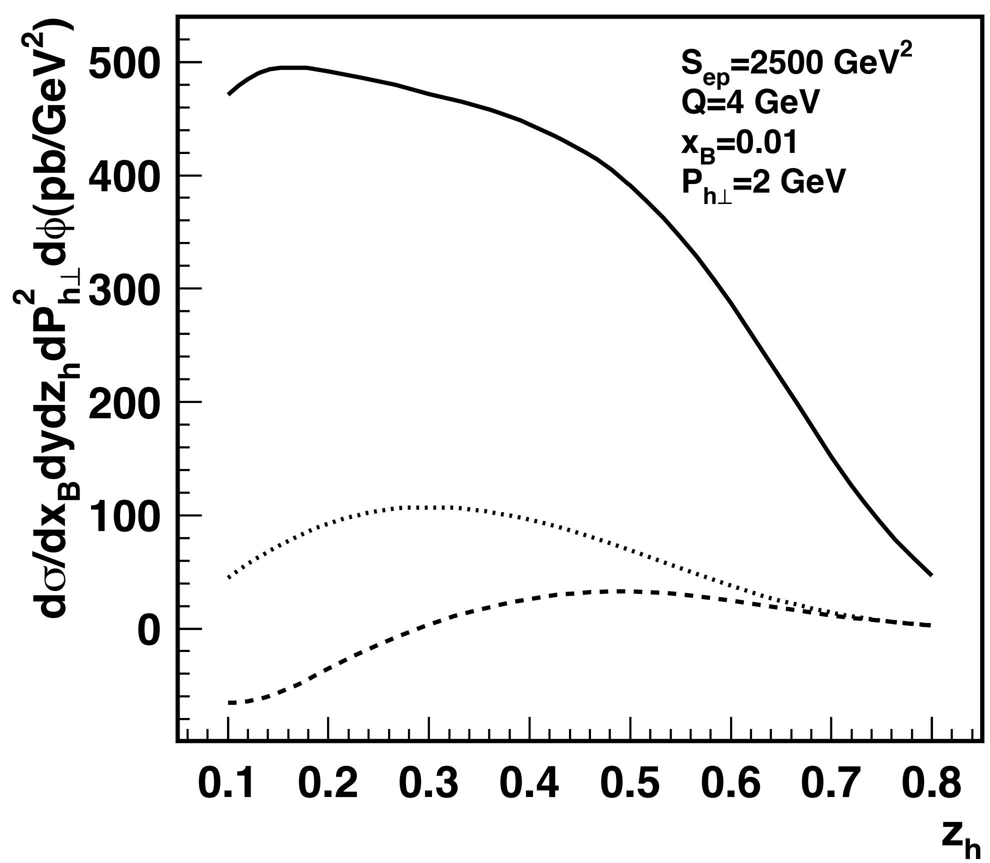
$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi} = \sigma_0^U + \sigma_1^U \cos\phi + \sigma_2^U \cos 2\phi$$

$$z_h = \frac{P \cdot P_h}{P \cdot q} = E_h/\nu$$

z_h: Energy fraction of photon carried by D-meson



COMPASS



EIC

reasonable production rate, small φ dependence

Features of the SSA in SIDIS

- Dependence of tri-gluon correlation functions:

$$D\text{−meson} \;\propto\; T_G^{(f)} + T_G^{(d)} \qquad\qquad \overline{D}\text{−meson} \;\propto\; T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- A_N depends on $T_G(x,x)$ and its derivative:

$$A_N \quad \propto \quad \epsilon^{P_h s_T n \bar{n}} \frac{1}{t} \frac{-x\frac{d}{dx}T_G(x,x)}{G(x)} \rightarrow 1/(1-x)$$

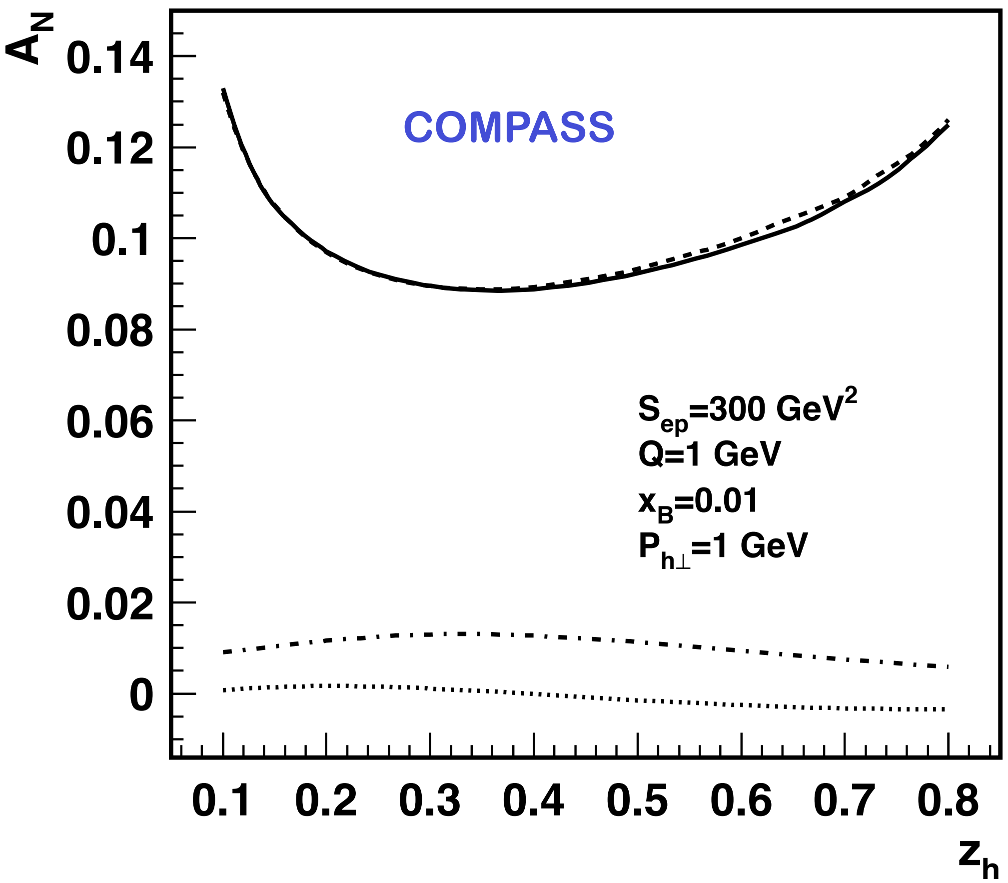
Since x has a minimum at $z_h \sim 0.5$ (from kinematics constrain), SSA should have a minimum if the derivative term dominates

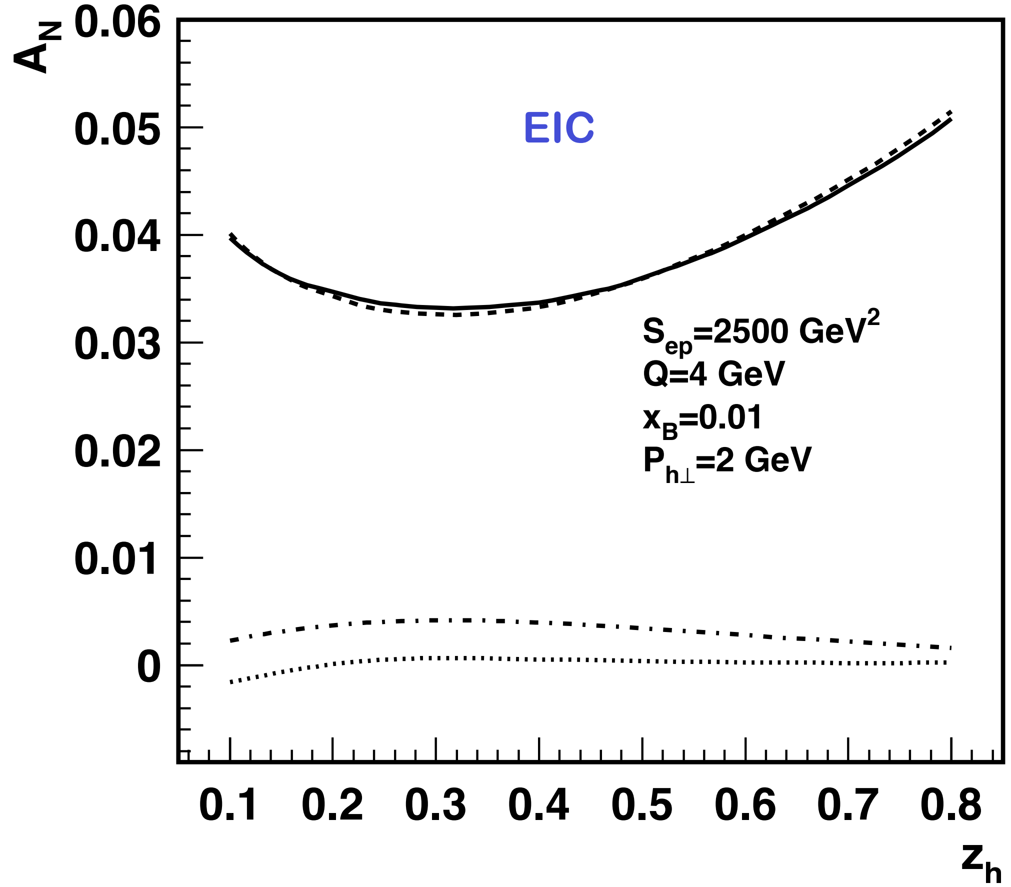
- Model for tri-gluon correlation functions:

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad\qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07 \text{ GeV}$$

Estimation of SSA in D-meson production in SIDIS

SSA for D^0 production ($T_G^{(f)}$ only)





- **Derivative term dominates, and small φ dependence**
- **Asymmetry is twice if $T_G^{(d)} = +T_G^{(f)}$, or zero if $T_G^{(d)} = -T_G^{(f)}$**
- **Opposite for the $\bar{D}$ meson**
- **Asymmetry has a minimum ~ z_h ~ 0.5**

Measure the SSAs ⇒ extract tri-gluon correlations

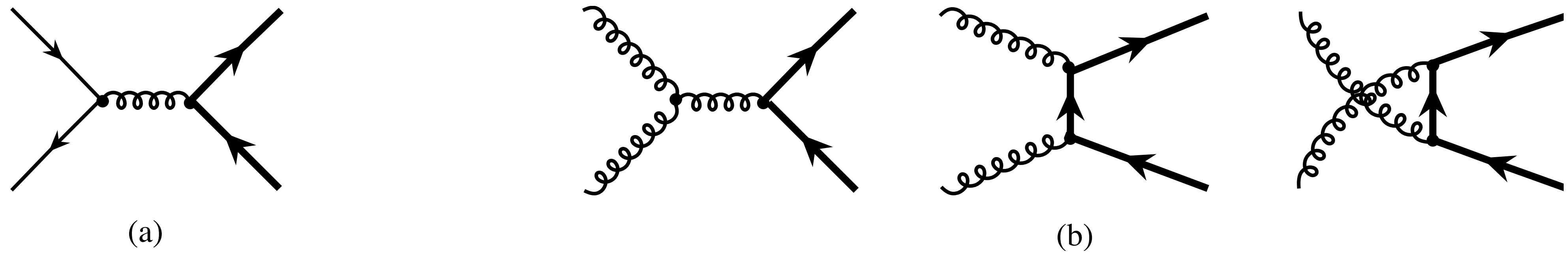
Test QCD: universality of tri-gluon correlations?

D-meson production in hadronic collisions: $p^{\uparrow}p \to D + X$

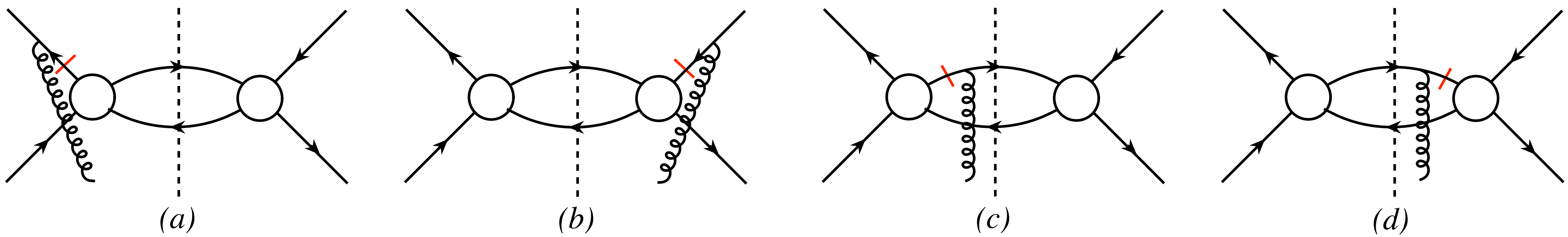
- Two partonic subprocesses:

Kang, Qiu, Vogelsang, Yuan, PRD78, 114013 (2008)

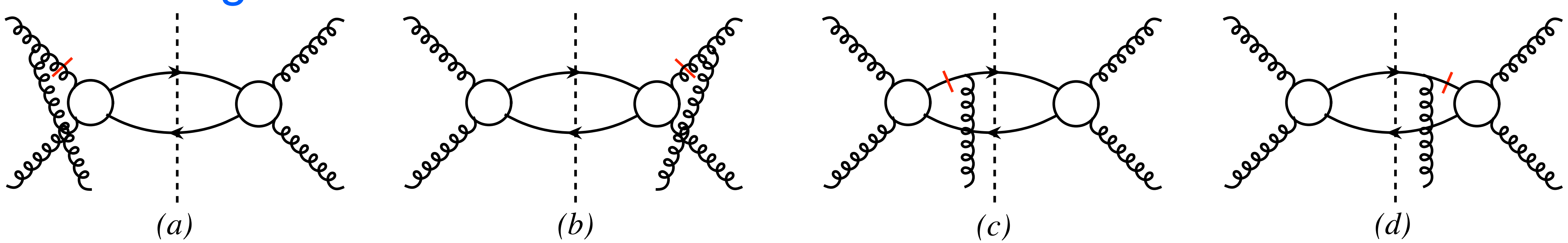
$p^{\uparrow}p \to D + X$



- Quark-antiquark annihilation:



- Gluon-gluon fusion:



Spin-dependent cross section for D-meson production

- SSA from both quark-gluon correlation $T_{q,F}(x,x)$ and tri-gluon correlation $T_G(x,x)$

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}\bigg|_{gg\to c\bar{c}} = \frac{\alpha_s^2}{S}\sum_{i=f,d}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\phi_{g/B}(x')\int\frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_h s_T n\bar{n}}}{z\tilde{u}}\right)\delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)$$
$$\times\left[\left(T_G^{(i)}(x,x)-x\frac{d}{dx}T_G^{(i)}(x,x)\right)H^{(i)}_{gg\to c}(\tilde{s},\tilde{t},\tilde{u})+T_G^{(i)}(x,x)\mathcal{H}^{(i)}_{gg\to c}(\tilde{s},\tilde{t},\tilde{u})\right]$$

- depends on correlation and its derivative
- same factorized form for $q\bar{q}$ subprocess

- Features of SSA:

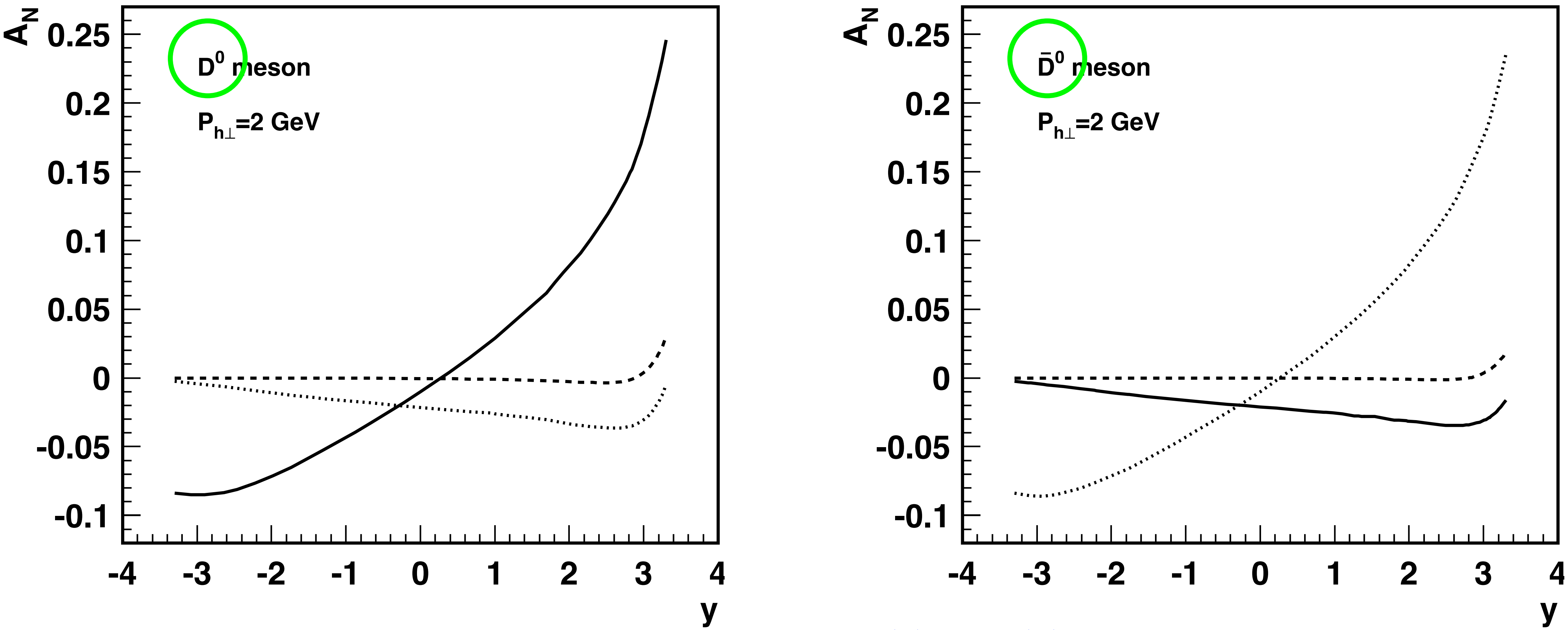
When $c\to\bar{c}$

- hard parts do NOT change sign for $T_{q,F}$ and $T_G^{(f)}$
- hard parts change sign for $T_G^{(d)}$

➡ SSA will be very different for D and $\bar{\text{D}}$ if $T_G^{(d)}\neq 0$
SSA will be very similar for D and $\bar{\text{D}}$ if $T_G^{(d)}=0$

Rapidity dependence of D-meson production

- SSA at RHIC: $\sqrt{s} = 200$ GeV $\mu = \sqrt{m_c^2 + P_{h\perp}^2}$ $m_c = 1.3$ GeV



Solid: $(1)\lambda_f = \lambda_d = 0.07$ GeV $T_G^{(d)} = T_G^{(f)}$

Dotted: $(2)\lambda_f = -\lambda_d = 0.07$ GeV $T_G^{(d)} = -T_G^{(f)}$

Dashed: $(3)\lambda_f = \lambda_d = 0$ $T_G^{(d)} = T_G^{(f)} = 0$

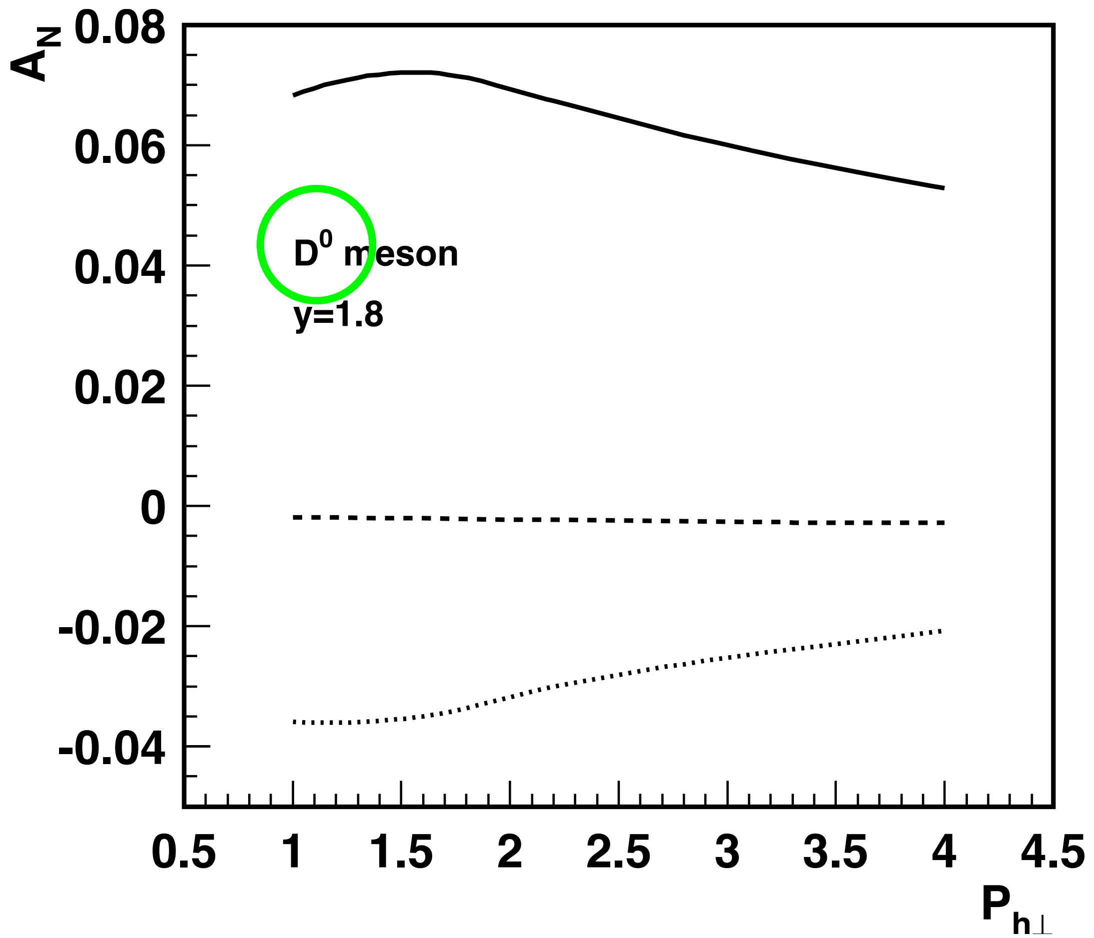
D meson : Largest A_N happens when $T_G^{(d)} = +T_G^{(f)}$

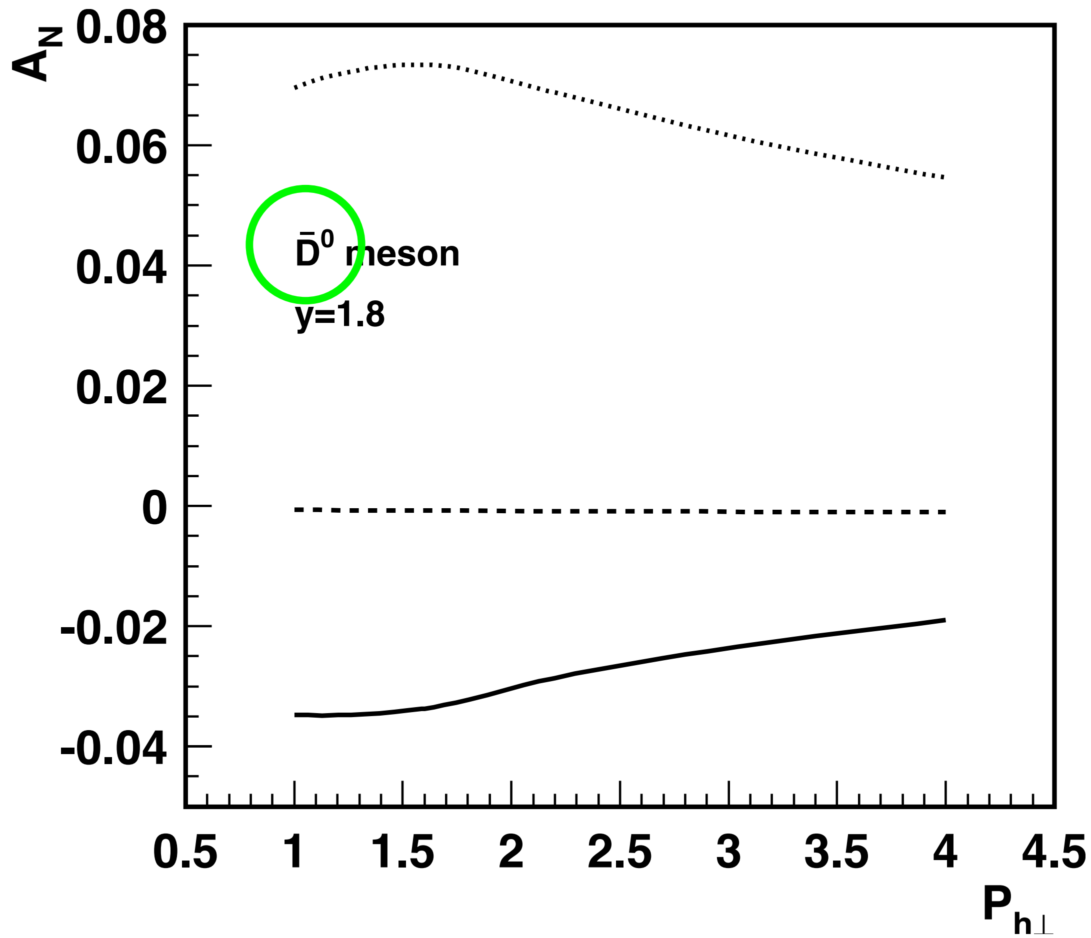
$\bar{D}$ meson : Largest A_N happens when $T_G^{(d)} = -T_G^{(f)}$

P_T-dependence of D-meson production

- SSA at RHIC

$\sqrt{s} = 200$ GeV $\quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3$ GeV





$(1)\lambda_f = \lambda_d = 0.07$ GeV $\quad T_G^{(d)} = T_G^{(f)}$

$(2)\lambda_f = -\lambda_d = 0.07$ GeV $\quad T_G^{(d)} = -T_G^{(f)}$

$(3)\lambda_f = \lambda_d = 0 \quad T_G^{(d)} = T_G^{(f)} = 0$

- Without tri-gluon correlation, SSA is too small to be observed
- As a twist-3 effect, the SSAs fall off as $1/P_T$ when $P_T >> m_c$

Strong scale dependence of SSA

- So far, all the calculations for SSA are at leading order (LO)

$$\Delta\sigma(Q, s_T) = \frac{1}{Q} H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- LO result has a strong scale dependence

  - Evolution of correlation function (long-distance distributions)
  - NLO correction (short-distance contribution beyond LO)

- Question: what are the complete set of correlation functions?

Recall: Leading twist (DGLAP evolution equation)

- unpolarized PDFs: $q(x)$, $G(x)$
- helicity distributions: $\Delta q(x)$, $\Delta G(x)$

twist-3 spin dependent correlations:

two sets of correlation functions

Transverse spin wish (to do) list

Experiment

- Drell-Yan
- Photon-jet
- Tensor charge (h_1)
- Large P_t SSA (1/P_t)
- Double spin asymmetry P_t depdence
- W and z production (reconstruction low pt)
- Flavor separation via He3 at RHIC
- large-x Sivers/Collins
- Polarized nucleon-nucleus experiments (nuclear effects in B-M function)

Theory

- Relation to OAM
- Evolution
- Soft gluon resummation
- Robust separation of Sivers and Collins in pp
- P_t behavior
- Explore More functional dependence (kt,x)

From Yuan's talk at SPIN 2008

- ❑ What if we don't see DY=-DIS
- ❑ Can we determine the sign of the transversity function?

BERKELEY LAB

RIKEN BNL Research Center

PKU - RBRC Workshop on Transverse Spin Physics

June 30th - July 4th, 2008

Twist-3 three-parton correlations

Qiu, Sterman, 1991, 1998
Ji, 1992, Kang, Qiu, 2008

- **Set I: spin-averaged twist-2 PDFs + an operator Insertion**

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0) \frac{\gamma^+}{2} \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle \qquad q(x)$$

$$\widetilde{T}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T|F^{+\rho}(0) \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda}) \qquad G(x)$$

- **Set II: spin-dependent twist-2 HDFs + an operator Insertion**

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T|\overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle \qquad \Delta q(x)$$

$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T|F^{+\rho}(0) \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle\, (i\epsilon_{\perp\rho\lambda}) \qquad \Delta G(x)$$

Two possible color contractions: if_{abc}, d_{abc}

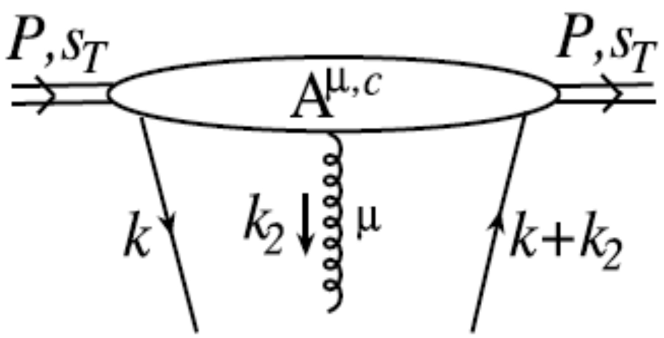
⇒Two tri-gluon correlation functions

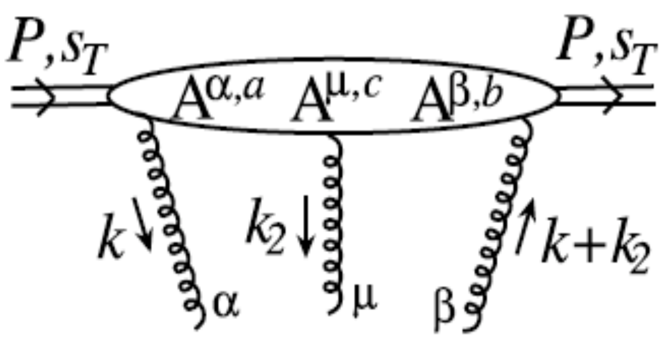
- **$T^{(f)}$ connects to gluon Sivers function**
- **$T^{(d)}$ has no connections to TMD distribution**

Feynman diagram reperesentation

Diagrams:

Kang, Qiu, arXiv: 0811.3101, 2008
to appear in PRD (2009)





$A^{\mu} \leftrightarrow F^{+\mu}$

Lagrangian → Feynman rules → Feynman diagrams → Cross section

⇒ Operator definition → Cut vertices → Feynman diagrams → Perturbative change

Same diagram can represent different correlation functions depending on cut vertices

Cut vertices in the light-cone gauge:

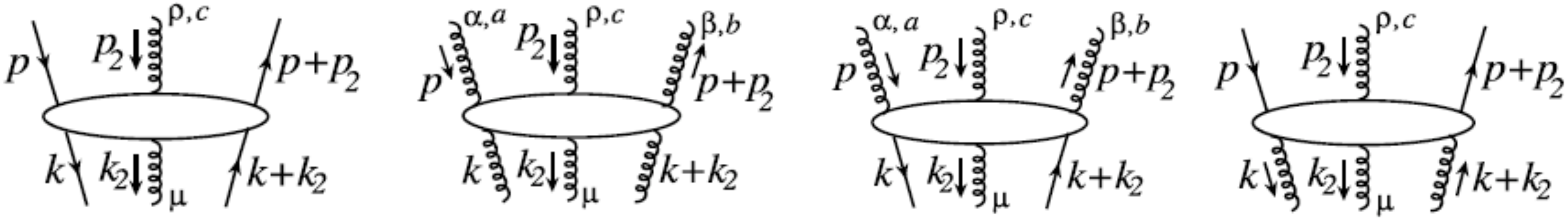
$$\mathcal{V}_{q,F}^{\rm LC} = \frac{\gamma^+}{2P^+}\,\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(i\,\epsilon^{s_T\sigma n\bar{n}}\right)[-g_{\sigma\mu}]\,\mathcal{C}_q\,,$$

$$\mathcal{V}_{G,F}^{\rm LC} = x(x+x_2)\,(-g_{\alpha\beta})\,\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(i\,\epsilon^{s_T\sigma n\bar{n}}\right)[-g_{\sigma\mu}]\,\mathcal{C}_g^{(f,d)}\,,$$

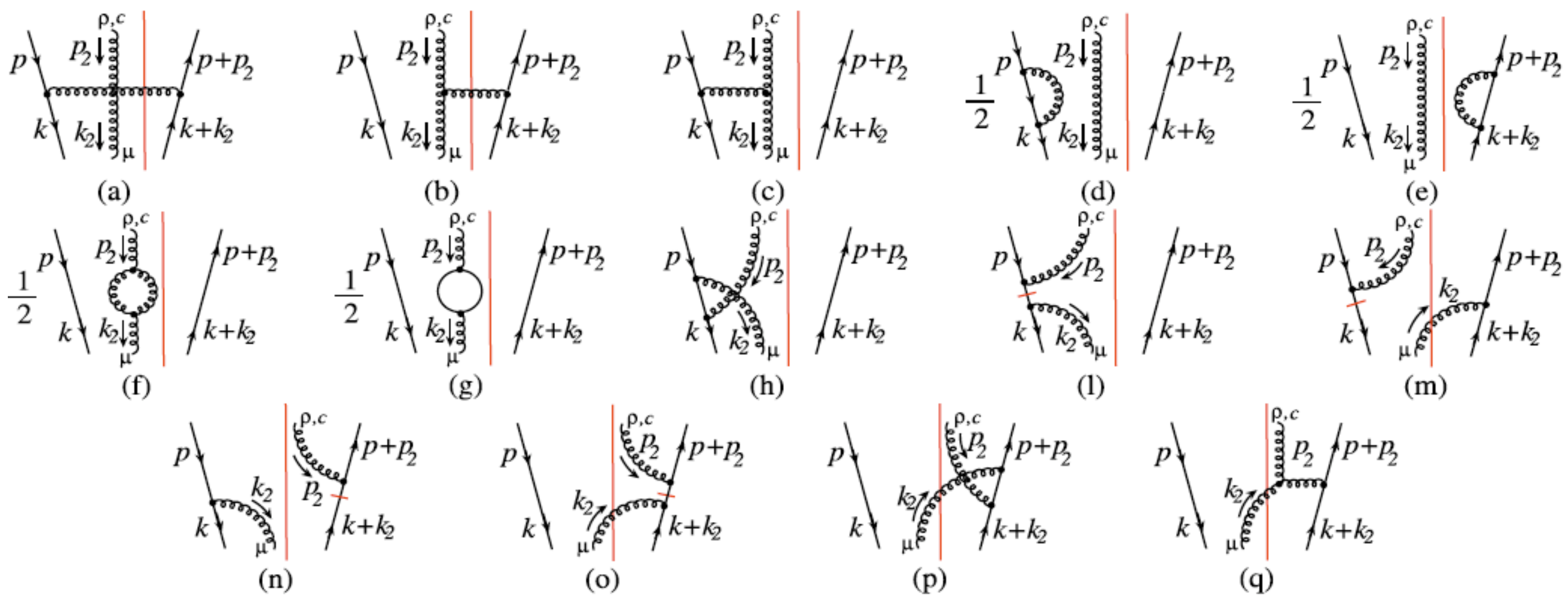
likewise for: $\mathcal{V}_{\Delta q,F}^{\rm LC}$ $\mathcal{V}_{\Delta G,F}^{\rm LC}$

Evolution kernels

- Feynman diagrams:



- Leading order for flavor non-singlet channel:



LO evolution equations - I

Diagonal contribution - Quarks:

relevant to single hadron production

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ P_{qq}(z)\, T_{q,F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F) - T_{q,F}(\xi,\xi,\mu_F)\right] + z\, T_{q,F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) + T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\Bigg\}$$

Diagonal contribution - Anti-quarks:

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ P_{qq}(z)\, T_{\bar{q},F}(\xi,\xi,\mu_F) + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F) - T_{\bar{q},F}(\xi,\xi,\mu_F)\right] + z\, T_{\bar{q},F}(\xi,x,\mu_F)\right] + \frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right] + P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F) - T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\Bigg\}$$

1. **All kernels are Infared safe**

2. **Diagonal term is the same as DGLAP**

3. **Singlet terms are different for quark and anti-quark**
   ⇒ they evolve differently (from tri-gluon correlations)

LO evolution equations - II

Diagonal contribution - Gluons:

$$
\begin{aligned}
\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ & P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right. \\
& \left. +2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right] \\
& + P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\sum_q \left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Bigg\}
\end{aligned}
$$

likewise for $T^{(f)}_{G,F}(x,x,\mu_F)$

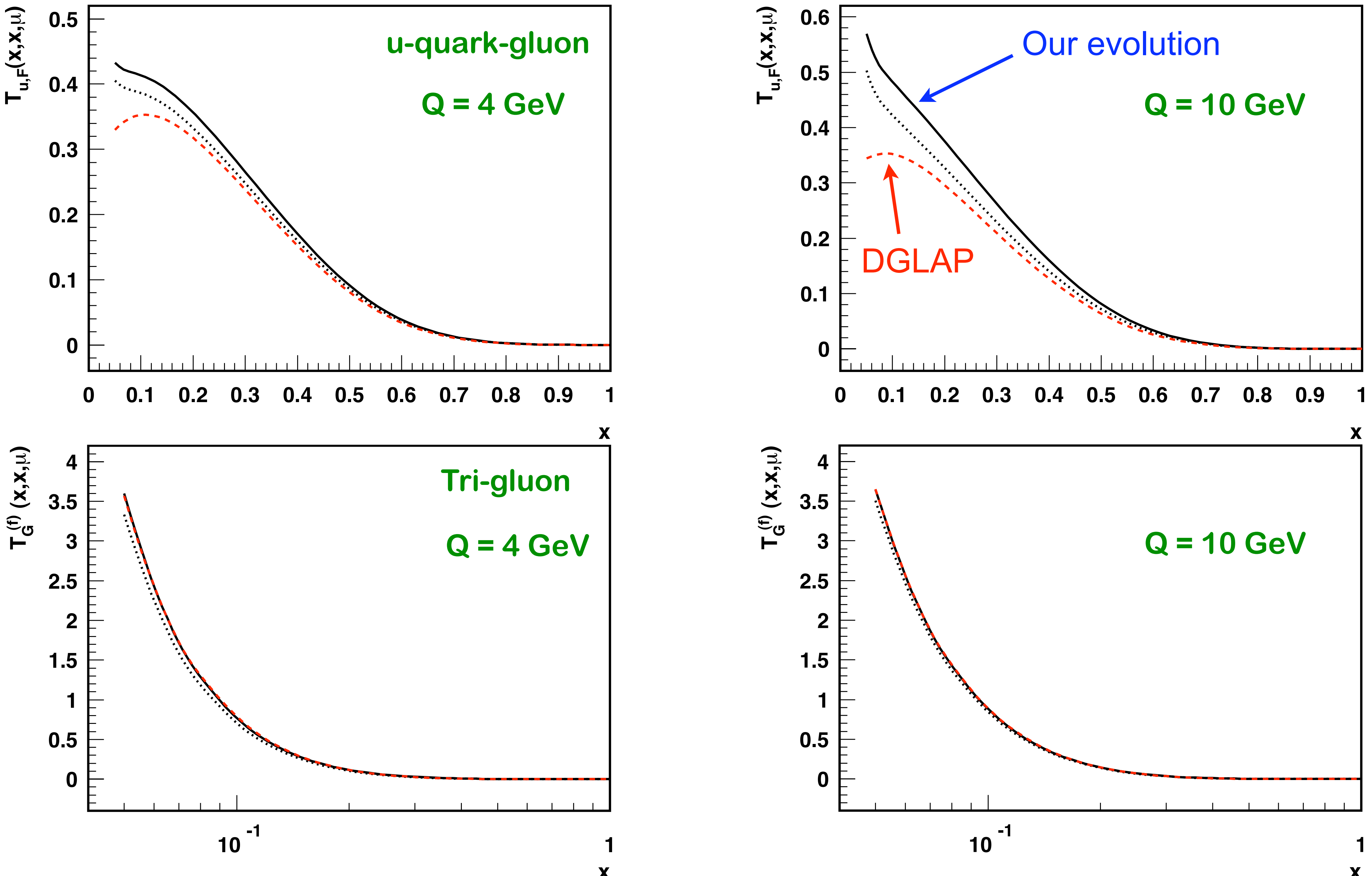
1. Similar features as in quark case:

IR safe, diagonal term same as DGLAP

2. $T_G^{(d)}$ **has no connection to TMD distribution. One may argue that** $T_G^{(d)}=0$ **, however,**

Evolution can generate $T_G^{(d)}$ **as long as** $\sum_q [T_{q,F}+T_{\bar{q},F}]\neq 0$

Q^2 - Dependence of correlation functions



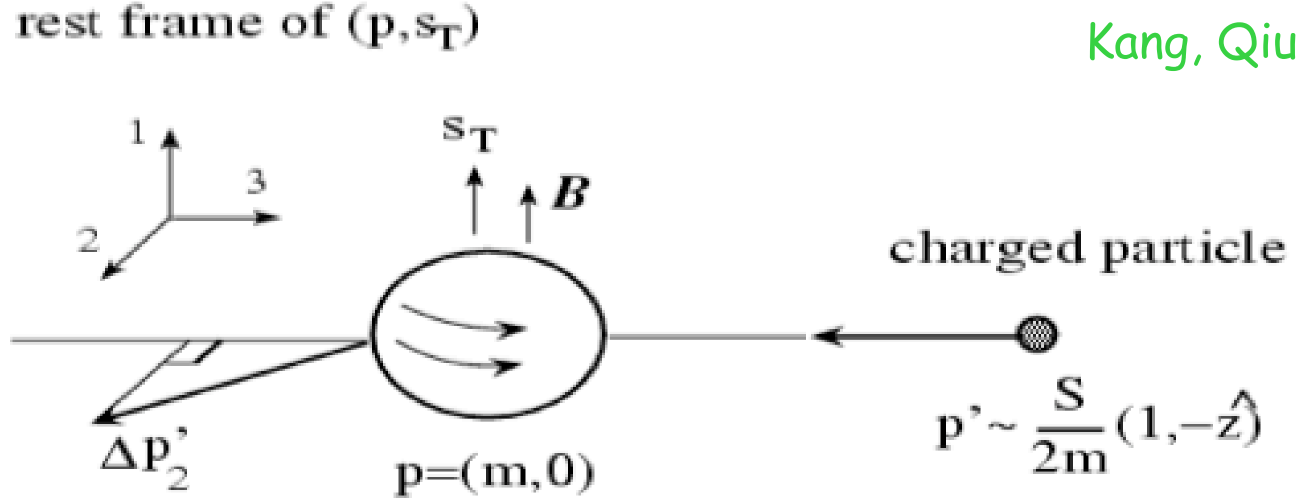
- Follow DGLAP for large x
- Large deviation in small x region (large coherence)

What can we learn from twist-3 correlation functions?

Set I: $\int dy^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y^-)$ Set II: $i \int dy_2^- e^{i x_2 P^+ y_2^-} \left[i\, s_T^\sigma F_\sigma{}^+(y_2^-) \right]$

Kang, Qiu, Sterman, in preparation

- Consider a classical (Abelian) situation:



– change of transverse momentum

$$\frac{d}{dt} p_2' = e(\vec{v}' \times \vec{B})_2 = -e v_3 B_1 = e v_3 F_{23}$$

– in the c.m. frame

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \frac{d}{dt} p_2' = e\, \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+$$

– total change: $\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y^-)$ → Set I

Set II: $\int dy_2^- e^{i x_2 P^+ y_2^-} \left[s_T^\sigma F_\sigma{}^+(y_2^-) \right] \propto \mu \cdot B$

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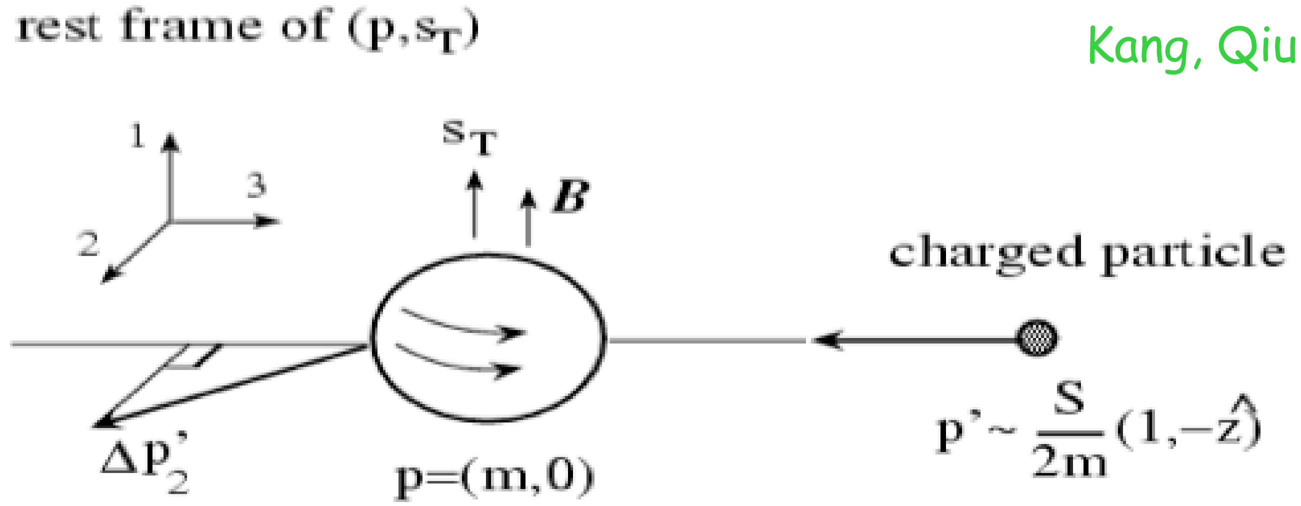

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Set II: $\int dy_2^- e^{ix_2 P^+ y_2^-} \left[s_T^\sigma\, F_\sigma{}^+(y_2^-) \right] \propto \mu \cdot B$

SSAs and three-parton correlations provide new information on hadron structure!

What about four parton correlations?

Outline

- Introduction: pQCD

- Go beyond probability distributions:

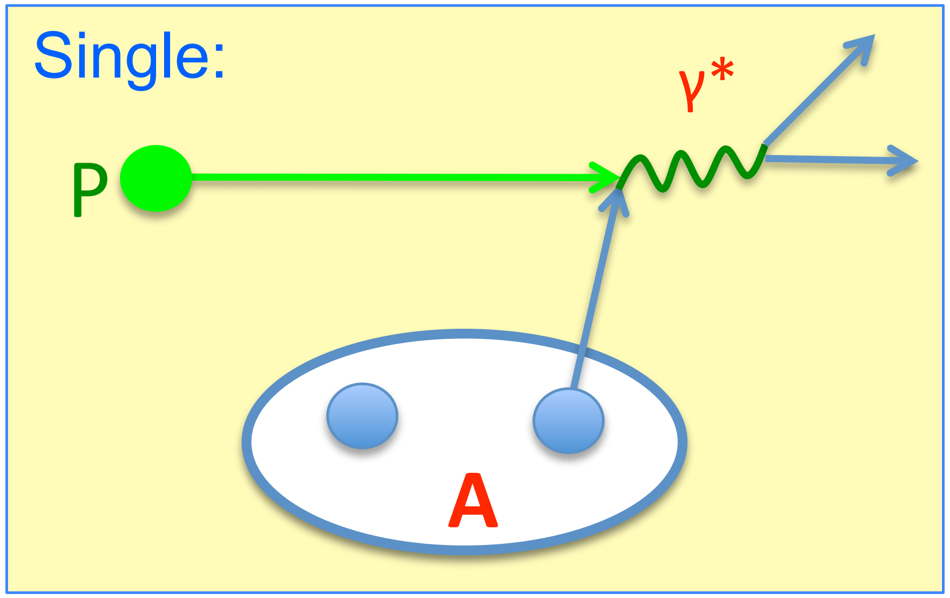
- Spin-dependent effect: three parton correlation
  - Tri-gluon correlation in ep and pp collision (LO)
  - Evolution of twist-3 correlations (beyond LO)
  - Physical meaning of twist-3 correlations

- Nuclear size dependent effect: four parton correlation
  - nuclear transverse momentum broadening of vector bosons

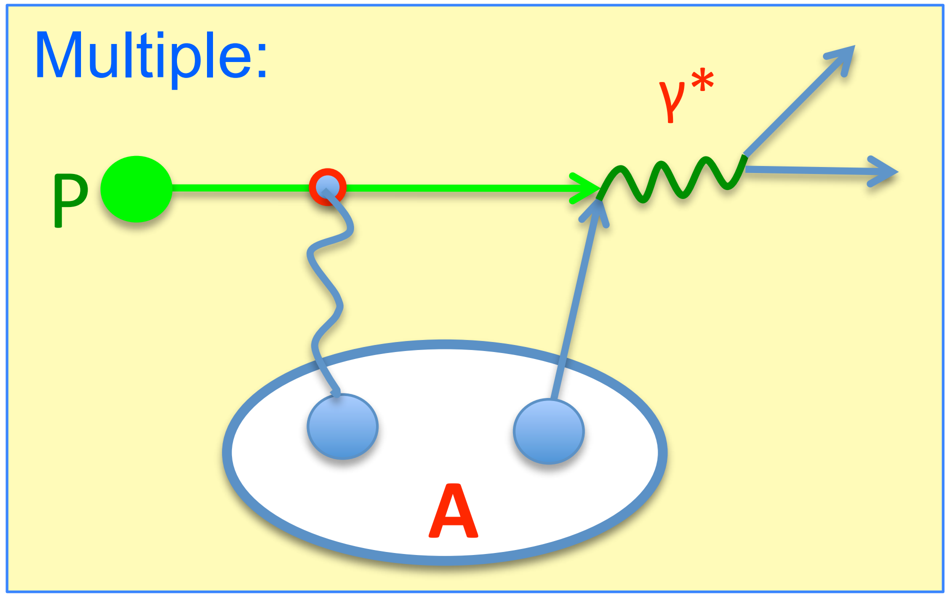
- Summary

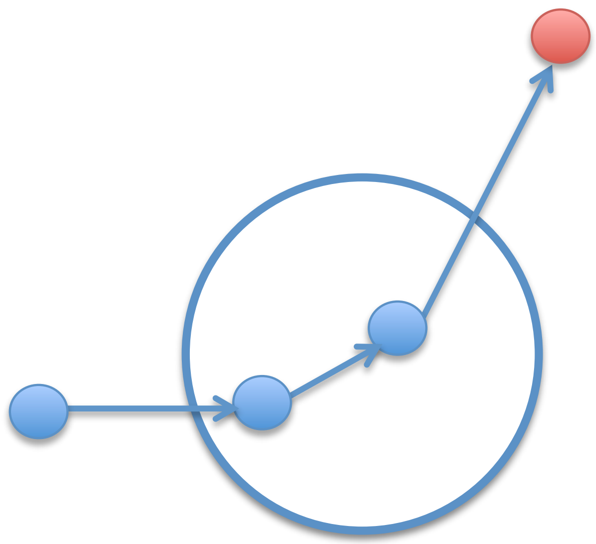
Nuclear dependence and four parton correlations

- **Single hard scattering:**
  - probes local parton densities
  - cannot tell the difference in target size



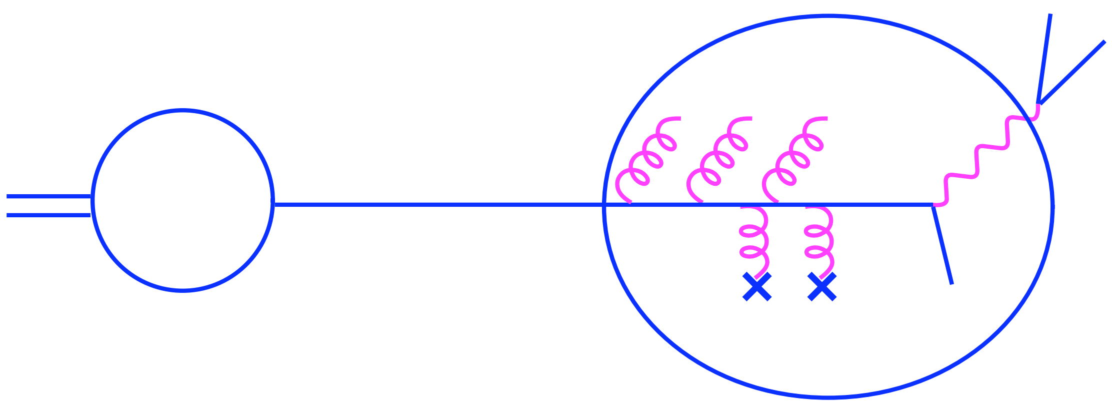
- **A-dependence (medium size dependence)**
  ⇔ multiple scattering
  (beyond single scattering)
  - probe four parton correlation functions
  - change production rate at given transverse momentum (eg, p_T spectrum)





Nuclear Modification to p_T Spectrum

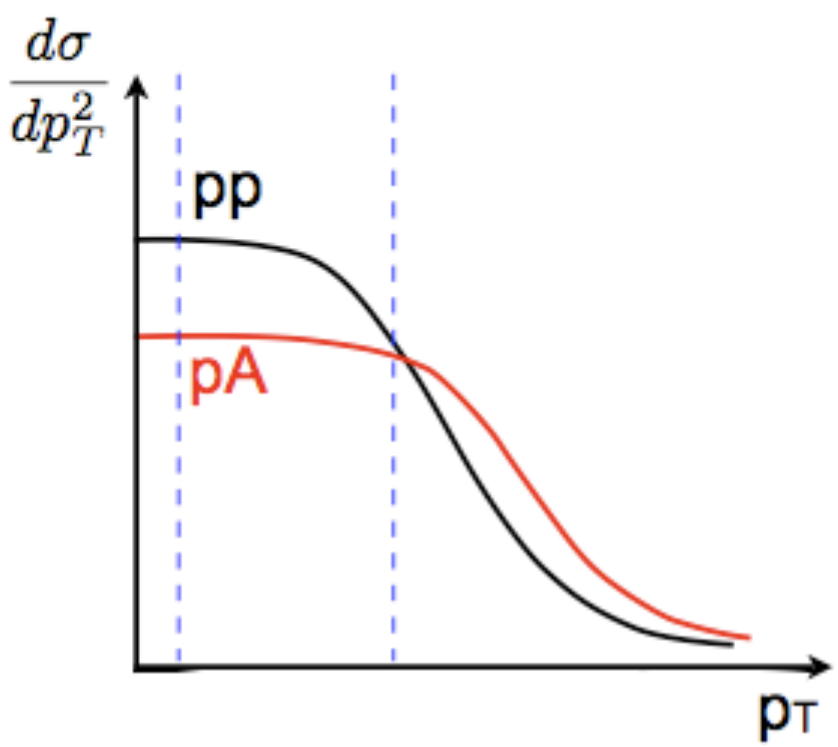
- Nuclear modification to p_T spectrum can lead to nuclear broadening

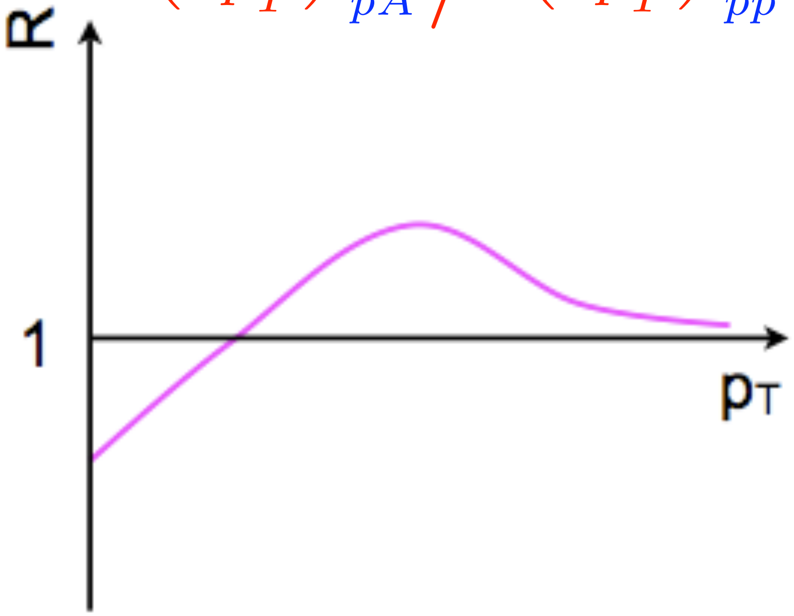


Vector bosons: γ*, W, Z, ...
— initial-state

- Enhancement in high p_T region and reduction in low p_T region
- Power suppressed: R→1 as p_T increases (Cronin type effect)

$$R = \left(\frac{d\sigma}{dp_T^2}\right)_{pA} \Big/ \left(\frac{d\sigma}{dp_T^2}\right)_{pp}$$



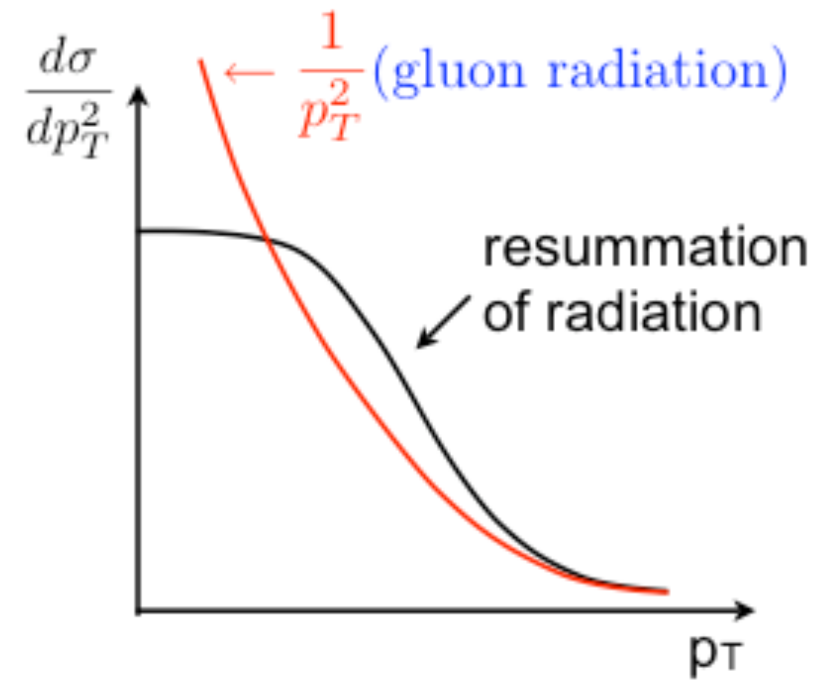


A-Dependence of p_T spectrum for vector bosons

- **pp collision:**

  p_T distribution at low p_T (for DY, W/Z, ...) is ill-defined in fixed order perturbative calculation

  ❖ Resummation (CSS in pp collisions)



- **pA collision:**

  ❖ Each scattering is too soft to calculate perturbatively

  ❖ Ideal solution: Resummation+multiple scattering

Ideal solution has not been achieved yet.

Instead, since moments of p_T distribution is much less sensitive to low p_T

Define: $$\langle p_T^2 \rangle = \int dp_T^2 p_T^2 \frac{d\sigma}{dp_T^2} \Big/ \int dp_T^2 \frac{d\sigma}{dp_T^2}$$

Broadening defined as: $$\Delta\langle p_T^2 \rangle = \langle p_T^2 \rangle_{pA} - \langle p_T^2 \rangle_{pp}$$

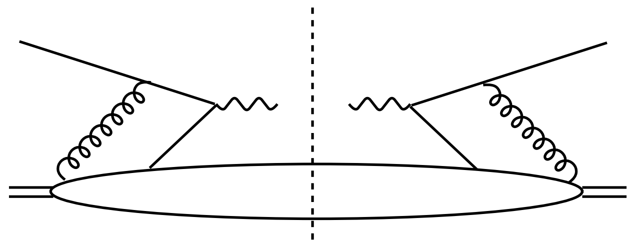
❖ perturbatively calculable

❖ sensitive to medium property

Broadening for Drell-Yan: initial-state

Broadening for Drell-Yan: $q\bar{q}$

Guo, 2001



$$\Delta\langle q_T^2\rangle_{\rm DY} \approx \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,C_F\right)\frac{\sum_q \int dx'\,\phi_{\bar{q}/h}(x')\int dx\,T_{q/A}(x)\,\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2}}{\sum_q \int dx'\,\phi_{\bar{q}/h}(x')\int dx\,\phi_{q/A}(x)\,\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2}}$$

- nuclear dependence from four parton correlation function:

$$T_{q/A}(x) = \int \frac{dy^-}{2\pi}\,e^{ixp^+y^-}\int\frac{dy_1^-dy_2^-}{2\pi}\,\theta(y^- - y_1^-)\,\theta(-y_2^-)$$
$$\times\frac{1}{2}\langle p_A|F_\alpha{}^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|p_A\rangle$$
$$\approx \lambda^2\,A^{1/3}\,\phi_{q/A}(x)$$

$$\lambda^2 \approx \frac{9}{16\pi R^2}\langle F^{+\alpha}F_\alpha{}^+\rangle$$

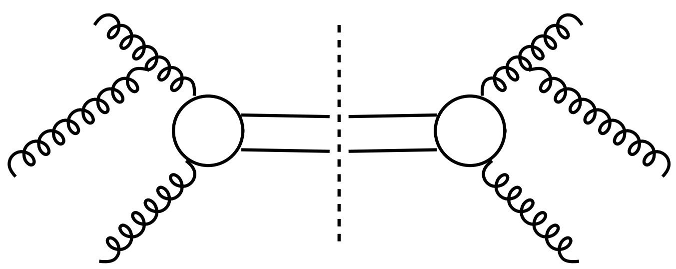
- prediction:

$$\Delta\langle p_T^2\rangle = C_F\left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\lambda^2A^{1/3}$$

Broadening depends on nuclear size $\propto A^{1/3}$

Broadening for heavy quarkonium: initial-state

- Heavy quarkoium: if the broadening is due to initial-state multiple scattering of the projectile partons in the nucleus only



J/ψ, ϒ: gg

one should expect:

$$\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \approx \frac{C_A}{C_F}\Delta\langle p_T^2\rangle_{DY} = 2.25\Delta\langle p_T^2\rangle_{DY}$$

Experimentally:

Broadening for heavy quarkonium: initial-state

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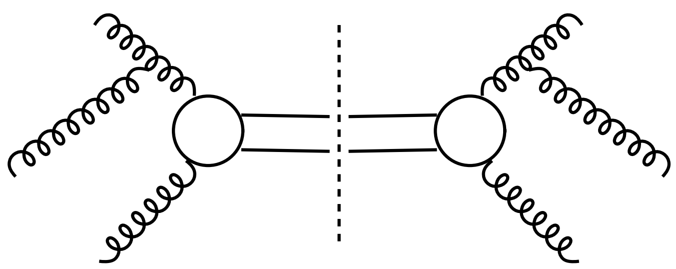
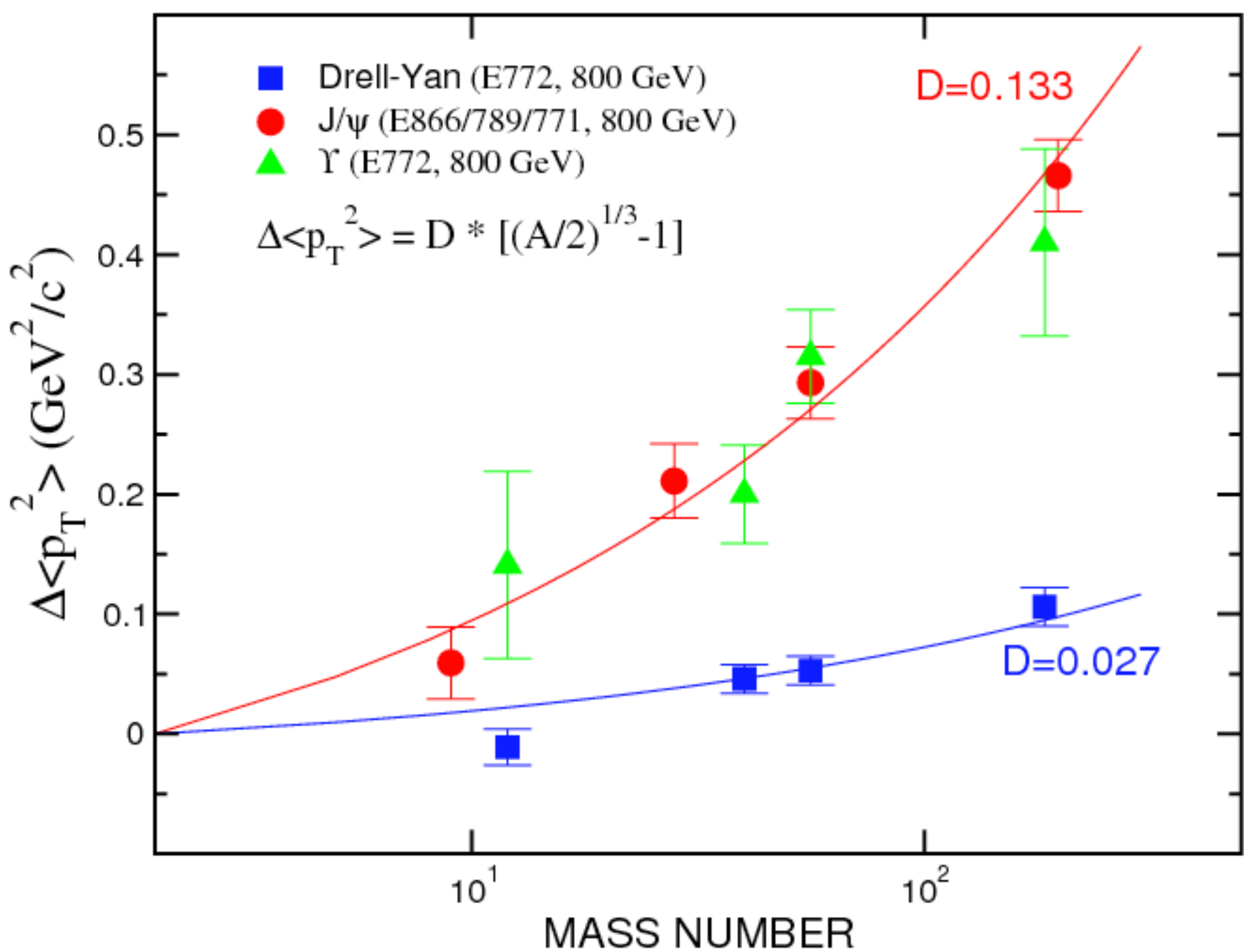


J/ψ, ϒ: gg

J. C. Peng, hep-ph/9912371



New analysis: M. B. Johnson, et.al. Phys. Rev. C 75, 035206 (2007)

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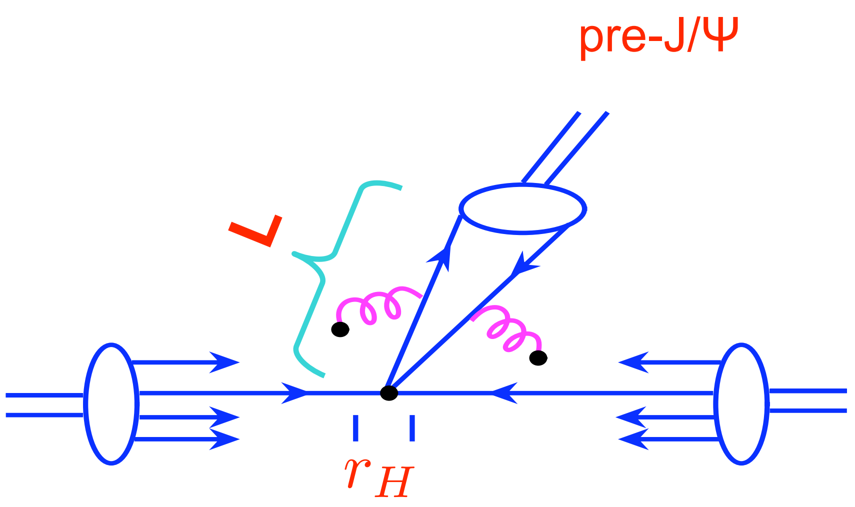
$$\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \approx \frac{C_A}{C_F}\Delta\langle p_T^2\rangle_{DY}$$
$$= 2.25\Delta\langle p_T^2\rangle_{DY}$$

Experimentally:

$$\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \sim 5\Delta\langle p_T^2\rangle_{DY}$$

Broadening due to Final State Multiple Scattering

- Parton model picture:



$$pA \to [c\bar{c} \to J/\psi] + X$$

- J/Ψ is unlikely to be formed at: $r_H \leq \frac{1}{2m_c} \sim \frac{1}{15}\text{fm}$
- Final state interaction changes the pair's momentum as well as color

$$\Delta \left\langle p_T^2 \right\rangle\Big|_{\text{final}} \propto L \lesssim bA^{1/3}$$

Length of ccbar pair undergoes multiple scattering before becoming pre-J/ψ

can be as small as zero

Broadening of heavy quarkonium at pA collision

- Net effect on broadening depends on how quarkonium is formed

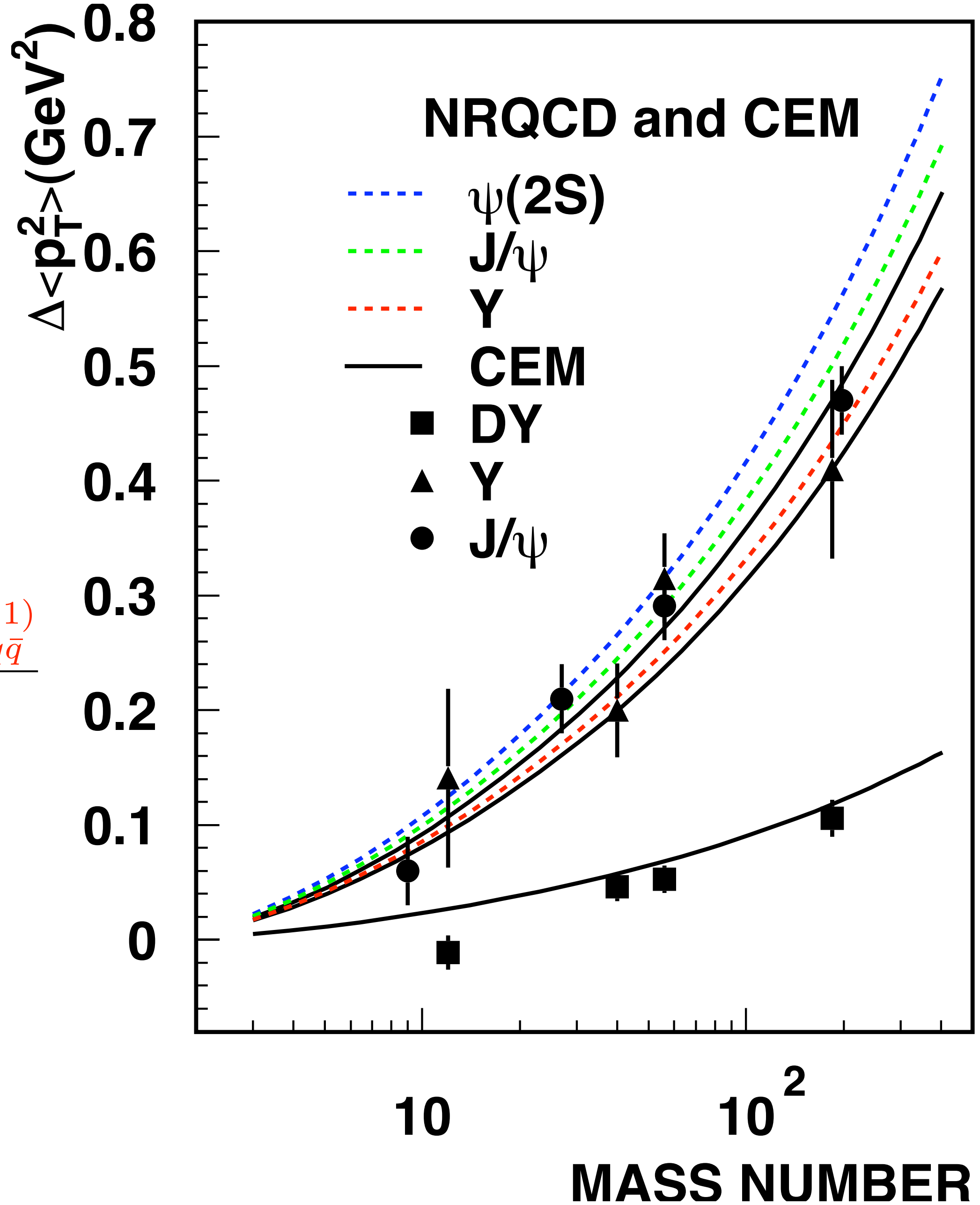
Kang, Qiu, PRD77, 2008

- ❖ Color Evaporation Model (CEM):
  - ✓ sensitive to the change of momentum but not the color
- ❖ NRQCD:
  - ✓ sensitive to both momentum and the color (→different matrix elements)

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm CEM} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}}+2\,C_A\,\sigma_{gg}}{\sigma_{q\bar{q}}+\sigma_{gg}}$$

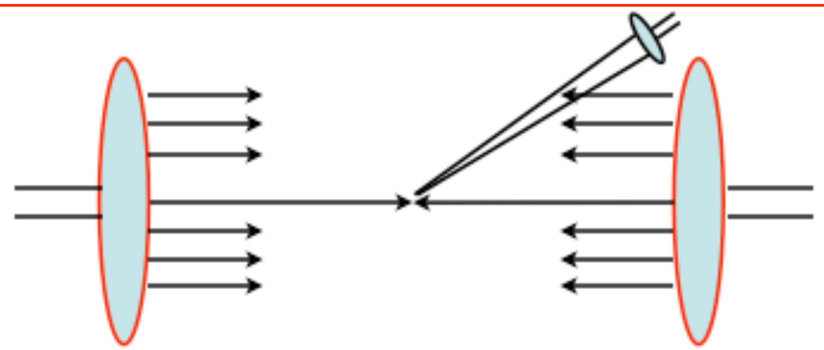
$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm NRQCD} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}}^{(0)}+2C_A\,\sigma_{gg}^{(0)}+\sigma_{q\bar{q}}^{(1)}}{\sigma_{q\bar{q}}^{(0)}+\sigma_{gg}^{(0)}}$$

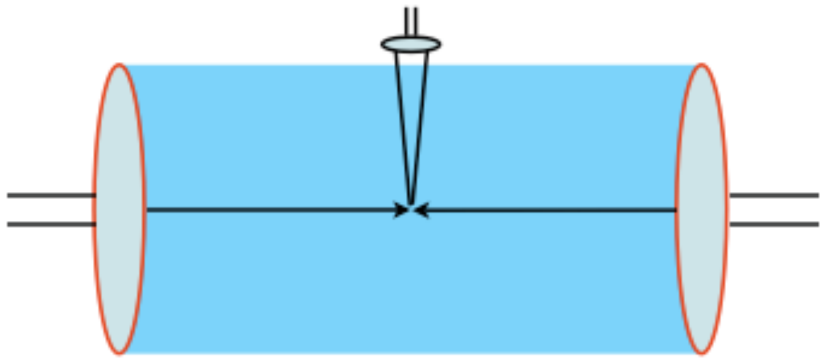
- ❖ different analytic expression but very similar numerical results
- ❖ $\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \sim 2\frac{C_A}{C_F}\Delta\langle p_T^2\rangle_{DY}$ can describe data well with final-state interaction



Extended to AA collision

- If NO hot medium is formed
  - broadening in AA ≈ superposition of pA
  - $\Delta\langle p_T^2\rangle_{AA} \propto L_{eff}$
- If hot medium is formed



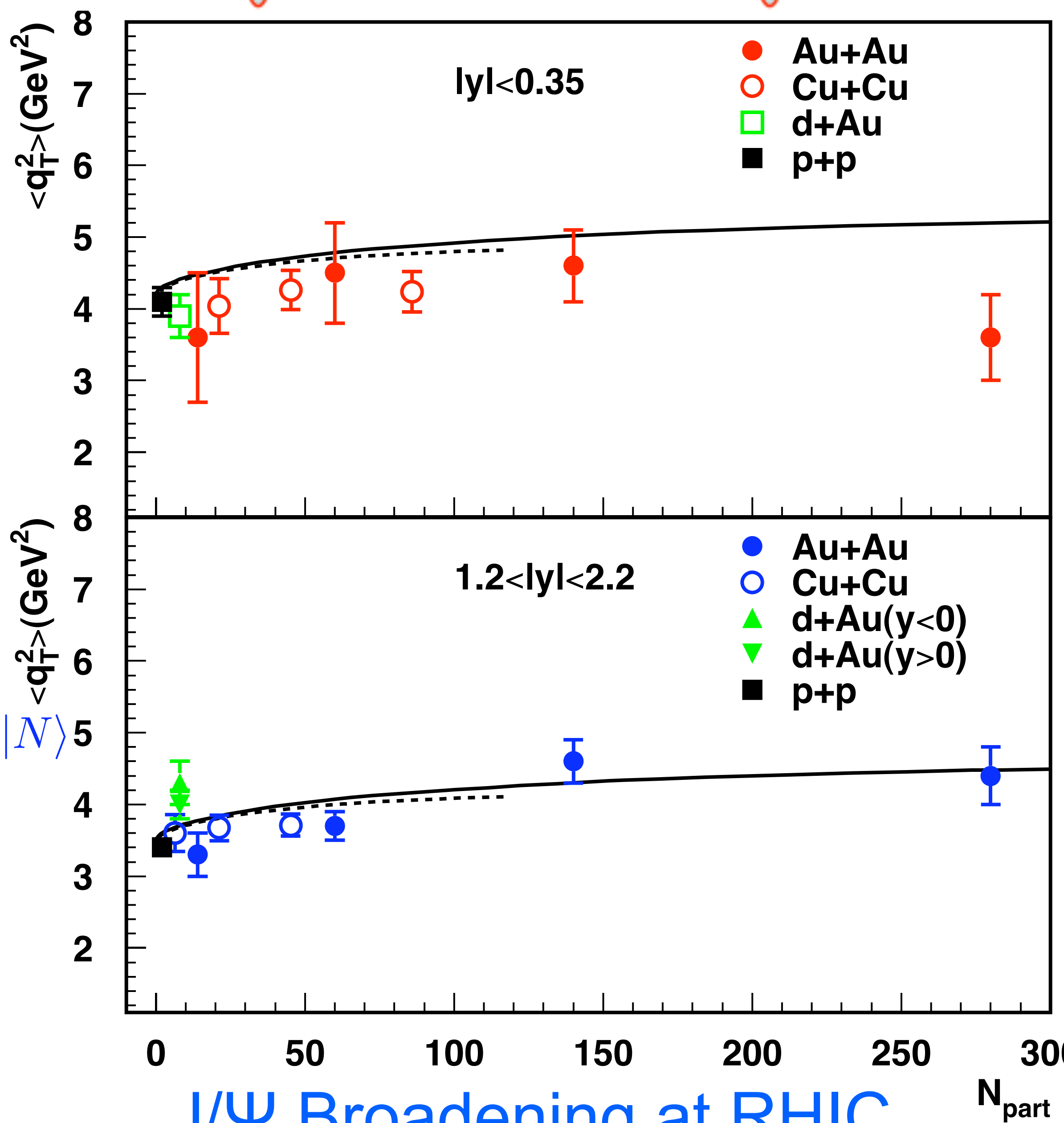


- $\Delta\langle p_T^2\rangle_{final} \sim 0$ final state energy loss $\rightarrow$ reduce $\langle p_T^2\rangle$
- $\Delta\langle p_T^2\rangle_{initial} \lesssim \text{superposition of } \Delta\langle p_T^2\rangle_{pA}$

$$\lambda^2 \approx \frac{9}{16\pi R^2}\langle F^{+\alpha}F_\alpha{}^+\rangle \sim \frac{1}{p^+}\int\frac{dy^-}{2\pi}\langle N|F^{+\alpha}(0)F_\alpha{}^+(y^-)|N\rangle$$

$$\rightarrow \frac{1}{2}\lim_{x\to 0} xG(x,Q^2)$$

recombination of gluons?



J/Ψ Broadening at RHIC

⇒ independent test initial-state interaction: broadening of W/Z at LHC

Summary

- Spin and/or nuclear dependence enable us to explore the hadron structure beyond the probability distributions
  - Quantum correlations between quarks and gluons

- For the first time, we derive the evolution equations of the twist-3 correlation functions that are responsible for generating the SSAs in the QCD collinear factorization approach
  - Consistent calculations beyond LO are now possible

- pQCD can be used to calculate the anomalous nuclear dependence
  - Broadening of vector bosons: good observable for four-parton correlations

Summary

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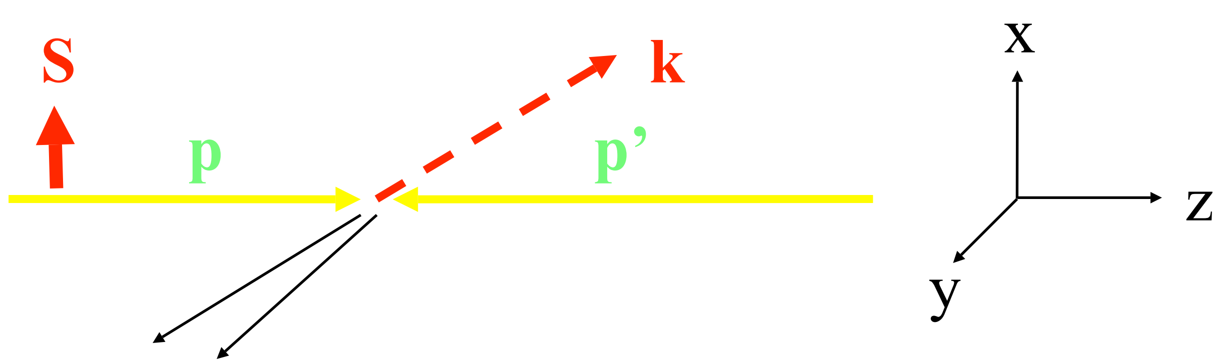
  - Broadening of vector bosons: good observable for four-parton correlations

Thank you!

Backup Slides

Single spin asymmetry corresponds to a T-odd triplet product

- Scattering of a transversely-polarized spin-1/2 hadron (S, p) with another hadron (or photon), observing a particle of momentum k



The cross section can have a term depending on the azimuthal angle of **k** which produce an asymmetry A_N when S flips:

$$d\sigma \;\sim\; \vec{S}\cdot(\vec{p}\times\vec{k}) \qquad\qquad A_N \propto i\,\vec{S}\cdot(\vec{p}\times\vec{k})$$

Nonvanishing A_N requires a phase, a helicity flip

✓ the phase "i" is needed because the structure S·(p×k) violates the naive time-reversal invariance

✓ one must have a reaction mechanism for the hadron to change its helicity (in a cut diagram)

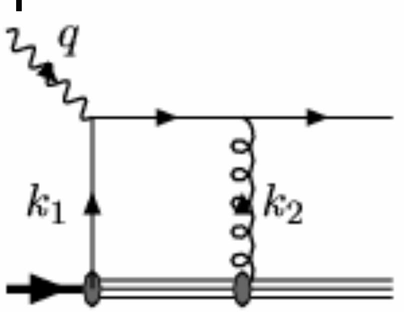
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \mathrm{Im}\left[M_{\rightarrow} M_{\leftarrow}^{*}\right]$$

Why does SSA exist?

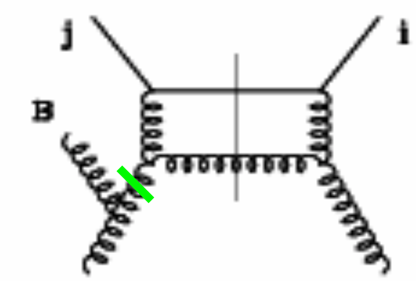
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \mathrm{Im}\left[M_{\rightarrow} M^{*}_{\leftarrow}\right]$$

- SSA requires
  - Helicity flip: one must have a reaction mechanism for the hadron to change its helicity (in a cut diagram)
  - A phase: the phase is needed because the structure S·(p×k) violate the naive time-reversal invariance

- TMD: the quark orbital angular momentum leads to hadron helicity flip
- The factorizable final state interactions --- the gauge link provides the phase



- Twist-three: the gluon carries spin, flipping hadron helicity
- The phase comes from the poles in the hard scattering amplitudes



Connection to twist-2 PDFs

- **Set I:**

  Spin-averaged twist-2 PDFs + an operator Insertion

$$\int \frac{dy_2^-}{2\pi}\, e^{ix_2P^+y_2^-}\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right] = i\int \frac{dy_2^-}{2\pi}\, e^{ix_2P^+y_2^-}\left[i\,\epsilon_\perp^{\rho\sigma}\, s_{T\rho}\, F_\sigma{}^+(y_2^-)\right]$$

- **Set II:**

  Spin-dependent twist-2 HDFs + an operator Insertion

$$i\int \frac{dy_2^-}{2\pi}\, e^{ix_2P^+y_2^-}\left[s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]$$

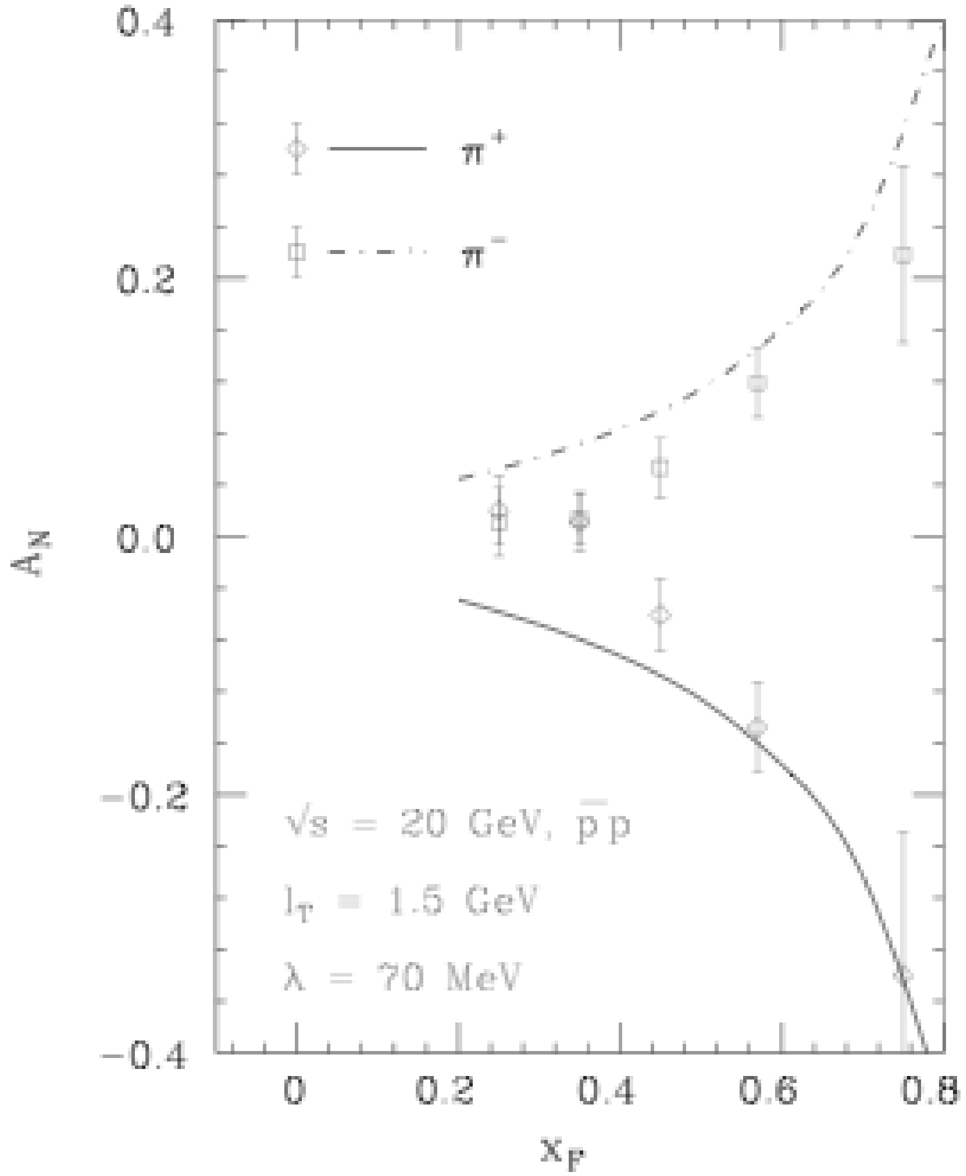
- **Extra "i":**

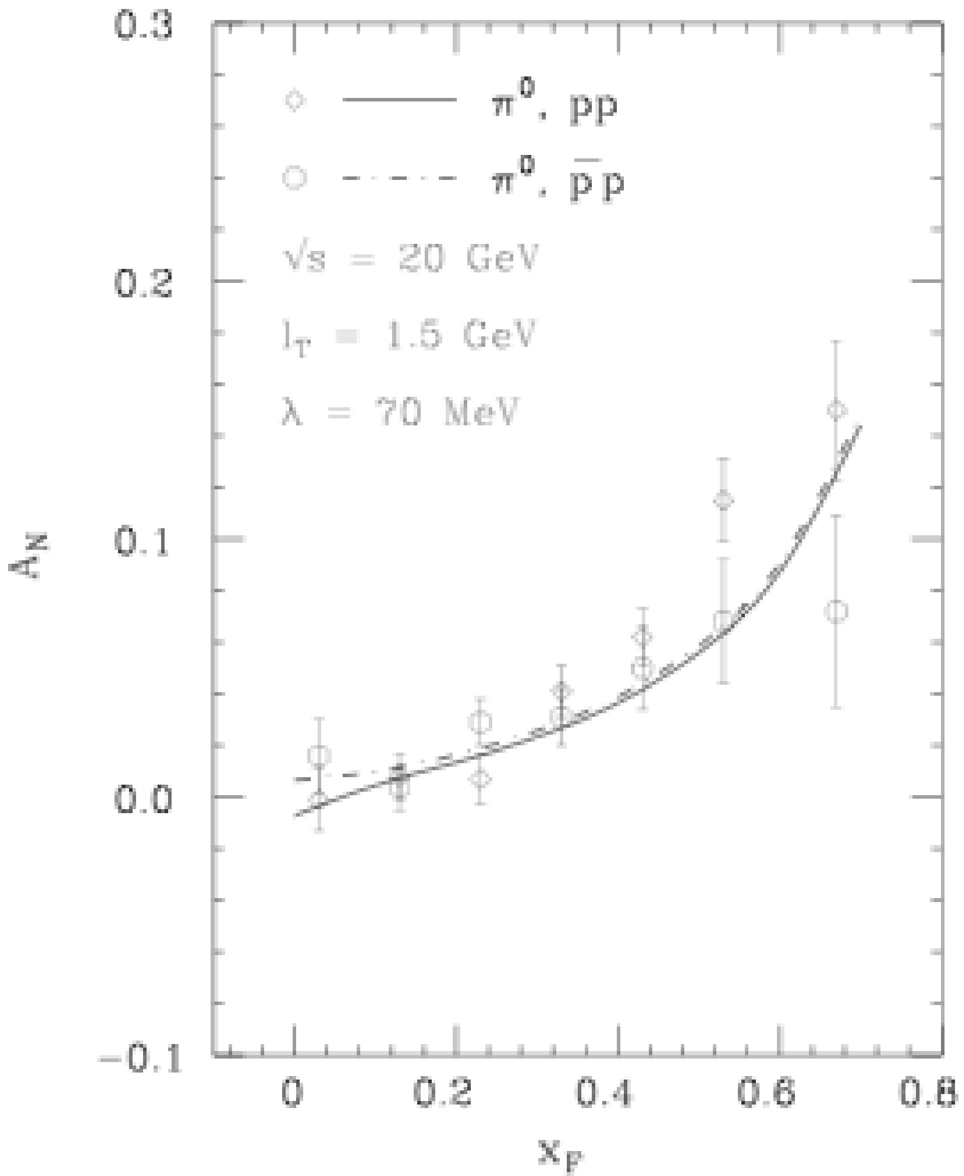
  Phase needed for the nonvanishing SSAs

  Do not contribute to parity conserving double-spin asymmetry, like g_2!

Model for $T_{q,F}(x,x)$

Qiu, Sterman, 1999



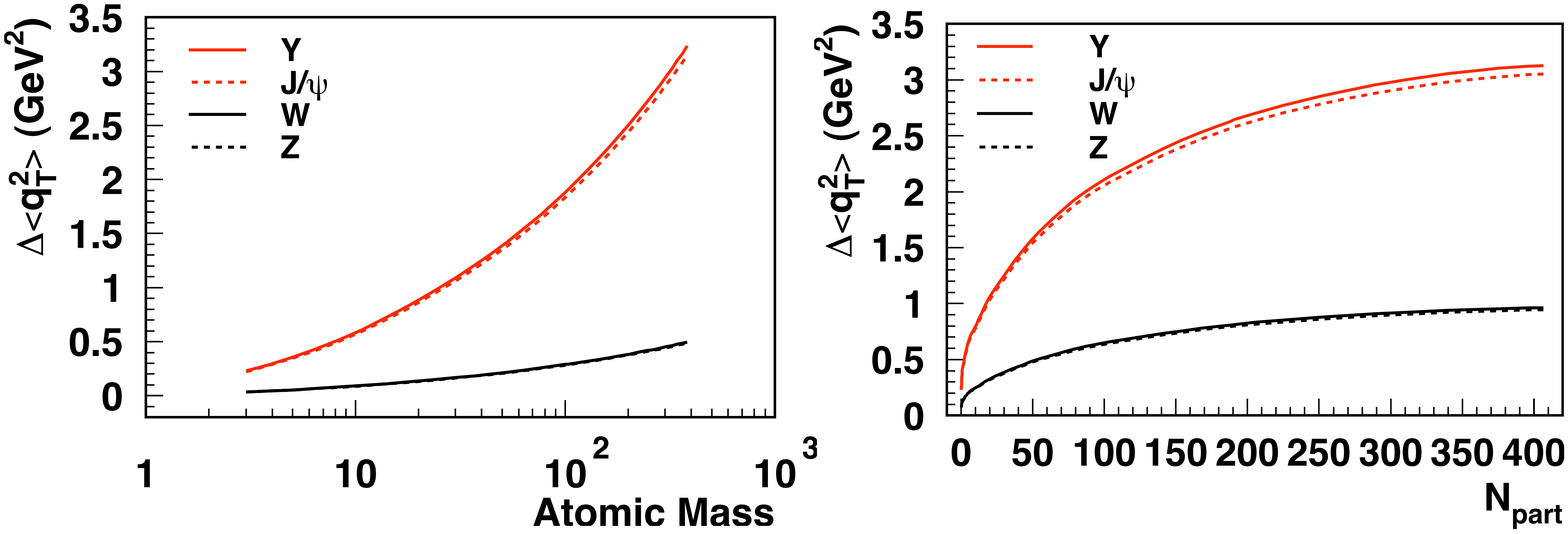


$$T_{u,F}(x,x) = \lambda_F \phi_u(x) \qquad T_{d,F}(x,x) = -\lambda_F \phi_d(x)$$

$$\lambda_F = 0.07 \text{ GeV}$$

Ideal probe for correlations: broadening of W/Z at LHC

- If W/Z is constructed from leptonic decays:
  - purely initial-state interactions, ideal for medium density (black lines)
- If W/Z can be constructed from hadronic decays (very difficult):
  - Mass shift $\Delta M^2 \sim \Delta\langle p_T^2\rangle_{\rm FinalState}$



- If hot medium is produced,heavy quarkonium broadening can approach C_A/C_F=9/4 (DY, Z initial state only)