^{12}C Background Systematic

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This is note about the background from ^{12}C due to the diamond foils in PREX. The PREX target consists of a sandwich of lead (0.5 mm) squeezed between two diamond foils (each 0.15 mm). See also the PREX web site for detailed information about the target. In this note we estimate the systematic error from the background subtraction.

Let us define the following terms.

- $A_{\text{raw}-\text{meas}}$ = Asymmetry actually measured
- A_{Pb} = Raw asymmetry from Lead = **0.509 ppm** = $p_e * A_{\text{phy}-\text{Pb}}$
- $A_{\text{phy}-\text{Pb}}$ = Physics asymmetry from Lead
- p_e = beam polarization, assumed $p_e = 0.8$.
- A_{C} = Raw asymmetry from ^{12}C = **0.874 ppm** = $p_e * A_{\text{phy}-\text{C}}$
- b = background fraction from ^{12}C = ratio of rate from Carbon to rate from Lead. $\mathbf{b = 0.053}$.
- $\epsilon_a = \frac{dA}{A}$ = relative error in the A_{C}.
- $\epsilon_b = \frac{db}{b}$ = relative error in b due to target thickness uncertainty

For this discussion it's assumed that $\epsilon_a = 0.05$ and $\epsilon_b = 0.05$. The estimate for ϵ_a is a reasonable guess and we will explore what will happen if it's a little smaller or bigger. The estimate for ϵ_b is fairly conservative and is based on the measured variations in thickness of the diamond and the sampling over these variations by the rastered beam. There may be a way of arguing these are smaller, but I'm not sure at this time.

The (raw) lead physics asymmetry $A_{\rm Pb}$ is related to the raw measured asymmetry $A_{\rm raw-meas}$ by

$$A_{\rm Pb} \approx A_{\rm raw-meas} - b\, A_{\rm C} \tag{1}$$

Taking a differential we get

$$dA_{\rm Pb} = dA_{\rm raw-meas} - (b\, dA_{\rm C} + db\, A_{\rm C}) \tag{2}$$

Dividing by $A_{\rm Pb}$ and assuming $\frac{dA_{\rm Pb}}{A_{\rm Pb}} \approx \frac{dA_{\rm raw-meas}}{A_{\rm raw-meas}}$ and using a definition $R = \frac{A_{\rm C}}{A_{\rm Pb}}$ (note $R = 1.72$) we obtain

$$\frac{dA_{\rm Pb}}{A_{\rm Pb}} = \frac{dA_{\rm raw-meas}}{A_{\rm raw-meas}} - Rb(\epsilon_a + \epsilon_b) \tag{3}$$

where in this context of this equation the ϵ's are like differentials but one will actually add these errors in quadrature because they are independent, i.e. to compute the total error the terms should be added like $\sqrt{(x^2 + y^2)}$.

The expected statistical error we will achieve is

$$\frac{dA_{\rm raw-meas}}{A_{\rm raw-meas}} = 0.03 \tag{4}$$

Putting in $R = 1.72$, $b = 0.053$, $\epsilon_a = 0.05$ and $\epsilon_b = 0.05$ and adding the ϵ terms in quadrature, the systematic error is

$$Rb(\epsilon_a + \epsilon_b) = 0.0064\,(0.64\%) \tag{5}$$

This adds in quadrature to the statistical error, leading to a blowup factor of the final error bar (i.e. the ratio of total error to statistical error) of 1.023 in this case. More estimates are shown in the table for various assumptions about σ_a and σ_b. The last line is probably our most optimistic result, a total systematic error of 0.39% and a blowup factor of 1.008. Note, the factors R and b will be measured during the experiment.

TABLE 1. ^{12}**C Background – Systematic Error**

Assumption	σ_a	σ_b	Syst. Error (%)	Error Blowup
1	0.05	0.05	0.64%	1.023
2	0.03	0.05	0.53%	1.016
3 (worst case)	0.10	0.05	1.0%	1.054
4 (best case)	0.03	0.03	0.39%	1.008