Rate Estimates and Acceptance Averaging for PREX

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This note explains simple estimates for the rates, average asymmetry, and sensitivity for PREX. The nuclei considered are ^{208}Pb, ^{120}Sn, and ^{48}Ca. We acknowledge Charles Horowitz [1] [2] for calculations of cross section, asymmetry, and sensitivity to R_N when stretched by 1%.

Two methods of the simple estimates are provided. One is a "point" kinematics method, the other is an integral over an acceptance function.

The rate R is how many electrons per second are seen in the detector.

I POINT KINEMATICS METHOD

The point kinematics method uses the cross section at a fixed energy E and scattering angle θ and assumes this can be used for the entire solid angle acceptance. This provides a simple estimate and a sanity check on the acceptance-averaging since they should not be very different. The basic equation for the rate is.

$$\mathrm{R}(\theta) \quad = \quad \frac{d\sigma}{d\Omega} \; I \;\; \xi \;\; T \;\; \rho \;\; d\Omega \;\; \frac{0.602}{A} \tag{1}$$

where

- $\frac{d\sigma}{d\Omega}$ is the differential cross section in barns/str

- I is the beam current in electrons per sec. Note, 50μA (typical beam current) corresponds to 3.125×10^{14} electrons per sec.
- ξ is a radiative loss factor (see formula below). One must integrate the radiative tail to find the fraction that gets into the detector, a bite of $\sim$4 MeV.
- T is the target length in units of centimeters (cm).
- ρ is the density in grams per cubic cm ($\frac{g}{\mathrm{cm}^3}$).
- $d\Omega$ is the solid angle in steradians (for 1 HRS + septum, $d\Omega = 0.0037$ str).
- The factor 0.602 arises from Avogadro's number and conversion factors.
- A is the atomic number.

A note about the radiative loss factor ξ. According to equation A.11 in Mo and Tsai [3] we may estimate

$$\xi \approx \left(\frac{\Delta}{E}\right)^{bt} \tag{2}$$

where $\Delta \sim 0.004$ GeV is the detector bite, E is the energy, $b \sim \frac{4}{3}$ and t is the fractional radiation length of the target ($t \sim 0.1$). This formula estimates the factor as $\xi \sim 0.5$ for $E = 1$ GeV, but numerical integration yields $\xi = 0.34$, and to be conservative this lower factor will be used.

II ACCEPTANCE FUNCTION INTEGRATION

The acceptance function method uses the basic formula for the rate $R(\theta)$ from equation 1 and integrates it over the scattering angle θ using an acceptance function $\epsilon(\theta)$. The function ϵ is defined as the relative probability that an electron starting at the target will reach the detector. The rate is normalized by the radiative loss factor ξ and the solid angle $d\Omega$. Hence we compute

$$R_{\mathrm{avg}} = \frac{1}{N} \int d\theta \ \epsilon(\theta) \ R(\theta) \tag{3}$$

where the normalization N is

$$N = \int d\theta \ \epsilon(\theta) \tag{4}$$

TABLE 1. ^{208}**Pb Quantities at E = 1.05 GeV,** $\theta = 5°$ $I = 50\mu$**A**

Quantity	Point Kine	Accept. Fcn	Hamc
$\frac{d\sigma}{d\Omega}$ [barns/str]	1.38		
T [cm]	0.05		
ρ [$\frac{\mathrm{g}}{\mathrm{cm}^3}$]	11.35		
Rate (1 HRS) [MHz]	890	930	1200
Asy ($P_e * A_{\mathrm{phys}}$) [ppm]	0.55	0.52	0.51
Sensitivity $\frac{dA}{A}$ [%]	2.9	2.6	2.6

We compute the average asymmetry A and sensitivity $S = \frac{dA}{A}$ to a 1% change in R_N in the following way which weights by the rate $R(\theta)$:

$$A_{\mathrm{avg}} = \frac{1}{M} \int d\theta \ \epsilon(\theta) \ R(\theta) \ A(\theta) \tag{5}$$

$$S_{\mathrm{avg}} = \frac{1}{M} \int d\theta \ \epsilon(\theta) \ R(\theta) \ S(\theta) \tag{6}$$

where the normalization M is

$$M = \int d\theta \ \epsilon(\theta) \ R(\theta) \tag{7}$$

The acceptance function $\epsilon(\theta)$ is shown in fig 1, and a table of values is available online at
http://hallaweb.jlab.org/parity/prex/accept/accept.txt.
The function is determined using the Monte Carlo code "hamc" described in the Appendix.

In table 1, 2, and 3 these quantities are shown for ^{208}Pb, ^{120}Sn, and ^{48}Ca respectively at a beam current of 50μA. The point-calculation, acceptance-averaged, and hamc-generated quantities are compared. The quantities shown are the rate (MHz) in 1 HRS, the raw asymmetry ($A = P_e * A_{\mathrm{phy}}$) where $P_e = 0.8$ is the beam polarization, and the sensitivity $\frac{dA}{A}$ to a 1% change in R_N. For convenience, I also show the cross section, target length T and density ρ. The choice of kinematics is, for each nucleus, chosen for $\theta = 5°$ and the energy E which minimizes the error in R_N which is equivalent to maximizing the product $R\,A^2\,S^2$.

For ^{208}Pb in 1-month of beam time, R_N can be measured to 1% accuracy using two HRS+septum spectrometers, if the error in polarimetry is 1%.

TABLE 2. ^{120}**Sn Quantities at E = 1.2 GeV,** $\theta = 5^\circ$ $I = 50\mu$**A**

Quantity	Point Kine	Accept. Fcn	Hamc
$\frac{d\sigma}{d\Omega}$ [barns/str]	0.31		
T [cm]	0.16		
ρ [$\frac{\mathrm{g}}{\mathrm{cm}^3}$]	5.8		
Rate (1 HRS) [MHz]	569	420	390
Asy ($P_e * A_{\mathrm{phys}}$) [ppm]	0.82	0.82	0.83
Sensitivity $\frac{dA}{A}$ [%]	2.8	2.9	3.0

TABLE 3. ^{48}**Ca Quantities at E = 1.7 GeV,** $\theta = 5^\circ$ $I = 50\mu$**A**

Quantity	Point Kine	Accept. Fcn	Hamc
$\frac{d\sigma}{d\Omega}$ [barns/str]	0.016		
T [cm]	0.66		
ρ [$\frac{\mathrm{g}}{\mathrm{cm}^3}$]	1.6		
Rate (1 HRS) [MHz]	83	53	49
Asy ($P_e * A_{\mathrm{phys}}$) [ppm]	1.7	1.7	1.7
Sensitivity $\frac{dA}{A}$ [%]	3.5	3.7	4.1

III APPENDIX: HAMC MONTE CARLO

The "hamc" Monte Carlo is used to determine the acceptance function $\epsilon(\theta)$ We emphasize that ultimately we won't need a Monte Carlo; we will determine $\epsilon(\theta)$ from experimental data once the septum magnets are built and installed.

Events are generated using a physics class that has information about the cross section and asymmetry. The tracks are generated uniformly in solid angle $d\Omega = sin(\theta)\, d\theta\, d\phi$ and the results later weighted by the differential cross section $\frac{d\sigma}{d\Omega}$. Energy loss in the target is found by a fast Monte Carlo method for estimating the Bremsstrahlung (both external and internal). This code produces a reasonable radiative tail shape, though the integral out to the maximum energy loss is less than predicted by theory. To compensate for this shortfall the target thickness is reduced by a factor which accounts for the fraction of events that fall out of the energy bite (4 MeV) of the detector and gets the radiative loss factor ξ defined above to be correct.

The generated four-vectors are transported to the detector in the HRS focal plane using a set of polynomials provided by John LeRose. Multiple scattering is included in the Monte Carlo for the target and the Be energy degrader. This affects the angles which causes smearing in the focal plane. The beam raster is simulated, which also produces a smearing. The events are transported to intermediate apertures such as the collimator or the entrance to quadrupoles.

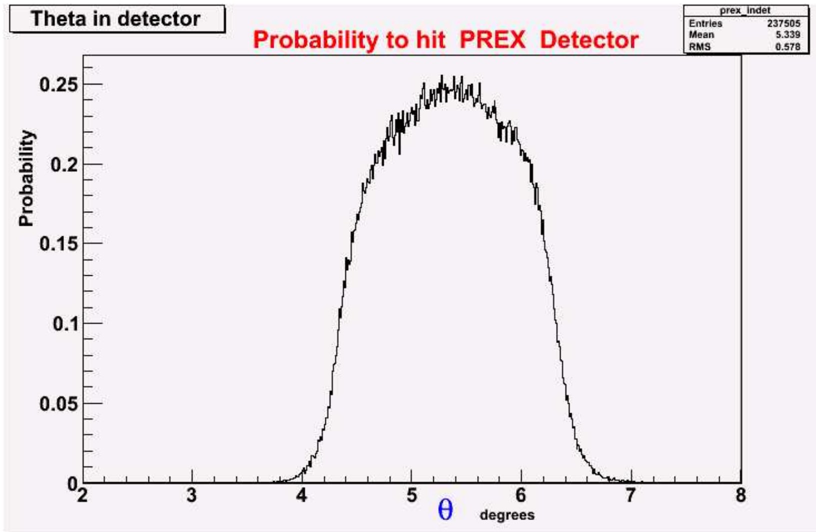

FIGURE 1. Simulated relative acceptance of the HRS with 5° septum magnet integrated over azimuth. This will also be measured.

For PREX, the collimator is designed such that only it, and no other aperture, defines the acceptance. Events that enter the focal plane and intersect the detector are integrated to make the total rate, average asymmetry, and average sensitivity to R_N.

How we obtain $\epsilon(\theta)$ and the "hamc" results: 1) $\epsilon(\theta)$: Fill a histogram with the generated scattering angle θ and fill another histogram with θ after requiring the electron passes throught the spectrometer and hits the detector (acceptance cut). The ratio of these two histograms is $\epsilon(\theta)$. 2) "hamc" averaging: Divide the acceptance into tiny bins in scattering angle θ and azimuthal angle ϕ, each bin corresponding to a small solid angle $sin(\theta)d\theta d\phi$. With acceptance cuts, count how many events occur in each bin and compute the average rate in each bin. Add the rates, and average the asymmetry A and sensitivity S by weighting by the rate in the bin, but taking only bins that have at least 5% of the average number of events in the central bins. The 5% cut defines the border of the acceptance and may be varied.

REFERENCES

1. C. J. Horowitz, Phys. Rev. C **57**, 3430 (1998).
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3. L.W. Mo and Y.S.Tsai, Rev. Mod. Phys **41** 205 (1969).