ON THE POSSIBILITY OF ACCURATE MEASUREMENTS OF ELECTRON BEAM ENERGY BY MEANS OF MØLLER SCATTERING

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Abstract:

The feasibility of using Møller scattering to measure the absolute value of the beam energy to $\sim 10^{-4}$ is discussed. The small emission angles typical in Møller scattering of high energy beams imply the need for very precise measurement of the particle angles. One of the primary limitations of this method arises from the effect of motion of the bound target electrons which distorts the shape of the angular distribution. Estimates of the size of this effect for iron and hydrogen targets are carried out. It is concluded that even for hydrogen the influence on the opening angle due to atomic binding is more than one order of magnitude larger than the angular accuracy requirement. Although, in principle, these effects can be calculated for hydrogen, it seems unlikely that the accuracy goal of 10^{-4} can be achieved.

Physics research programs for electron accelerators under construction or already operating require a high level of precision in beam energy measurements. In particular, for some of the experiments to be carried out at CEBAF [1] the absolute value of the beam energy E_0 must be determined with an accuracy of $\Delta E_0/E_0 \sim 10^{-4}$.

Several ways can be suggested to solve this problem (see, *e.g.*, [2,3]). Here, we consider the possibility of determining E_0 by means of Møller scattering (elastic scattering of beam electrons on free target electrons) [4].

From kinematics of electron–electron scattering it follows that the sum

$$\alpha = \tan\theta + \tan\theta_R, \tag{1}$$

where θ (θ_R) is the laboratory frame angle for the scattered (recoil) electron, reaches its minimum

$$\alpha_0 = 2\sqrt{\frac{2m}{E_0+m}} \tag{2}$$

at the center-of-mass (cm) angle $\theta_{\rm cm} = 90°$, when $\theta = \theta_R$. In principle, by measuring α_0 one can determine E_0 with an accuracy

$$\frac{\Delta E_0}{E_0} = \frac{\Delta\alpha_0^2}{\alpha_0^2}. \tag{3}$$

With this technique, errors due to the beam size and angular spread of incident electrons are completely or to a great extent eliminated.

In the idealized case of a monoenergetic beam and an infinitely thin target, the distribution of events in α is given by the expression

$$\frac{d^2\sigma}{d\alpha d\phi} = 2\frac{d\sigma}{d\Omega_{\rm cm}}\frac{\alpha_0^2}{\alpha^2}\left(\alpha^2-\alpha_0^2\right)^{-\frac{1}{2}} \tag{4}$$

where $d\sigma/d\Omega_{\rm cm}$ is the center-of-mass Møller cross-section, and $d\phi$ is the differential azimuthal angle. Under these conditions the uncertainty in the minimum value of α is the only systematic error (see Eq. (3)).

For $\theta_{cm} = 90°$ and for ultra-relativistic ($\gamma = E_0/m \gg 1$) electrons, α_0 is approximately equal to the opening angle in the laboratory between the recoil and scattered electrons: $\alpha_0 \approx |\theta| + |\theta_R|$. In this case $\alpha_0 \propto 1/\sqrt{\gamma}$, and a 10^{-4} measure of the beam energy requires measurement of the opening angle to 5×10^{-5}. For example, with a 4 GeV beam and $\theta_{cm} = 90°$, both electrons emerge at 0.92°. The opening angle is then 1.84° and must be measured with an accuracy of 1.6μrad for a 10^{-4} determination of the beam energy.

However, in reality the distribution of events in α can considerably differ from that given by (4) due to the motion of electrons bound in target atoms, the energy spread in the beam and multiple Coulomb scattering.

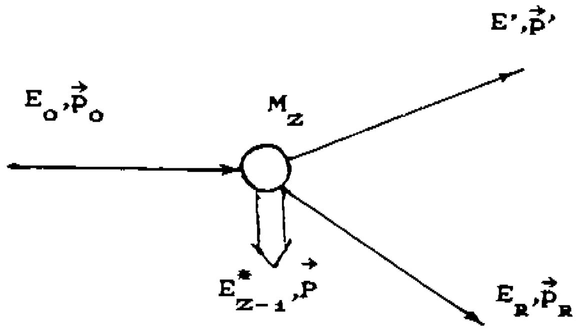

Figure 1. *The interaction of an electron with a target atom.*

In order to estimate the role of the first of these effects, let us consider the interaction of an electron with a target atom (see Fig. 1). As a result, the scattered and the knocked out electrons have energy (momentum) equal to $E'(\vec{p}\,')$ and E_R $(\vec{p}_R)$ respectively, whereas the recoil ion gains the energy

$$E^*_{Z-1} = M_Z - M_{Z-1} - m + |E_n| + \frac{\vec{P}^{\,2}}{2M_{Z-1}}, \tag{5}$$

where M_Z is the mass of an atom (ion) with Z electrons, $E_n < 0$ is the binding energy of the electron being knocked out from the atom, and $\vec{P}$ is the momentum of the recoil ion. Then for the sum (1) one has:

$$\alpha = \frac{2}{\gamma}\frac{\sqrt{1+\delta}\sin\theta_{\rm cm}}{1+\delta-\cos^2\theta_{\rm cm}} \tag{6}$$

$$\delta = \frac{4m^2}{s-4m^2}(1-\gamma^{-2}) - \gamma^{-2} \tag{7}$$

$$\gamma = \frac{E'+E_R}{\sqrt{s}}. \tag{8}$$

Here $s = (E'+E_R)^2 - (\vec{p}\,' + \vec{p}_R)^2$ is the invariant energy of the final-state electrons squared and $\theta_{\rm cm}$ is the angle of the scattered electron in the frame moving with the velocity

$$\vec{v} = \frac{\vec{p}\,' + \vec{p}_R}{E'+E_R}.$$

For our purposes it will be convenient to represent s as

$$s = s_0 + \Delta s, \tag{9}$$

where $s_0 = 2m(E_0 + m)$ is the invariant energy squared for two free electrons, with one of them being at rest and the other having the energy E_0. Taking into account energy/momentum conservation (see also Eq. (5)):

$$E_0 + m + E_n = E' + E_R + \frac{\vec{P}^{\,2}}{2M_{Z-1}} \tag{10}$$

$$\vec{p}_0 = \vec{p}\,' + \vec{p}_R + \vec{P}, \tag{11}$$

with $\vec{p}_0$ being the incident electron momentum. In the ultra-relativistic limit one finds:

$$\frac{\Delta s}{s_0} = \frac{E_n}{m} + \frac{|\vec{P}\,|}{m}\cos(\theta_{\vec{p}_0,\vec{P}}). \tag{12}$$

Since in this case

$$|\delta| \sim \frac{|\Delta s|}{s_0}\frac{2m}{E_0} \ll \frac{|\Delta s|_{\max}}{s_0}$$

and

$$\frac{E_0 + m - E' - E_R}{E' + E_R} \sim \frac{|E_n|}{E_0} \ll \frac{|\Delta s|_{\max}}{s_0}$$

then for angles θ_{cm} close to 90° one can write

$$\alpha^2 = \frac{\alpha_0^2}{\sin^2\theta_{\mathrm{cm}}}(1 + \Delta s/s_0), \tag{13}$$

where α_0 is determined by (2). Thus the corrections due to the binding and the intra-atomic motion of target electrons can be approximated by the extra-factor of $(1 + \Delta s/s_0)$ in the expression for α^2.

In order to estimate quantitatively the influence of these effects upon beam energy measurements, let us turn to the simple model, in which the cross-section of the process shown in Fig. 1 is represented as

$$Z\frac{d^2\sigma'}{d\alpha d\phi} = \int \frac{s}{2E_0 E}\frac{d^2\sigma}{d\alpha d\phi}(\vec{p}\,)\rho(p)d\vec{p} \tag{14}$$

$$E = \sqrt{p^2 + m^2},$$

with the momentum density of intra-atomic electrons

$$\rho(p) = \frac{1}{4\pi p^2}\sum_n N_n\delta(p - \sqrt{2mE_n}). \tag{15}$$

Here $d\sigma(\vec{p}\,)$ is the Møller cross-section for scattering on an electron moving with the momentum $\vec{p}$, and N_n is the number of electrons in the n-th atomic energy level. In our consideration we neglect the final-state interaction, assuming

$$\vec{P} = -\vec{p}. \tag{16}$$

Furthermore,

$$\frac{s}{2E_0E}\frac{d^2\sigma}{d\alpha d\phi}(\vec{p}\,) \approx \frac{d\sigma}{d\Omega_{\rm cm}}(0)\frac{d\cos\theta_{\rm cm}}{d\alpha}.$$

Then, integration of (14) can be performed analytically, and we obtain

$$Z\frac{d^2\sigma'}{d\alpha d\phi} = \sum_n \sigma_n \tag{17}$$

with

$$\begin{aligned}
\sigma_n &= 0, && \alpha^2-\alpha_0^2 \le -a_n-b_n \\
\sigma_n &= 2\frac{d\sigma}{d\Omega_{\rm cm}}\frac{\alpha_0^2}{\alpha^2}N_n\frac{\sqrt{\alpha^2-\alpha_0^2+a_n+b_n}}{b_n}, && -a_n-b_n < \alpha^2-\alpha_0^2 \le -a_n+b_n \\
\sigma_n &= 4\frac{d\sigma}{d\Omega_{\rm cm}}\frac{\alpha_0^2}{\alpha^2}\frac{N_n}{\sqrt{\alpha^2-\alpha_0^2+a_n+b_n}+\sqrt{\alpha^2-\alpha_0^2+a_n-b_n}}, && \alpha^2-\alpha_0^2 > -a_n+b_n
\end{aligned}$$

where

$$a_n = \frac{|E_n|}{m}\alpha_0^2, \qquad b_n = \sqrt{2E_n/m}\;\alpha_0^2.$$

The results of the calculations for iron and hydrogen using the above relations are presented in Fig. 2 and Fig. 3 respectively. The solid lines are the values of $Z\alpha_0(d^2\sigma'/d\alpha d\phi)/(d\sigma/d\Omega_{\rm cm})$ plotted against $(\alpha-\alpha_0)/\alpha_0$; the dotted lines are the corresponding calculation for an unbound electron (according to Eq. (4)). The energies E_n were taken from Ref. [5].

It is seen that the effects of binding and intra-atomic motion of target electrons essentially distort the "idealized" distribution (4). This result is a consequence of relativistic kinematics. Namely, in the ultra-relativistic limit the compression factor γ corresponding to the transformation from the laboratory to center-of-mass frame of the two interacting electrons appears to be sensitive to even small fluctuations of the velocity, $|\vec{v}\,|$. In our case, such fluctuations result from the motion of electrons bound in the scatterer atoms.

In order to minimize the effects of binding on the angular distribution of the final state electrons, one could choose a hydrogen target, which has a relatively small binding energy and for which calculations should be more reliable. Taking $|\vec{P}\,| = \sqrt{2mE_n}$, and $E_n = 13.6$ eV, momenta of order 3.73 KeV/c result. In this case the second term in Eq.(12) dominates and we have:

$$\Delta s/s_0 \approx \frac{|\vec{P}\,|}{m}\cos(\theta_{\vec{p}_0,\vec{p}}).$$

From Eq.(13), we have for $\theta_{\rm cm} = 90°$:

$$\Delta\alpha^2/\alpha_0^2 = \Delta s/s_0.$$

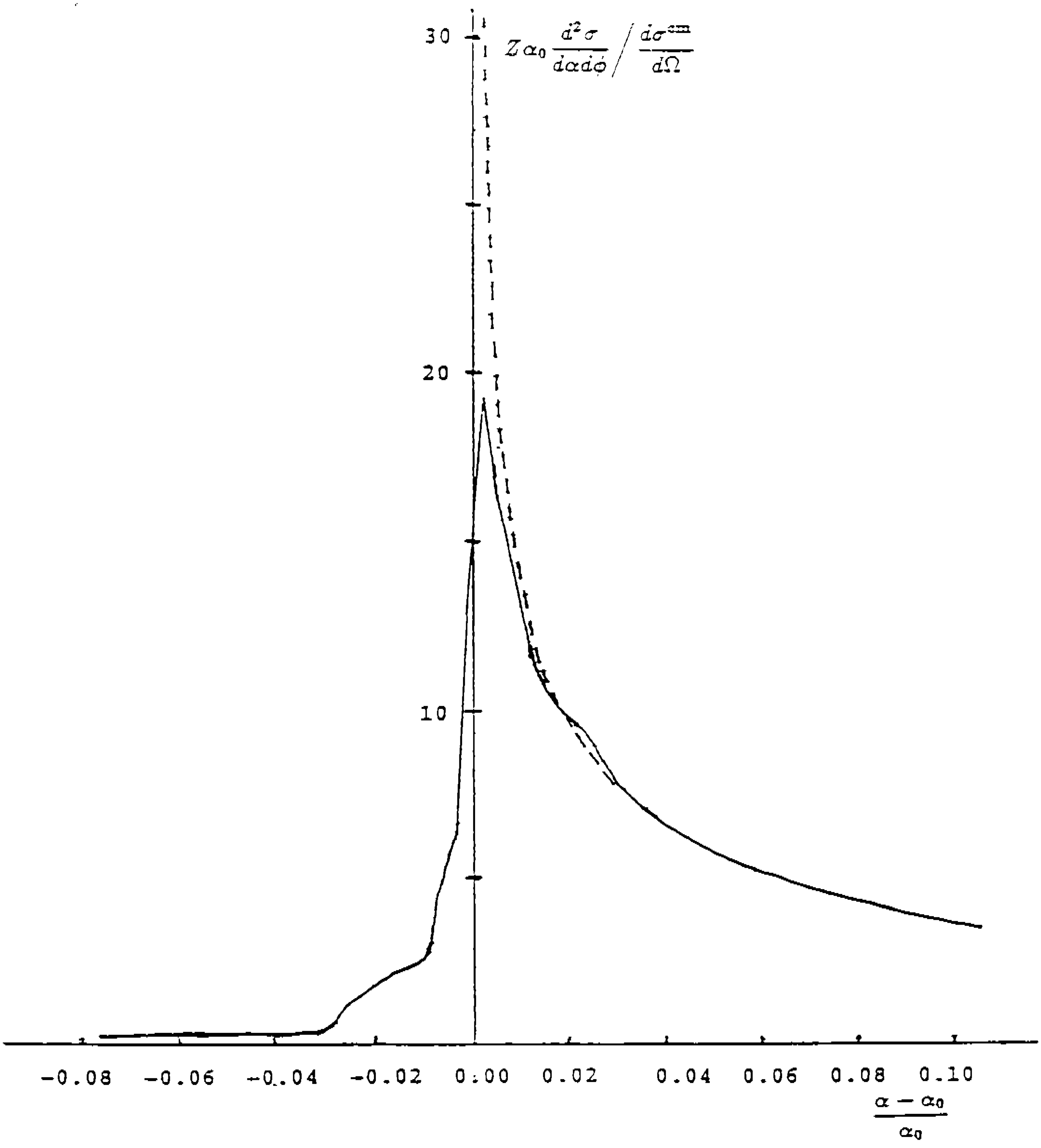

Figure 2. *Angular distribution for iron based on calculation described in the text. The solid line includes the effect of binding and intra-atomic motion in the target whereas the dashed line is the result assuming the target electron is unbound.*

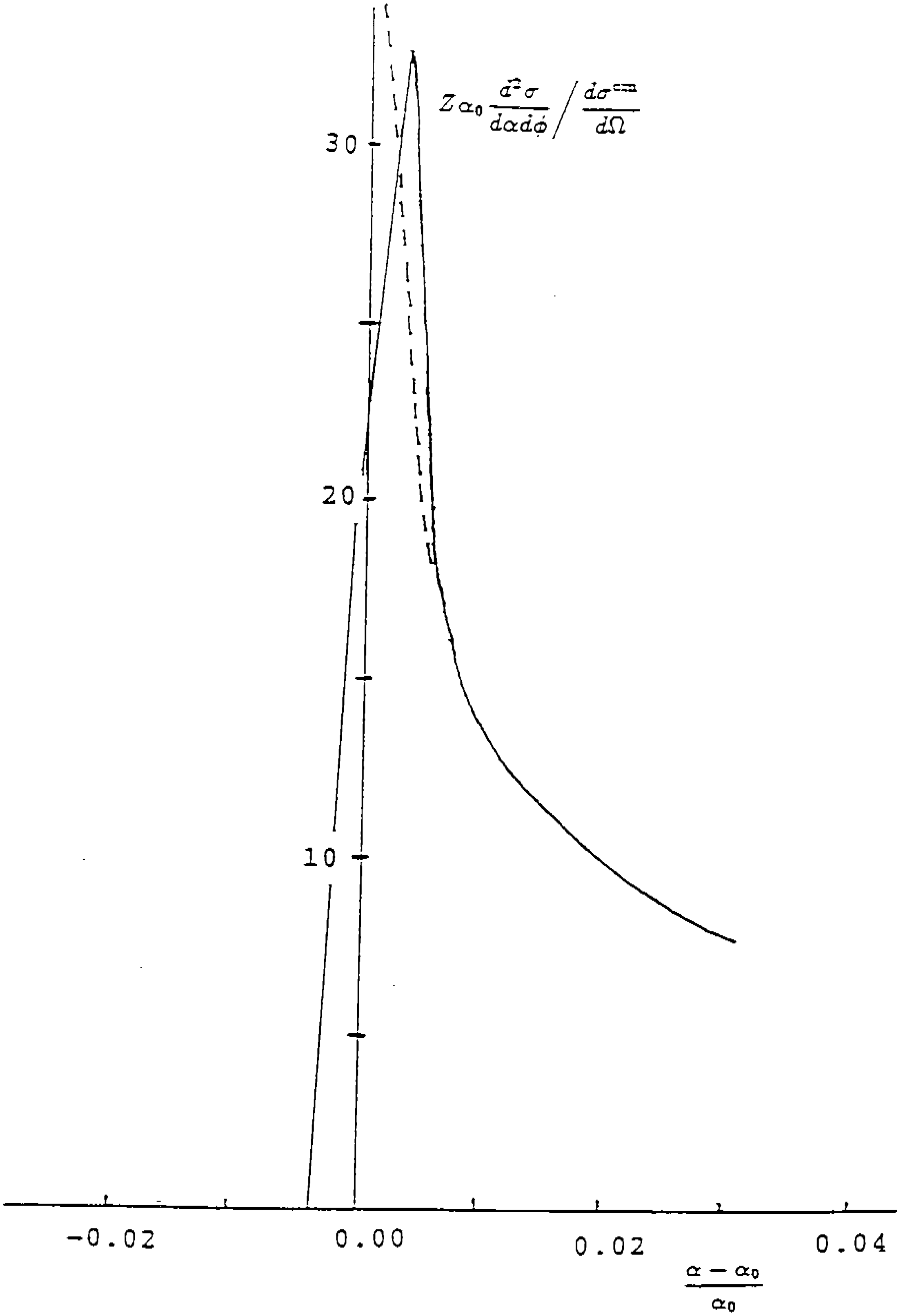

Figure 3. *Same as Fig. 2. except for a hydrogen target.*

Finally, from Eq.(3), $\Delta E_0/E_0 = \Delta\alpha^2/\alpha_0^2$ so that

$$\Delta E_0/E_0 \approx \frac{|\vec{P}\,|}{m}\cos(\theta_{\vec{p}_0,\vec{p}}).$$

There will be a distribution of momenta, both in magnitude and angle, which will give rise to a spread in the deduced value of E_0. Taking the coefficient of the cosine in the above relation as a measure of this spread, we find for $|\vec{P}\,| = 3.73$ KeV/c, $\Delta E_0/E_0 \sim 7 \times 10^{-3}$. Therefore, the effect from the bound electron's motion is ~ 70 times larger than the accuracy requirement of 10^{-4}. Although, in principle, one can calculate the distribution for hydrogen, it seems unlikely that the required accuracy can be achieved.

References

[1] P.E. Ulmer *et al.*, "Physics Requirements on the Determination and Stability of the Parameters of the Beam", CEBAF Technical Note 90-255 (1990).

[2] J. Kent *et al.*, SLAC-PUB-4922 (1989).

[3] S.I. Serednjakov *et al.*, Phys. Lett. **B66**, 102 (1977).

[4] P. Galumyan, private communication.

[5] M.A. Blokhin, I.G. Shveitzer, "Rentgenospectral'nyi spravochnik".M., Nauka (1982).