Jlab Experiment E93050
Virtual Compton Scattering

Technical Notes on Simulation Studies
1st Pass Analysis 1998-2000

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VCS-E93050

Beam profile for the simulation of E93050 Experiment

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At Gent (Dec. 1-3, 1999) we discussed on how to incorporate realistic beam profiles (*beamx*, *beamy*) in the simulation. This memo summarizes what has been proposed: to use flat beam profiles to generate the simulated events, and afterwards to use these events with a weight taking into account experimental shapes.

This paper is the sum of two independent memos from Clermont and Gent, which have been simply joined together. They correspond to Parts I and II.

1 PART I - principle of re-weighing simulated events

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1.1 Luminosity and Solid angle

We first remind how to use the simulation to get the experimental cross section in one physics bin j (= one bin in the physics variables: $\cos\theta_{\gamma\gamma}$, q'_{cm}, etc. Up to five variables can be defined, as the $ep \rightarrow ep\gamma$ cross section is 5-fold differential).

When running VCSSIM one generates N_0 events ($ep \rightarrow ep\gamma$) in the target, i.e. before any acceptance cuts. N_{acc} events go into the acceptance of the spectrometers, and $N_{acc}(j) = N_{sim}(j)$ events are found finally in the physics bin j after all reconstruction cuts. L_{sim} is the integrated simulated luminosity that has been necessary to create all these events. Then one calculates the solid angle (or "geometrical efficiency" as named by Luc) in the physics bin j by the equation:

$$d\Omega_{sim}(j) = \frac{N_{sim}(j)}{L_{sim} \times d^5\sigma_{BH+Born}(j)} \tag{1}$$

where $d^5\sigma_{BH+Born}(j)$ is a theoretical cross section. For VCS-MAMI the choice was to evaluate this theoretical cross section at the central point of the physics bin (but other choices can be made). The integrated luminosity L_{sim} is calculated by the code, and its value is printed in an outputfile [1].

• **Remark about luminosity:** For what follows (cf. eq.(6)), we just need to explicitely state that the luminosity is proportional to the number of generated events:

$$L_{sim} = N_0 \times T \qquad \text{in picobarn}^{-1} \tag{2}$$

T is a global factor that can be seen as depending on target length, target density, etc: this is the experimental definition of luminosity. Actually Luc's definition of the luminosity in VCSSIM is different, although not in conflict: it is the sum over all events in a well defined phase space, devided by the cross section integral over that phase space.

[1] inversely, one cannot ask for a given luminosity as an input.

1.2 Beam profile

The transverse beam profile (x and y) changes, or may change, from run to run. What is the most economic way to take that into account in the simulation? we want to avoid creating many "simulated runs" corresponding to all possible observed beam profiles.

One can propose the following method.

- **simulated beam profile**

For a given setting, we propose to run VCSSIM with a FLAT beam profile in x and y, within large bounds $[A, B]$ and $[C, D]$ covering the full experimental range. Example: $[A, B] = [C, D] = [-8, +8]$ mm. This is the "vertex" generation level, i.e. before any spectrometer acceptance cuts. Figure 1-a shows vertex spectrum of x=*beamx* and figure 1-b shows the same spectrum for the simulated events which passed the acceptance cuts (warning: it's a figure of principle, the shape does not pretend to be realistic). So in figure 1 the number of events is N_0 (left) and N_{acc} (right), and the corresponding luminosity is L_{sim}.

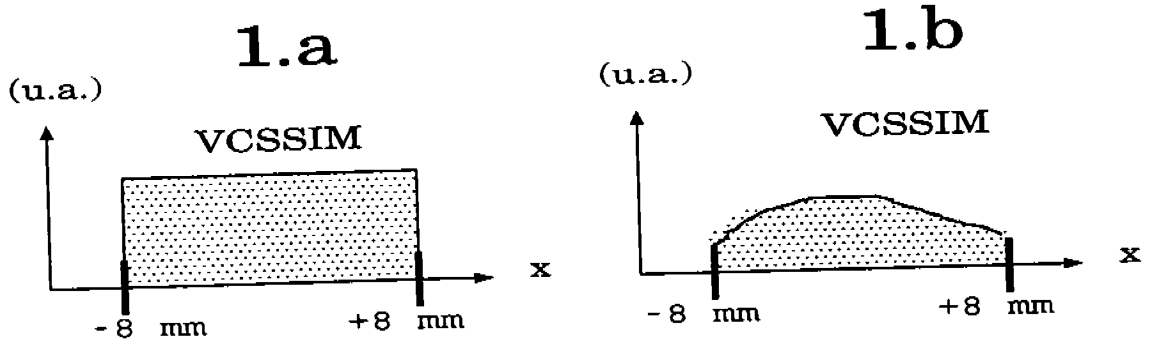

Figure 1:

- **experimental beam profile**

Figure 2-a shows *beamx* and *beamy* spectra for a given run. Let's denote these experimental distributions $F_{hor}(x)dx$ and $F_{ver}(y)dy$. One may also sum up a series of runs with different beam profiles; the cumulated distributions of *beamx* and *beamy* may become more complex, cf. figure 2-b, but the notations and method still hold.

N.B. in principle the true beam profile should be obtained most directly from T8 events. Beam profiles obtained from T1 or T5 events are biased because they include convolution by spectrometer acceptance and cross section. But maybe this effect can be neglected? it has to be checked.

- **Re-weighing the simulation**

In order for the simulation to reproduce an experimental profile, one can just change the weight w of each simulated event: divide by the "old weight" and multiply by a

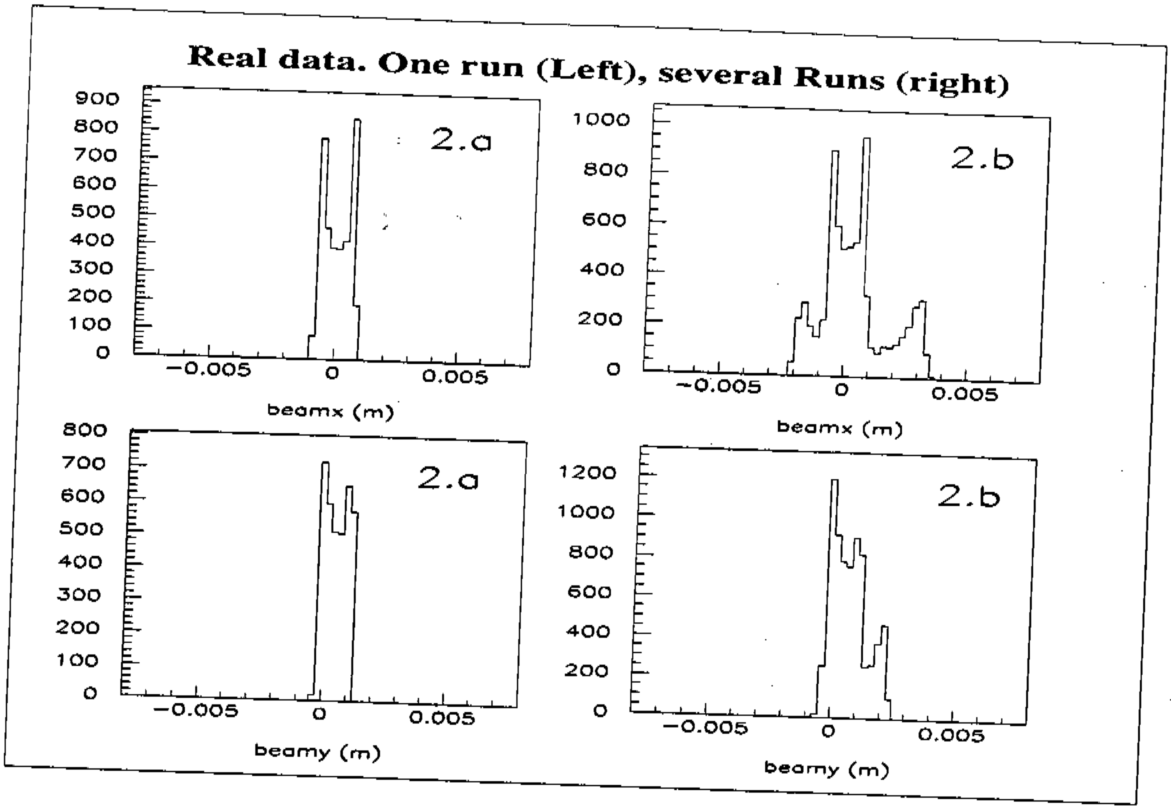

Figure 2:

new one.

$$w(\text{ event } \# \, i) = \frac{w_{new}(i)}{w_{old}(i)} \tag{3}$$

The "old weight" is the implicit weight attached to each event in VCSSIM; for a flat beam profile, it is a constant, it does not depend on (x_i, y_i)= beam coordinates on target for simulated event # i. The new weight takes into account the real beam profile, and its value depends on (x_i, y_i). To ensure proper normalization, we choose the following definition of the weights for simulated event # i:

$$\begin{aligned} w_{new}(i) &= \frac{F_{hor}(x_i)\ dx}{\int_A^B F_{hor}(x)dx} \times \frac{F_{ver}(y_i)\ dy}{\int_C^D F_{ver}(y)dy} \\ w_{old}(i) &= \frac{P_{hor}(x_i)\ dx}{\int_A^B P_{hor}(x)dx} \times \frac{P_{ver}(y_i)\ dy}{\int_C^D P_{ver}(y)dy} \end{aligned} \tag{4}$$

i.e. each distribution function is divided by its own integral. $P_{hor}(x)dx$ is a uniform distribution between A and B (same for y) :

$$P_{hor}(x)dx \quad = \quad \frac{K_x\, dx}{(B-A)} \quad \text{with } K_x = \text{ arbitrary constant factor.}$$

$$\Rightarrow w_{old}(i) \quad = \quad \left[\frac{K_x\, dx}{(B-A)} \cdot \frac{1}{\int_A^B \frac{K_x dx}{(B-A)}} \right] \times \left[\frac{K_y\, dy}{(D-C)} \cdot \frac{1}{\int_C^D \frac{K_y dy}{(D-C)}} \right] \qquad (5)$$

$$= \quad \frac{1 \times dxdy}{(B-A)(D-C)}$$

Thus we propose to use the simulated event # i with a weight equal to:

$$\boxed{w(i) \quad = \quad \frac{F_{hor}(x_i)}{\int_A^B F_{hor}(x)dx} \times (B-A) \times \frac{F_{ver}(y_i)}{\int_C^D F_{ver}(y)dy} \times (D-C) \qquad \text{(5-bis)}}$$

To get the solid angle $d\Omega_{sim}(j)$ in physics bin j, the re-weighing means:

$$\begin{array}{lll} \text{replace} \quad N_{sim}(j) = \sum_{i=0}^{N_{acc}(j)} (1) & \text{by} & N_{sim}(j) = \sum_{i=1}^{N_{acc}(j)} (w(i) \neq 1) \\ \text{replace} \quad L_{sim} = [\ \sum_{i=1}^{N_0} (1)\ ] \times T & \text{by} & L_{sim} = [\ \sum_{i=1}^{N_0} (w(i) \neq 1)\ ] \times T \end{array} \qquad (6)$$

It turns out that the re-weighing leaves L_{sim} unchanged. This is due to the choice of weight normalizations. To show this intuitively: some of the $w(i)$ are smaller than 1, others are larger than 1, and the whole sum over N_0 is unchanged. To show this quantitatively, one just has to remind the usual equivalence between a mathematical integral and a Monte-Carlo sum (here in 2D):

$$\int_A^B F_{hor}(x)dx \times \int_C^D F_{ver}(y)dy \ \equiv \ [\ \sum_{i=1}^{N_0} F_{hor}(x_i).F_{ver}(y_i)\ ] \times [\ \frac{(B-A).(D-C)}{N_0}\ ] \qquad (7)$$

This relation holds provided that the Monte-Carlo events have been generated in a flat distribution, between A and B for x_i and between C and D for y_i (this is the case here).

On the one hand the re-weighing does not change the denominator of the solid angle, but on the other hand it may change the value of the numerator $N_{sim}(j)$. This is indeed what we want: it expresses the fact that the acceptance may (slightly) change for different beam positions.

One last point: the functions F_{hor}, F_{ver} can be fitted to the experimental data. The difficulty may be to find a good parametric function, that can be integrated analytically. Alternatively, the functions F_{hor}, F_{ver} can be taken to be just the content of experimental distributions, bin per bin in x and y with a fine enough bin width. This is simpler and does not need any fit nor analytical integration. *I would probably favor this solution.* (see example 2).

Below are some examples to get familiar with the re-weighing procedure. Example 1 is a simple mathematical case, it is mentioned because we discussed it at Gent. Example 2 deals with a realistic beam profile obtained in one run. Example 3 is a generalization to cumulated runs with possibly different beam profiles.

- **Example 1.** The experimental beam profile is flat but in reduced windows $[x_1, x_2]$ and $[y_1, y_2]$ instead of $[A, B]$ and $[C, D]$. This was discussed at Gent as a study case.

$$
\begin{aligned}
F_{hor}(x)dx &= G_x/(x_2 - x_1) && \text{for } x \in [x_1, x_2] \\
F_{hor}(x)dx &= 0 && \text{otherwise} \\
G_x &= && \text{arbitrary constant.}
\end{aligned}
$$

$$
\int_A^B F_{hor}(x)dx = \int_{x_1}^{x_2} F_{hor}(x)dx = \int_{x_1}^{x_2} \frac{G_x}{x_2 - x_1} dx = G_x
$$

(same in y). So:

$$
\begin{aligned}
w(i) &= \tfrac{B-A}{x_2-x_1} \times \tfrac{D-C}{y_2-y_1} && \text{for } x_i \in [x_1, x_2] \text{ and } y_i \in [y_1, y_2] \\
w(i) &= 0 \text{ otherwise.}
\end{aligned}
$$

Example of application: consider $[A, B] = [C, D] = [-8, +8]$ mm and $[x_1, x_2] = [y_1, y_2] = [-4, +4]$ mm. $\Rightarrow$ $w(i) = 4$ or 0, depending on the (x_i, y_i) of the event; see figure 3.

In the sum giving $N_{sim}(j)$: assign weight=0 to events which have x_i outside $[x_1, x_2]$ or y_i outside $[y_1, y_2]$, and assign weight=4 to the other events. If the simulated beam profile, after acceptance, is like in figure 1-b, this will result in:
- a little less than 3/4 of the events having zero weight;
- a little more than 1/4 of the events having weight =4.

Consequently, the solid angle $d\Omega_{sim}(j)$ corresponding to the reduced beam windows will be slightly larger than the solid angle computed for the large beam windows $[A, B], [C, D]$.

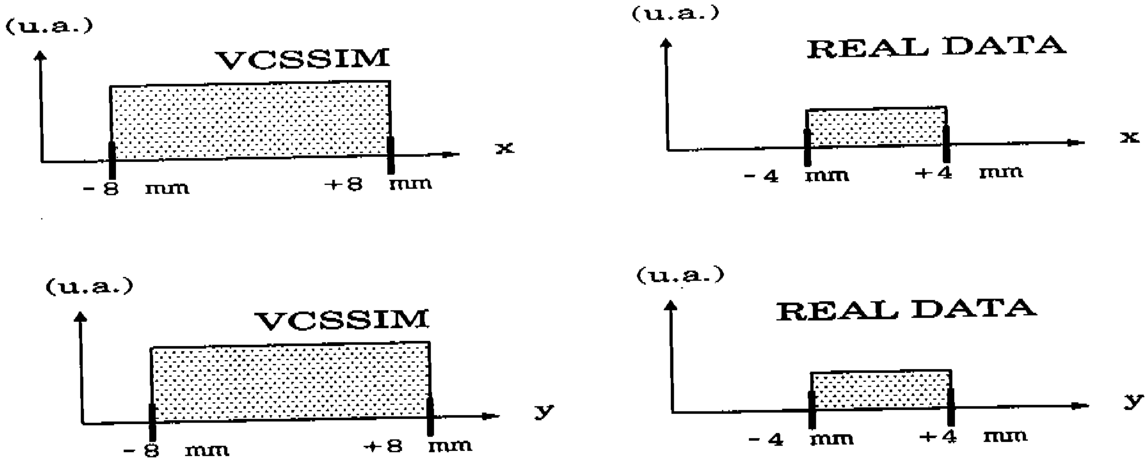

Figure 3:

• **Example 2.** General case where the experimental beam profile is given bin per bin in x and y. Numbers per bin that are indicated below correspond to the spectra of figure 2-a.

Profile Scan: from Lower Edge= -8mm, to Upper Edge= +8 mm
bin width= 0.25 mm ; number of bins= 64 (in x or y)

Experimental distribution														
x-bin #	1	...	29	30	31	32	33	34	35	36	37	38	...	64
counts/bin	0	0	83	783	469	398	394	423	855	195	0	0	0	0
Total number of events= 3600														
y-bin #	1	...	29	30	31	32	33	34	35	36	37	38	...	64
counts/bin	0	0	0	0	18	725	600	515	510	654	576	2	0	0
Total number of events= 3600														

For a simulated event pertaining to a certain bin, the distribution functions F_{hor} and F_{ver} take the value:

$$F_{hor} = \frac{\#\text{ counts in the bin in } x}{0.25} \ (mm^{-1})$$

$$F_{ver} = \frac{\#\text{ counts in the bin in } y}{0.25} \ (mm^{-1})$$

Their integrals are:

$$\int F_{hor}(x)dx \equiv \sum_{bins} \frac{\#\text{ counts per bin}}{0.25} \times (\text{bin width}=0.25\text{ mm}) = 3600$$

$$\int F_{ver}(y)dy \equiv \sum_{bins} \frac{\#\text{ counts per bin}}{0.25} \times (\text{bin width}=0.25\text{ mm}) = 3600$$

For example, a simulated event of beam coordinates (x_i, y_i) falling in bin # 34 in x and # 31 in y has the weight:

$$w(i) = [\ \frac{423}{3600} \cdot \frac{16(mm)}{0.25(mm)}\ ] \times [\ \frac{18}{3600} \cdot \frac{16(mm)}{0.25(mm)}\ ] = 2.406$$

The resulting weights per bin in a 2D (x, y) table are shown below. ix is the bin number in$x$, iy is the bin number in y. These weights have to be used with L_{sim} unchanged, i.e. as it comes out of VCSSIM.

ix \ iy -->	29	30	31	32	33	34	35	36	37	38	39
27	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
28	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
29	.0	.0	.5	19.0	15.7	13.5	13.4	17.2	15.1	.1	.0
30	.0	.0	4.5	179.4	148.5	127.4	126.2	161.8	142.5	.5	.0
31	.0	.0	2.7	107.5	88.9	76.3	75.6	96.9	85.4	.3	.0
32	.0	.0	2.3	91.2	75.5	64.8	64.2	82.3	72.5	.3	.0
33	.0	.0	2.2	90.3	74.7	64.1	63.5	81.4	71.7	.2	.0
34	.0	.0	2.4	96.9	80.2	68.8	68.2	87.4	77.0	.3	.0
35	.0	.0	4.9	195.9	162.1	139.2	137.8	176.7	155.6	.5	.0
36	.0	.0	1.1	44.7	37.0	31.7	31.4	40.3	35.5	.1	.0
37	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
38	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

- **Example 3.** We now want to cumulate several runs with different beam profiles.

For the beam profile cumulated over runs, care must be taken to add the beam histograms proportionally to the incoming charge of each run. Then the method explained in example 2 applies to this case in a straightforward way. From the point of view of beam profile, this method allows to treat all runs of a given VCS setting of da-1 or da-2 as one single big run, to be associated to one single simulation setting from VCSSIM.

It would be nice if we could find such a unified method to treat other problems, such as the variations of E-arm mispointing during a VCS setting...

• **Application 4:** Details of the procedure adopted at present time in our analysis code.

Experimental beam profiles are built from T5 events. This choice is made for practical purposes: i) the T5 are the input events of the VCS analysis ; ii) we take all T5 (no cuts), this provides large statistics and hence smooth distributions in $beamx, beamy$.

The steps for building the cumulated distributions $F_{hor-cumul}(x)$ and $F_{ver-cumul}(y)$ over several runs are the following:

- define the bounds A, B, C, D and the binning in $beamx, beamy$. For simplicity, the bounds should match the ones used in VCSSIM.
- each **experimental T5 event** is put in a bin (ix, iy) according to its values of $beamx$, $beamy$. One count is added in $F_{hor}(ix)$ and in $F_{ver}(iy)$.
- at the end of the run, the possibility is offered to "symmetrize" the distributions F_{hor} and F_{ver}, i.e. to change them in such a way that the two peaks[2] reach the same height. This is done using a slope factor versus $beamx$ or $beamy$.
- then, at the end of the run, a renormalization factor is applied to the horizontal distribution:

  $F_{hor}(x) = F_{hor}(x) * Q/(\text{integral of } F_{hor})$

  in order to make proper addition of runs with different charges Q. This factor is not applied to the vertical distribution, because it is a renormalization of the weight, and the weight will always be computed as the product $F_{hor} \times F_{ver}$.
- then, at the end of the run, one adds the distribution obtained in the run to the distribution cumulated over runs: $F_{hor-cumul}(x) = F_{hor-cumul}(x) + F_{hor}(x)$ (same for y).

  Reset F_{hor}, F_{ver} for next run.
- when all the runs of a given setting have been analyzed, one has in $F_{hor-cumul}$ and $F_{ver-cumul}$ the distributions representing the beam profile of cumulated runs. Their integrals are calculated: I_x and I_y.
- in the analysis of **simulated events** of this setting; each event is put in a bin (ix, iy) according to its values of $beamx, beamy$. The event weight is computed as:

  $$weight = \frac{F_{hor-cumul}(ix) \times F_{ver-cumul}(iy) \times (B-A)\ (D-C)}{I_x \times I_y}\ .$$

[2]Usually, if the beam position and raster amplitude were stable during the run, the beam profile in x or y displays two peaks and a valley in-between.

2 PART II - Determination of the geometrical efficiency for realistic beam profiles

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I present a different view on the way to obtain geometrical efficiencies for realistic beam profiles, giving exactly the same result as Helene's approach

2.1 General definition of geometrical efficiency

In general, if one performs a simulation containing the cross section behaviour one can write for the corrected geometrical efficiency (I try to use as much as possible the same notations as Helene, so see her text for the definition of notations. I drop however the index (j) for the bin, all what I write below is true for any bin in phase space):

$$\Delta\Omega_{sim} = \frac{N_{sim}}{L_{sim}.d^5\sigma_{BH+Born}} \tag{8}$$

The above expression is true for any beam profile. We would like to express the above expression for any beam profile using results obtained with a flat beam profile. This would allow us to perform simulations using a flat beam profile only, and prevent a lot of simulations each with another beam profile. Indeed, it is a fact that $\Delta\Omega_{sim}$ is a quantity which depends on the beam profile, certainly when the cross section behaviour is taken into account.

2.2 Introducing a flat beam profile

To simplify things I work in one dimension only, the x-dimension. One has to consider the fact that $\Delta\Omega_{sim}$ is a quantity that depends on x. If one considers $\Delta\Omega_{sim}$ to be a continuous function of x, one can write the number of counts in the expression above in a general way:

$$N_{sim} = \int_{x1}^{x2} d^5\sigma_{BH+Born}.\Delta\Omega_{sim}(x).\frac{dL_{sim}(x)}{dx}dx \tag{9}$$

where $\frac{dL_{sim}(x)}{dx}$ represents the actual beam profile used in the simulation, since the integrated luminosity is the product of integrated charge and target thickness and the target thickness is independent of x. $x1$ and $x2$ are the limits of the charge distribution on the target.

Taking infinitesimal bins in x, one can define $\Delta\Omega_{sim}(x)$ as:

$$\Delta\Omega_{sim}(x) = \frac{\frac{dN_{sim}(x)}{dx}dx}{\frac{dL_{sim}(x)}{dx}dx.d^5\sigma_{BH+Born}} \tag{10}$$

Remark that the above expression is true for any beam profile. In particular, it is also true for a flat beam profile. So, if one performs a simulation using a flat beam profile, as we want to do, one can write for $\Delta\Omega_{sim}(x)$:

$$\Delta\Omega_{sim}(x) = \frac{\frac{dN_{sim,flat}(x)}{dx}dx}{\frac{dL_{sim,flat}(x)}{dx}dx.d^5\sigma_{BH+Born}} \tag{11}$$

Now we can rewrite formula (8), which represents the geometrical efficiency obtained with a simulation using any beam profile, using expression (11), by substituting (11) in (9) and (9) in (8):

$$\Delta\Omega_{sim} = \frac{\int_{x1}^{x2} d^5\sigma_{BH+Born}\cdot\frac{\frac{dN_{sim,flat}(x)}{dx}dx}{\frac{dL_{sim,flat}(x)}{dx}dx.d^5\sigma_{BH+Born}}.\frac{dL_{sim}(x)}{dx}dx}{L_{sim}.d^5\sigma_{BH+Born}} \tag{12}$$

or

$$\Delta\Omega_{sim} = \frac{\int_{x1}^{x2}\cdot\frac{\frac{dN_{sim,flat}(x)}{dx}dx}{\frac{dL_{sim,flat}(x)}{dx}dx}.\frac{dL_{sim}(x)}{dx}dx}{L_{sim}.d^5\sigma_{BH+Born}} \tag{13}$$

which becomes, dropping out a few dx's and replacing some factors:

$$\Delta\Omega_{sim} = \frac{\int_{x1}^{x2}\frac{\frac{1}{L_{sim}}\frac{dL_{sim}(x)}{dx}}{\frac{dL_{sim,flat}(x)}{dx}}.\frac{dN_{sim,flat}(x)}{dx}dx}{d^5\sigma_{BH+Born}} \tag{14}$$

The above expression allows to obtain the geometrical efficiency a simulation with an actual beam profile would yield, using the result from a simulation with a flat beam profile. Indeed, let us consider all "components" in expression (14):

- $\frac{dN_{sim,flat}(x)}{dx}dx$ is the number of counts from the flat simulation present in an infinetisimal bin dx at position x. One can generalize this to the number of counts in a bin around x, in which case the integral will become a sum over bins.

- $\frac{1}{L_{sim}}\frac{dL_{sim}(x)}{dx}$ is the normalized actual beam profile, since of course $L_{sim} = \int_{x1}^{x2}\frac{dL_{sim}(x)}{dx}dx$
- $\frac{dL_{sim,flat}(x)}{dx}$ is simply given by $\frac{L_{sim,flat}}{(B-A)}$, where $L_{sim,flat}$ is the integrated luminosity from the flat profile simulation, and A and B are the x-borders of this simulation (which should of course include $x1$ and $x2$).
- $d^5\sigma_{BH+Born}$ is the cross section in a point in the phase space bin.

Conclusion: given the results of a simulation with a flat beam profile, and given an actual beam profile, all ingredients are there to apply equation (14) to obtain the asked result. Remark that only the shape of the actual beam profile is necessary, since the normalized actual beam profile is used in equation (14). So there is no problem with "profile height" here, any constant factor drops out anyway!

2.3 The link with Helene's results

Given the fact that $\frac{dL_{sim,flat}(x)}{dx} = \frac{L_{sim,flat}}{(B-A)}$, we can rewrite equation (14) as:

$$\Delta\Omega_{sim} = \frac{\int_{x1}^{x2}\frac{1}{L_{sim}}\frac{dL_{sim}(x)}{dx}(B-A)\frac{dN_{sim,flat}(x)}{dx}dx}{L_{sim,flat}.d^5\sigma_{BH+Born}} \tag{15}$$

So, one notices that the events $\frac{dN_{sim,flat}(x)}{dx}dx$ at position x are weighted by $\frac{1}{L_{sim}}\frac{dL_{sim}(x)}{dx}(B-A)$ which is exactly the x-component of the weight factor w which is given by Helene in eq.(5-bis). Indeed, we have

$$\frac{F_{hor}(x)}{\int_A^B F_{hor}(x)dx} = \frac{1}{L_{sim}}\frac{dL_{sim}(x)}{dx} \tag{16}$$

The left and right hand of the above equation represent the normalized shape of the beam profile, and Helene's result is for the case $x1 = A$ and $x2 = B$

It is clear that everything above is also true for the y-component part.

So, both views yield the same result!

VCS-E93050

A study of the influence of horizontal beam position on solid angle for coincidence VCS events

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January 2000
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We have studied how the solid angle for $p(e,e'p)\gamma$ coincidence events behaves as a function of the horizontal beam position on target (*beamx*). This gives an idea of what will be the systematic error on the measured $(ep \rightarrow ep\gamma)$ cross sections due to systematic uncertainty on *beamx* in the experiment.

Experimentally, the systematic error on beam coordinates at target is mostly due to limited knowledge of the absolute position of the Beam Position Monitors BPM-A and BPM-B in Hall A Frame. The main conclusion of the present study is that with "BPM offsets" known to (?) $\pm$ 0.5 mm, one gets systematic effects on the solid angle that can reach a few % locally in the VCS phase space.

1 Principle

Such systematic studies are most suitably done with a simulation, in which the variation of parameters is under control. We have used VCSSIM simulation code and generated coincidence events of one VCS setting: da-2-12.

VCSSIM allows flexibility in beam profile definition. We choosed a delta function in *beamx* (also in *beamy*), and the position of this delta-function was moved by steps of 2.5 mm. The solid angle was determined for each step.

This solid angle can be:

- an **integral** one, i.e. integrated over the whole acceptance
- a **differential** one, i.e. determined in one phase space bin j. By **bin** we mean a bin in $\cos\theta_{\gamma\gamma cm}$ and q'_{cm} which are the main variables for the extraction of polarizabilities.

So a first part will be devoted to integral solid angles, and a second part to solid angles per bin. We define the (differential) solid angle $\Delta\Omega(j)$ in physics bin j by:

$$\Delta\Omega(j) \;=\; \frac{N_{sim}(j)}{L_{sim} \times d^5\sigma_{theo}(j)} \tag{1}$$

where $N_{sim}(j)$ is the number of (simulated) events going through the spectrometer acceptance and being reconstructed in bin j. L_{sim} is the (simulated) luminosity that was necessary to generate this number of events. $d^5\sigma_{theo}(j)$ is the theoretical cross section that was used to generate the events; it is computed at one particular (and arbitrary) point in phase space. The integral solid angle is the sum of the $\Delta\Omega(j)$ over all bins j. It has a formal expression similar to eq.(1):

$$\Delta\Omega^{tot} \;=\; \frac{N^{tot}_{sim}}{L_{sim}\times < d^5\sigma_{theo} >} \tag{2}$$

where N^{tot}_{sim} is the total number of accepted events and $< d^5\sigma_{theo} >$ is a cross section integrated in the accepted phase space.

VCSSIM proposes 2 choices of theoretical cross section shape $d^5\sigma(ep \to ep\gamma)$ to generate events:

1. *constant* cross section in the whole phase space
2. *[Bethe-Heitler + Born]* cross section.

• **Difference between the two choices of cross section:**

The first test we made is summarized in table 1: we moved *beamx* from 0 to +5 mm, asking for the same number of accepted events N_{sim}^{tot}. We did that for both cross section choices.

cross section choice	beamx (delta funct.)	N_{sim}^{tot}	L_{sim} (pb^{-1})	relative change in luminosity w.r.t. the case $beamx = 0$
1) $d^5\sigma$ = constante	0 mm	4×10^4	2.15×10^5 pb^{-1}	--------
1) $d^5\sigma$ = constante	+5 mm	4×10^4	2.18×10^5 pb^{-1}	$\frac{2.18-2.15}{2.15} = +1.4$ %
2) $d^5\sigma$ = BH+B	0 mm	10^4	4.59×10^6 pb^{-1}	--------
2) $d^5\sigma$ = BH+B	+5 mm	10^4	4.25×10^6 pb^{-1}	$\frac{4.25-4.59}{4.59} = -7.4$ %

Table 1: Choices of cross section and first test on integral quantities.

One sees that the two choices of cross section do not lead to the same relative change in L_{sim}. This is normal, because when one takes option 2) one adds the effects due to cross section variation itself. Indeed the change in *beamx* induces a change in the accepted range of horizontal angles in each arm[1], and the (BH+B) cross section varies with these polar angles θ_e, θ_p. See figure 1.

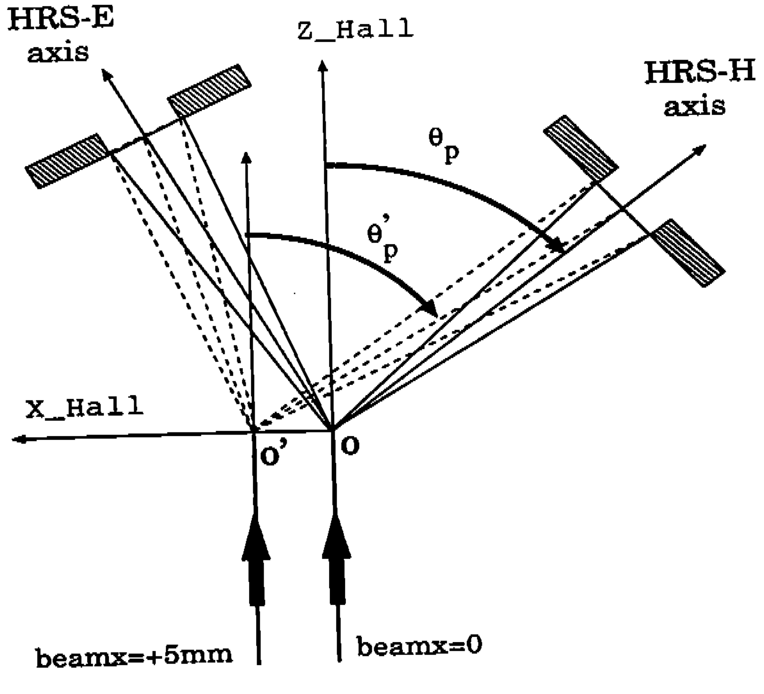

Figure 1: *View in horizontal plane (X,Z) of Hall A frame. Change in polar angle θ_p when horizontal beam position is changed by 5 mm.*

[1] angles measured w.r.t. the beam axis

The quantity $\Delta\Omega$ that we have defined may not be exactly a "solid angle" but more a "geometrical efficiency", as Luc calls it. So the use of a cross section or another may lead to different solid angles. But their relative change with *beamx* should be the same.

• **Our choice of cross section:** based on the above considerations, we have chosen to use option 1) = constant cross section in VCSSIM. First we are interested in geometrical effects and not in cross section behaviour. Second, this option requires much less computer time than the (BH+B) option, for the same number of accepted events.

2 Results on Integral solid angle

As we have chosen $d^5\sigma$ to be a constant, the ratio N_{sim}/L_{sim} is equal to a solid angle $\Delta\Omega$, up to a constant factor. In table 2 we also keep the number of accepted events N^{tot}_{sim} constant. So we have a very simple property: when we move *beamx*, the relative change observed on L_{sim} is directly equal to the relative change in solid angle, with opposite sign. This is how the last column of table 2 is computed. It shows R= relative change in solid angle w.r.t. its value at $beamx = 0$.

$$R = \frac{\delta\Delta\Omega}{\Delta\Omega} = \frac{\Delta\Omega(beamx) - \Delta\Omega(0)}{\Delta\Omega(0)}$$

beamx (delta funct.)	N^{tot}_{sim}	L_{sim} (pb^{-1})	$R(beamx) = \frac{\Delta\Omega(\text{beamx}) - \Delta\Omega(0)}{\Delta\Omega(0)}$
-5.0 mm	4×10^4	2.174×10^5	-0.0102 ± 0.0070
-2.5 mm	4×10^4	2.153×10^5	-0.0005 ± 0.0070
0 mm	4×10^4	2.152×10^5	0
+2.5 mm	4×10^4	2.165×10^5	-0.0060 ± 0.0070
+5.0 mm	4×10^4	2.178×10^5	-0.0121 ± 0.0070

Table 2: Relative change in integral solid angle obtained with constant cross section.

Figure 2 displays the values of the last column as a function of *beamx*. The error bar on R is statistical:

$$\frac{\Delta R}{R+1} = \left[\left(\frac{\Delta N^{tot}_{sim}(beamx)}{N^{tot}_{sim}(beamx)}\right)^2 + \left(\frac{\Delta N^{tot}_{sim}(0)}{N^{tot}_{sim}(0)}\right)^2 \right]^{1/2}, \qquad \text{with } \Delta N = \sqrt{N}. \tag{3}$$

Conclusion: effects are small. The integral solid angle does not change by more than 1 % when beamx changes by as much as 5 mm.

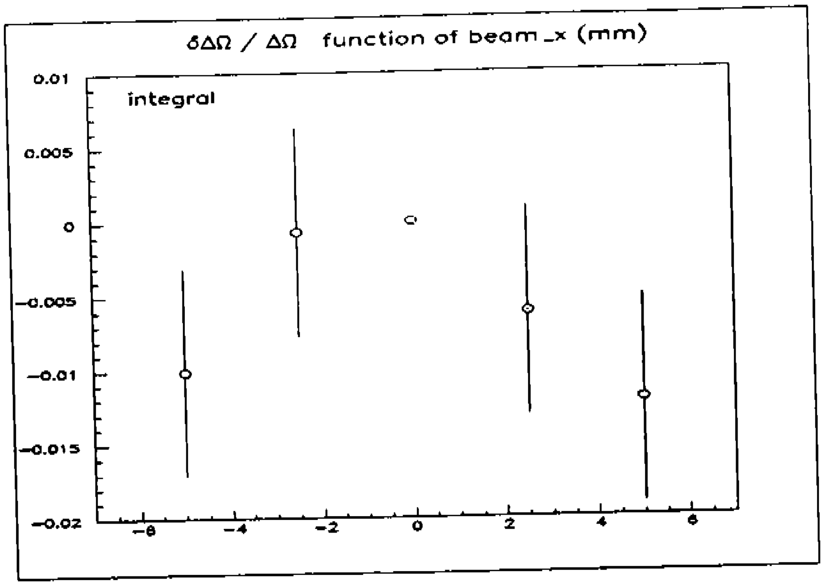

Figure 2: *Relative change of integral solid angle versus beamx.*

3 Results on Differential solid angle

We use the same samples of simulated events as in previous section, but now reconstructed events are put in physics bins in $\cos\theta_{\gamma\gamma cm}$ and q'_{cm}. Results per bin are shown in figures 3 -7 in terms of the previously defined ratio R = $\delta\Delta\Omega/\Delta\Omega$. Each figure 3,4,5,6, corresponds to a different value of *beamx* (-5, -2.5, +2.5, 5 mm), and particular physics bins have been selected. The statistical error bar on R is computed as in eq.(3).

As expected, variations of solid angle are much more important locally in phase space that when integrated over phase space. They can reach up to 10-40 % for *beamx* = 5 mm, the highest values corresponding to events close to acceptance edge.

Figure 7 shows another more or less expected feature, namely that locally in phase space the solid angle varies linearly with *beamx* to first order. The slope of variation changes sign from one physics bin to another, as shown in the figure. That explains why the integral solid angle is not so sensitive to beam position variations.

4 Conclusions

1.

This set of plots (3-7) can serve as calibration curves of systematic error on solid angle (and hence on experimental cross sections) due to *beamx* uncertainty.

2.

For an experimental $\Delta beamx = \pm 0.5$ mm due to BPM offsets uncertainty, we expect to have an induced systematic error on $d^5\sigma_{exp}$ that can reach a few percents locally in phase space ($\sim$ 0-4 %).

3.

The variation of solid angle with vertical beam position *beamy* should be studied as well. The procedure explained in this paper could be applied. Also, one should probably extend the study to other VCS DA-settings. Finally, the variation of solid angle with spectrometer mispointing could also be studied along the same lines (always for coincidence events in VCS kinematics).

Many thanks to Luc Van Hoorebeke for discussions and critical inputs to this document.

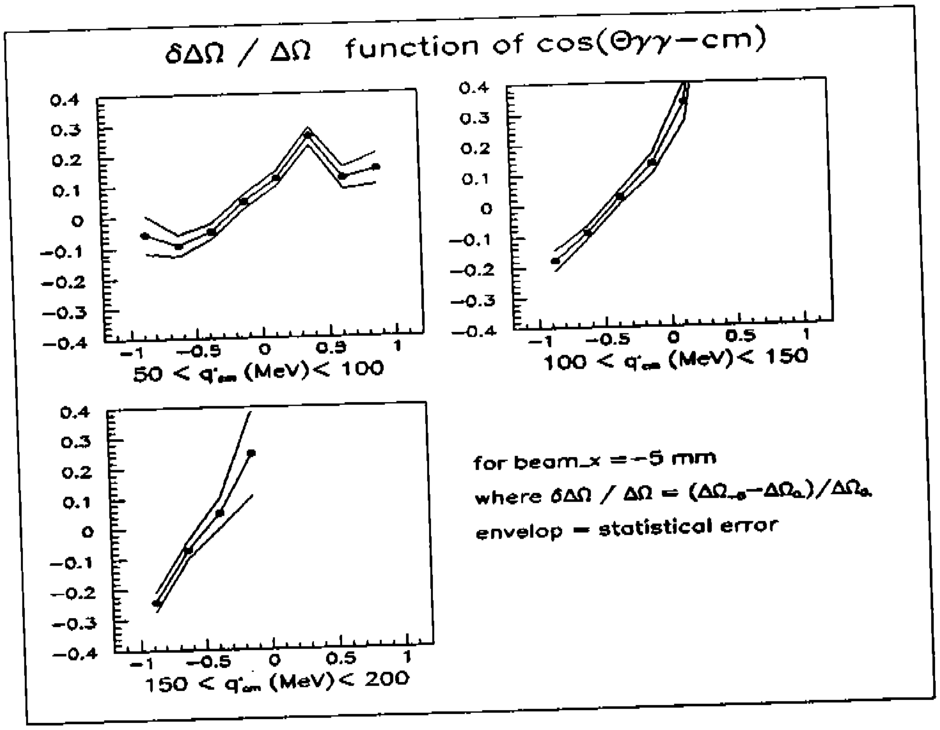

Figure 3: *Relative change in differential solid angle for beamx = -5.0 mm.*

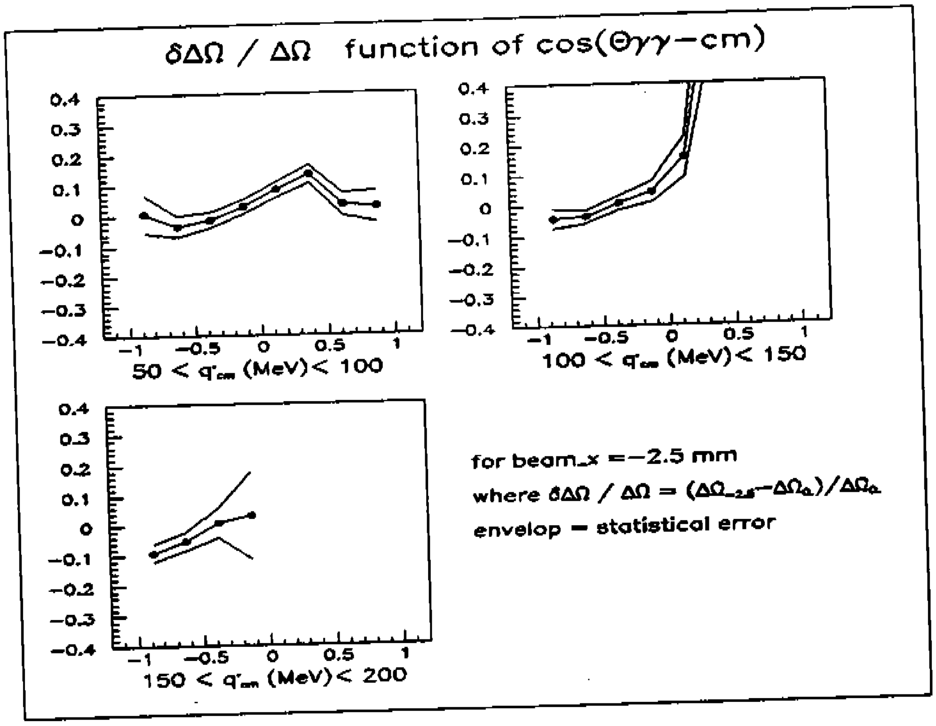

Figure 4: *Relative change in differential solid angle for beamx = -2.5 mm.*

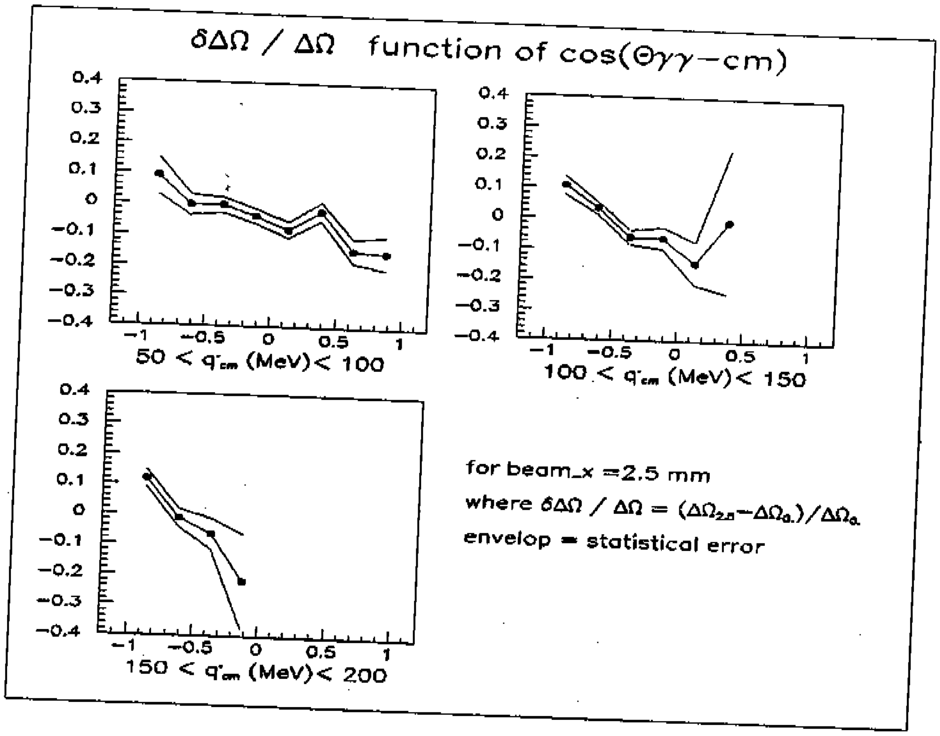

Figure 5: *Relative change in differential solid angle for beamx = +2.5 mm.*

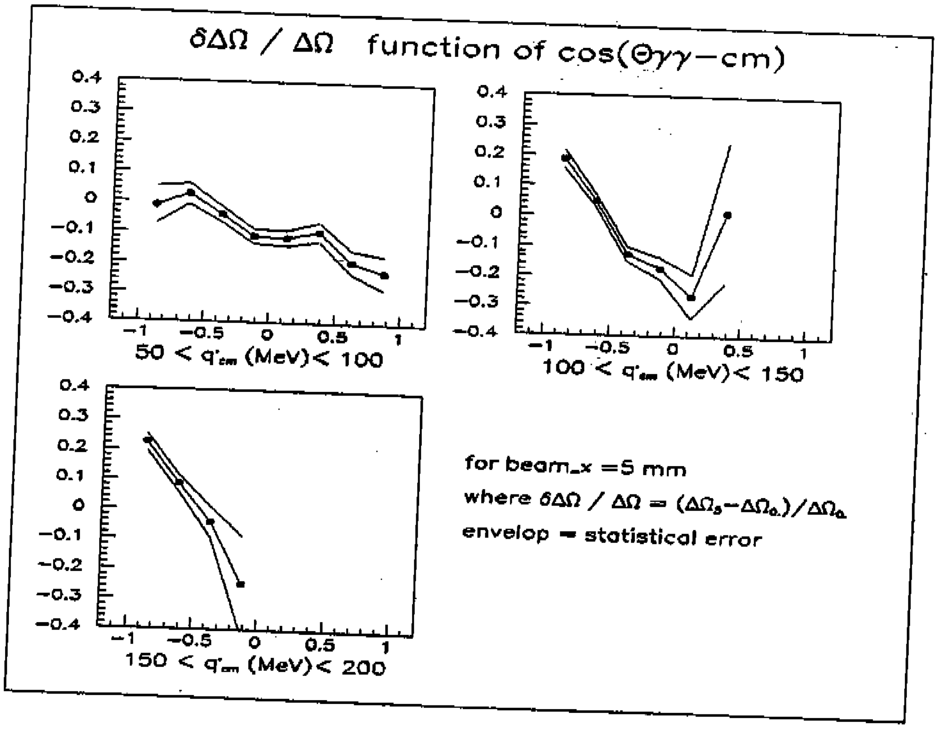

Figure 6: *Relative change in differential solid angle for beamx = +5.0 mm.*

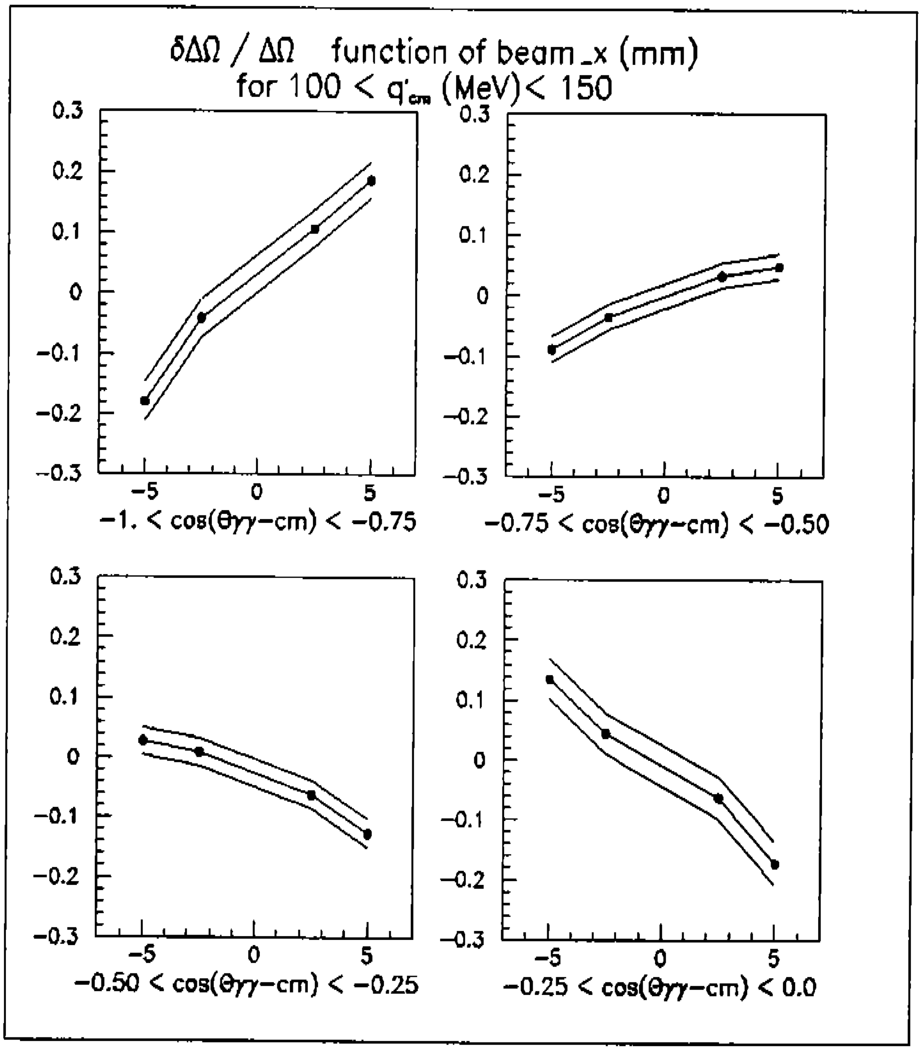

Figure 7: *Relative change in differential solid angle as a function of beamx, for 4 selected physics bins: [one bin in* q'_{cm}*]* $\oplus$ *[4 bins in* $\cos\theta_{\gamma\gamma\ cm}$*].*