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Spectrometer constant determination for the Hall-A High Resolution Spectrometer pair

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The constant Γ for a spectrometer relates the central momentum measured by the spectrometer (P_0) to the central magnetic field (B_0).

$$P_0 = \Gamma B_0 \tag{1}$$

The spectrometer constants of the Hall-A High Resolution Spectrometer pair have been known only to about 2×10^{-3} level. The precision beam energy measurements using EP and Arc have made it possible to measure the spectrometer constants much more accurately. Therefore, whenever a beam energy measurement was performed we have gathered data necessary for the calculation of Γ.

We use two methods to calculate the Γ. Both methods involve scattering electrons from a thin ^{12}C target. The energy loss inside such a thin target is negligible compared to the measured energies. Furthermore, due to the relatively high mass of the ^{12}C nucleus, target recoil energy is negligible. As a result, these measurements are rather insensitive to spectrometer angle measurements.

The two methods used were,

1. The direct method, where we measured elastic scattered electrons from ^{12}C accompanied by a beam energy measurement to directly calculate Γ for that spectrometer,
2. The indirect method, where we measured the missing energy of the $1p_{1/2}$ state in ^{12}C(e,e'p) coincidence data. We then used this information with the already measured spectrometer constant of one spectrometer to derive Γ for the other spectrometer.

1 Direct calculation of Γ using elastic ^{12}C(e,e) reaction

At low beam energies around 1 GeV, elastic scattering from ^{12}C is prominent for spectrometer angles less than $\sim 20^\circ$. Elastic data from such kinematics are routinely used for optics offset calibration of the HRS pair. For these calibration runs both spectrometers are set in negative polarity to detect electrons, and are placed around 15°. These elastic data combined with EP and Arc beam energy measurements were used for Γ calculations of the two spectrometers. The spectrometer magnetic fields were set so that the elastic peak appears close to the center of the effective "focal plane" defined by the lowest VDC plane. Furthermore, software cuts were used to select tracks making small angles (< 4 mrad) with the central ray. This ensured that the Γ calculated here is not sensitive to the optics matrix elements.

The first determination of Γ using ^{12}C elastic data was performed on 09/22/99 during the Gammap experiment. However only the Left HRS was available for this measurement. A symmetric ^{12}C elastic measurement was also performed during the Gammap experiment without a beam energy measurement. Such a symmetric elastic measurement between the two spectrometers without

a beam energy measurement can be used to deduce the ratio of the two spectrometer constants and to calculate the constant for one spectrometer given the constant of the other.

Two sets of symmetric measurements with beam energy measurements were performed before and after the detector swap between the two spectrometers in September 2000. The first measurement was performed on 12/06/99 at the beginning of experiment 89044 and the second measurement on 10/01/00 at the beginning of experiment 97111.

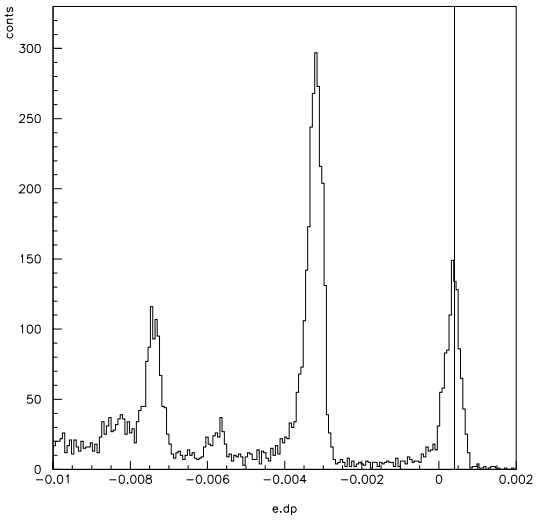

Figure 1: The ^{12}C(e,e) **dp** spectrum. The line indicates the peak position for the elastic peak at **dp** $=4 \times 10^{-4}$.

Fig. 1 shows the **dp**[1]spectrum from an elastic ^{12}C run. The location of the elastic peak in **dp** is related to the scattered electron momentum (P_f) by,

$$E_f = P_f = \Gamma B_0 (1 + dp) \tag{2}$$

where B_0 [kG] is the dipole field. In this calculation we use the dipole field measured by the low-field probes in both spectrometers since the low probes are used to set the spectrometer momenta[2].

For the ground state of ^{12}C, the energy E_f is related to the beam energy E_i by

$$E_f = \frac{E_i - E_{loss1}}{1 + \frac{2E_i \sin^2(\theta/2)}{M_t}} - E_{loss2} \tag{3}$$

where M_t is the mass of the target nucleus,
E_{loss1} and E_{loss2} denote the energy loss in the target before and after scattering.

Since we used a thin target, $E_{loss1} \approx E_{loss2} \approx 0.1$ MeV. This energy loss can be calculated to ~ 0.01 MeV level ($\sim 1 \times 10^{-5}$ of E_f). The dipole field was measured using the NMR probe to better than 1×10^{-4} accuracy. The narrowness of the elastic peak (3×10^{-4} FWHM) allows for the determination of the peak position to better than 1×10^{-4}. The scattering angles in the HRS pair are measured with an accuracy of ~ 0.7 mrad. Due to relatively high mass of the ^{12}C nucleus, this angular uncertainty contributes less than 1×10^{-5} to the uncertainty in Γ. The dominant uncertainty in Γ calculated using this method directly comes from the uncertainty in the beam energy measurement at the level of $\sim 3 \times 10^{-4}$.

[1] $dp = \frac{P - P_0}{P_0}$

[2] Note that the HRS NMR probe readings are in Tesla, thus they must be multiplied by 10 to convert to kG.

2 Indirect calculation of Γ

We used the ^{12}C(e,e'p) coincidence data gathered in several different settings to derive Γ for one spectrometer using a previously calculated Γ for the other spectrometer. The uncertainty in Γ obtained using this method suffers due to propagation of errors. However this method allows the measurement at higher momenta where vanishing elastic cross sections for ^{12}C prevents the use of the previous method.

For the ^{12}C(e,e'p) reaction,

$$E_{miss} = E_i - E_f - T_p - T_B - E_{loss1} - E_{loss2} \tag{4}$$

$$T_B = \frac{p_{miss}^2}{2M_B} \tag{5}$$

and conservation of energy requires

$$E_{miss} = M_B + M_p - M_A \tag{6}$$

where
E_{miss} is the missing energy,
T_B is the kinetic energy of the recoiling nucleus,
p_{miss} is the missing momentum,
M_A and M_B are the masses of target and recoil nuclei respectively.

Thus E_{miss} is just the binding energy of the knocked-out proton. For ^{12}C, the 1p proton binding energy is 16 MeV. Using the 1p protons the energy (momentum) of the corresponding electrons can be reconstructed using

$$E_f = E_i - 16.0 - T_p - T_B - E_{loss1} - E_{loss2}\text{MeV} \tag{7}$$

T_p is obtained by using previously calculated Γ for the second spectrometer.

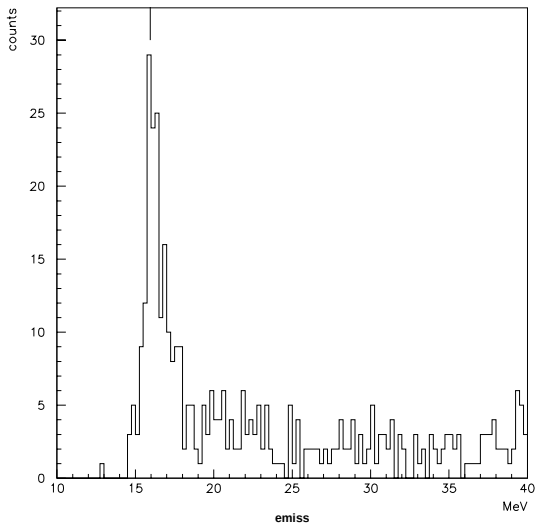

Figure 2: A ^{12}C(e,e'p) missing energy spectrum. the line indicates the 1p peak position at 16.0 MeV

A series ^{12}C(e,e'p) runs accompanied by precision beam energy measurements were taken between 07/99 and 11/00. These runs cover a large range of momenta for both spectrometers; 0.8 GeV - 4.0 GeV for HRSL and 0.5 GeV - 3.0 GeV for HRSR. This provided the opportunity to calculate Γs as a function of spectrometer momenta and to deduce the functional dependence of Γs on momentum.

2.1 Results

Tables 1 and 2 give the calculated values of Γ below 2 GeV for the Left HRS (HRS-L) and the Right HRS (HRS-R) respectively. As the tables indicate, all Γ measurements for each spectrometer agree well.

Date	Momentum (GeV)	Γ_{HRS-L} (MeV/kG)	Method
09/22/99	1.26	270.1 ± 0.1	^{12}C(e,e) direct
12/06/99	0.84	270.2 ± 0.1	^{12}C(e,e) direct
10/01/00	0.83	270.2 ± 0.1	^{12}C(e,e) direct

Table 1: Γ_{HRS-L}. All three were direct Γ calculations from ^{12}C(e,e) runs accompanied by energy measurements

Date	Momentum (GeV)	Γ_{HRS-R} (MeV/kG)	Method
08/12/99	0.86	269.8 ± 0.2	^{12}C(e,e) indirect
12/06/99	0.84	269.8 ± 0.1	^{12}C(e,e) direct
10/01/00	0.83	269.9 ± 0.1	^{12}C(e,e) direct

Table 2: Γ_{HRS-R}. The first measurement is from a ^{12}C(e,e) run with both spectrometers in symmetric configuration, but without an energy measurement. In this case a previously calculated Γ_{HRS-L} was used to calculate Γ_{HRS-R}. The last two were direct Γ calculations from ^{12}C(e,e) runs accompanied by energy measurements.

Data from the series of ^{12}C(e,e'p) measurements indicate that Γ for each spectrometer goes down slowly with momentum. Furthermore, this data indicate that the momentum dependence of Γs can be absorbed in to a 3rd order polynomial in magnetic field B_0.

$$P_0 = \sum_{i=0}^{2} \Gamma_i \mathrm{B}_0^{\mathrm{i}} = \Gamma_0 + \Gamma_1 B_0 + \Gamma_2 B_0^2 + \Gamma_3 B_0^3 \tag{8}$$

Where B_0 is measured in kG and the Γ_i for each spectrometer are:

Spectrometer	Γ_0	Γ_1	Γ_2	Γ_3
HRS-L	0.0	270.2 ± 0.15	0.0	-1.6 E-03 ± 0.7 E-03
HRS-R	0.0	269.8 ± 0.15	0.0	-1.6 E-03 ± 0.7 E-03

Table 3: Γ coefficients for the two spectrometers.

Fig. 3 shows the missing energy spectra for these coincidence runs calculated with Γ coefficients given in table 3.

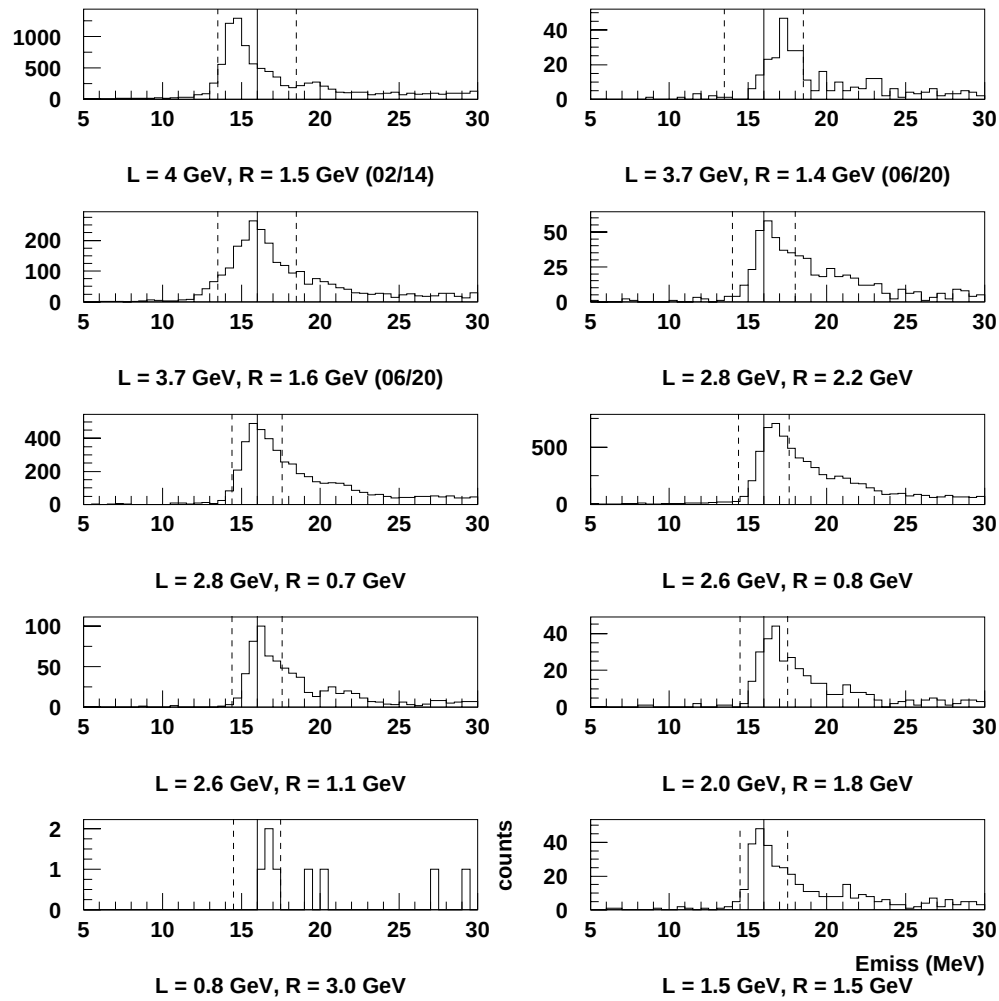

Figure 3: ^{12}C(e,e'p) E_{miss} spectra for a series of runs covering the momentum ranges of the two spectrometers. Γ coefficients given in table 3 have been used to obtain these spectra. The solid line in each spectrum indicates the expected location of the peak, while the dashed lines show the uncertainty interval arising from the uncertainty in Γ coefficients and in beam energy measurements.

The calculated spectrometer constants agree well with the spectrometer constant estimations by John LeRose based on the spectrometer magnet field maps [1].

3 conclusion

The spectrometer constants have been calculated over the full momentum range of the the Hall-A High Resolution Spectrometer pair with an accuracy of $\sim 5 \times 10^{-4}$. ^{12}C(e,e'p) data accumulated over 15 months show that these constants are stable at this level.

References

[1] J. J. LeRose, Jefferson lab tech-note 97-042.