Non-Factorized Form of Radiative Corrections to DVCS

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I. INTRODUCTION

These notes follow the paper of M. Vanderhaeghen *et al.* [1], and the code of D. Lhuillier, *et al.,* as modified by P. Guichon to include the DVCS amplitude with a fully factorized GPD ansatz.

In the code, the analytic parts of the scattering amplitude (including first order virtual radiative corrections) are written as

$$i\mathcal{M} = \mathcal{T}_{BH}\frac{1}{1-\Pi(-t)} + \mathcal{T}_{BH}\delta_{v,BH} + \mathcal{T}_{BH,s.e.} + \mathcal{T}_{VCS}\frac{1}{1-\Pi(Q^2)} + \mathcal{T}_{VCS}\delta_{v,VCS} \tag{1}$$

(the subscripts v, $s.e.$ refer to vertex and self-energy corrections, respectively). In this form, it would appear that the vacuum polarization correction Π has been iterated to all orders. However, it seems to me that the factor $1/[1-\Pi]$ should be included with the vertex (v) and self energy ($s.e.$) terms as well.

The separation of radiative corrections into virtual and real terms is dependent on the renormalization scheme used to cancel the divergences. However, the final result, including both real and virtual photon effects must be independent of the renormalization procedure. The real photon effects, themselves, have terms that depend on the radiative cutoff in the experimental analysis, and other terms which are independent of this cutoff. The former I label $\delta_R(\Delta)$ and the later $\delta_{R,0}$. Since $\delta_R(\Delta)$ is directly observable from the shape of the spectrum, I infer that it must be independent of the renormalization scheme. Therefore, the sum

$$\delta_{\text{Virtual}} + \delta_{R,0} \tag{2}$$

must also be independent of renormalization.

II. VIRTUAL RADIATIVE CORRECTIONS

The BH and VCS amplitudes both have Vacuum Polarization and Vertex corrections from virtual photon or electron loops. The BH amplitude also has self energy corrections to the off-shell electron propagators.

A. Vacuum Polarization

The Vacuum Polarization terms are purely analytic, and are given by:

$$\begin{aligned} \Pi(-t) &= \frac{\alpha}{3\pi}\left[\ln\left(\frac{-t}{m_e^2}\right) - \frac{5}{3}\right] \\ \Pi(Q^2) &= \frac{\alpha}{3\pi}\left[\ln\left(\frac{Q^2}{m_e^2}\right) - \frac{5}{3}\right]. \end{aligned} \tag{3}$$

B. Vertex Corrections

For the vertex terms, define the variable

$$v = \sqrt{1 + \frac{4m_e^2}{Q^2}} \tag{4}$$

The analytic parts of the vertex terms are:

$$\begin{aligned} \delta_{v,BH} &= \frac{\alpha}{2\pi}\left\{\left[\frac{v^2+1}{4v}\right]\ln\left[\frac{v+1}{v-1}\right]\ln\left[\frac{v^2-1}{4v^2}\right]\right. \\ &\quad \left. + \left[\frac{v^2+1}{2v}\right]\left[\mathrm{Sp}\left(\frac{v+1}{2v}\right) - \mathrm{Sp}\left(\frac{v-1}{2v}\right)\right] - 3 - v\ln\left[\frac{v+1}{v-1}\right]\right\} \end{aligned} \tag{5}$$

$$\begin{aligned} \mathcal{T}_{VCS}\delta_{v,VCS} &= \epsilon_\mu(q')^\dagger \Gamma^{\mu\nu}_{VCS}\overline{u}(k',h)\gamma_\nu u(k,h) \\ &\quad \frac{\alpha}{2\pi}\left\{\left[\frac{v^2+1}{4v}\right]\ln\left[\frac{v+1}{v-1}\right]\ln\left[\frac{v^2-1}{4v^2}\right]\right. \\ &\quad \left. + \left[\frac{v^2+1}{2v}\right]\left[\mathrm{Sp}\left(\frac{v+1}{2v}\right) - \mathrm{Sp}\left(\frac{v-1}{2v}\right)\right] - 2 + \left[\frac{2v^2+1}{2v}\right]\ln\left[\frac{v+1}{v-1}\right]\right\} \\ &\quad + \epsilon_\mu(q')^\dagger \Gamma^{\mu\nu}_{VCS}\overline{u}(k',h)\frac{\sigma_{\nu\lambda}q^\lambda}{2m_e}u(k,h) \\ &\quad \frac{\alpha}{2\pi}\left[\frac{v^2-1}{2v}\right]\ln\left[\frac{v+1}{v-1}\right] \end{aligned} \tag{6}$$

The Spence function is defined as:

$$\mathrm{Sp}(z) = -\int_0^z \frac{\ln(1-t)}{t}dt \tag{7}$$

This function as the following limits:

$$\begin{aligned} \mathrm{Sp}(z) &\to z \qquad \text{as} \quad z \to 0 \\ \mathrm{Sp}(z) &\to \frac{\pi^2}{6} \qquad \text{as} \quad z \to 1^- \end{aligned} \tag{8}$$

Taking the limit $Q^2 \gg m_e^2$:

$$\begin{aligned} \delta_{v,BH} &\to \frac{\alpha}{2\pi}\left\{\frac{1}{2}\ln\left[\frac{Q^2}{m_e^2}\right]\ln\left[\frac{m_e^2}{Q^2}\right] + \frac{\pi^2}{6} - 3 - \ln\left[\frac{Q^2}{m_e^2}\right]\right\} \\ &\to -\frac{\alpha}{2\pi}\left\{\frac{1}{2}\ln^2\left[\frac{Q^2}{m_e^2}\right] - \frac{\pi^2}{6} + 3 + \ln\left[\frac{Q^2}{m_e^2}\right]\right\} \end{aligned} \tag{9}$$

$$\begin{aligned} \mathcal{T}_{VCS}\delta_{v,VCS} &= \epsilon_\mu(q')^\dagger \Gamma^{\mu\nu}_{VCS}\overline{u}(k',h)\gamma_\nu u(k,h) \\ &\quad \frac{\alpha}{2\pi}\left\{-\frac{1}{2}\ln^2\left[\frac{Q^2}{m_e^2}\right] + \frac{\pi^2}{6} - 2 + \frac{3}{2}\ln\left[\frac{Q^2}{m_e^2}\right]\right\} \\ &\quad + \epsilon_\mu(q')^\dagger \Gamma^{\mu\nu}_{VCS}\overline{u}(k',h)\frac{\sigma_{\nu\lambda}q^\lambda m_e}{Q^2}u(k,h) \\ &\quad \frac{\alpha}{2\pi}\left\{\frac{1}{2}\ln\left[\frac{Q^2}{m_e^2}\right]\right\} \end{aligned} \tag{10}$$

Notice that the Pauli term in the VCS correction does not factorize. However, in our kinematics, the Pauli term can be safely neglected, since it is of order $m_e/(x_B M_p)$, relative to the other terms.

Neglecting the Pauli term, we can write

$$\delta_{v,VCS} = -\frac{\alpha}{2\pi}\left\{\frac{1}{2}\ln^2\left[\frac{Q^2}{m_e^2}\right] - \frac{\pi^2}{6} + 2 - \frac{3}{2}\ln\left[\frac{Q^2}{m_e^2}\right]\right\} \tag{11}$$

and

$$\delta_{v,BH} - \delta_{v,VCS} = -\frac{\alpha}{2\pi}\left\{1 + \frac{5}{2}\ln\left[\frac{Q^2}{m_e^2}\right]\right\} \tag{12}$$

This difference has the value

$$\delta_{v,BH} - \delta_{v,VCS} = -0.047 \qquad \text{at} \quad Q^2 = 2.0\ \mathrm{GeV}^2. \tag{13}$$

C. Electron Self Energy

The electron self energy terms are more complicated, as they do not directly factorize. Neglecting terms of order $m_e M x_B/Q^2$, the analytic self-energy terms (taken from the Lhuillier, Vanderhaeghen, Guichon code) are

$$T_{BH,s.e.} = \epsilon^{\mu}(q')^{\dagger}\Gamma^{\nu}(p',p)\overline{u}(k',h)\left\{\gamma_{\nu}\frac{(k-q')\cdot\gamma}{-2k\cdot q'}\gamma_{\mu}\delta_d + \gamma_{\mu}\frac{(k'+q')\cdot\gamma}{2k'\cdot q'}\gamma_{\nu}\delta_c\right\}u(k,h) \quad (14)$$

with

$$\delta_d = \frac{\alpha}{4\pi}\left[3+\ln\left(\frac{2k\cdot q'}{m_e^2}\right)\right], \qquad \text{and}$$
$$\delta_c = \frac{\alpha}{4\pi}\left[3+\ln\left(\frac{2k'\cdot q'}{m_e^2}\right)+i\pi\right] \quad (15)$$

III. REAL RADIATIVE CORRECTIONS

The emission of additional soft photons is described in Eq. 58–62 of [1]:

$$d^4\sigma(M^2_{X,Max},\alpha^4) = d^4\sigma(\alpha^3)\left[1+\delta_R\left(M^2_{X,Max}\right)\right] \quad (16)$$

The correction term is:

$$\begin{aligned}\delta_R\left(M^2_{X,Max}\right) &= \frac{\alpha}{\pi}\ln\left[\frac{(\Delta E_s)^2}{\widetilde{E}_e\widetilde{E}'_e}\right]\left[\ln\left(\frac{Q^2}{m_e^2}\right)-1\right] \\ &\quad + \frac{\alpha}{\pi}\left[\frac{1}{2}\ln^2\left(\frac{Q^2}{m_e^2}\right)-\frac{1}{2}\ln^2\left(\frac{\widetilde{E}_e}{\widetilde{E}'_e}\right)-\frac{\pi^2}{3}+\mathrm{Sp}\left(\cos^2\frac{\widetilde{\theta}_e}{2}\right)\right] \\ &= \frac{\alpha}{\pi}\ln\left[\frac{(\Delta E_s)^2}{\widetilde{E}_e\widetilde{E}'_e}\right]\left[\ln\left(\frac{Q^2}{m_e^2}\right)-1\right]+\delta_R^{(0)} \end{aligned} \quad (17)$$

The kinematic variables are defined in the 's'-frame such that the emitted photon energy is independent of its direction. In terms of invariant variables, or alternatively target rest variables, they are defined as:

$$\begin{aligned}\widetilde{E}_e &= \frac{k\cdot(p+q-q')}{\sqrt{M_X^2}} = \frac{M_p}{\sqrt{M_X^2}}\left(E_e - \frac{Q^2}{2M_p} - \frac{k\cdot q'}{M_p}\right) \\ \widetilde{E}'_e &= \frac{k'\cdot(p+q-q')}{\sqrt{M_X^2}} = \frac{M_p}{\sqrt{M_X^2}}\left(E'_e + \frac{Q^2}{2M_p} - \frac{k'\cdot q'}{M_p}\right) \\ \sin^2\frac{\widetilde{\theta}_e}{2} &= \frac{E_eE'_e}{\widetilde{E}_e\widetilde{E}'_e}\sin^2\frac{\theta_e}{2}. \end{aligned} \quad (18)$$

The first term in Eq. 17 is incorporated into our Monte Carlo simulation. *In the future, it would be appropriate to incorporate the factor* $\exp(-\delta_R^{(0)})$ *as an event-by-event cross section* *weight in the simulation* The second term is approximately independent of M_X^2, and must be applied to our extracted results, together with the virtual radiative corrections.

The correction term $\delta_R^{(0)}$ is plotted in Fig. 1 for the three central kinematics of E00-110. There is a tiny variation over $\phi_{\gamma\gamma}$ and with the choice of $M_X^2(Max)$. There is also a small variation (not shown) over the range of Δ^2 of the experiment. The largest variation is due to the acceptance in $[x_B, Q^2]$ of each HRS setting. The variation of the correction factor $1 + \delta_R^{(0)}$ is $\pm 1\%$ over the HRS acceptance. The plots also show the elastic approximation:

$$\delta_{R,\,\mathrm{El}}^{(0)} = \frac{\alpha}{\pi}\left[\frac{1}{2}\ln^2\left(\frac{Q^2}{m_e^2}\right) - \frac{1}{2}\ln^2\left(\frac{E_e'}{E_e}\right) - \frac{\pi^2}{3} + \mathrm{Sp}\left(\cos^2\frac{\theta_e}{2}\right)\right] \tag{19}$$

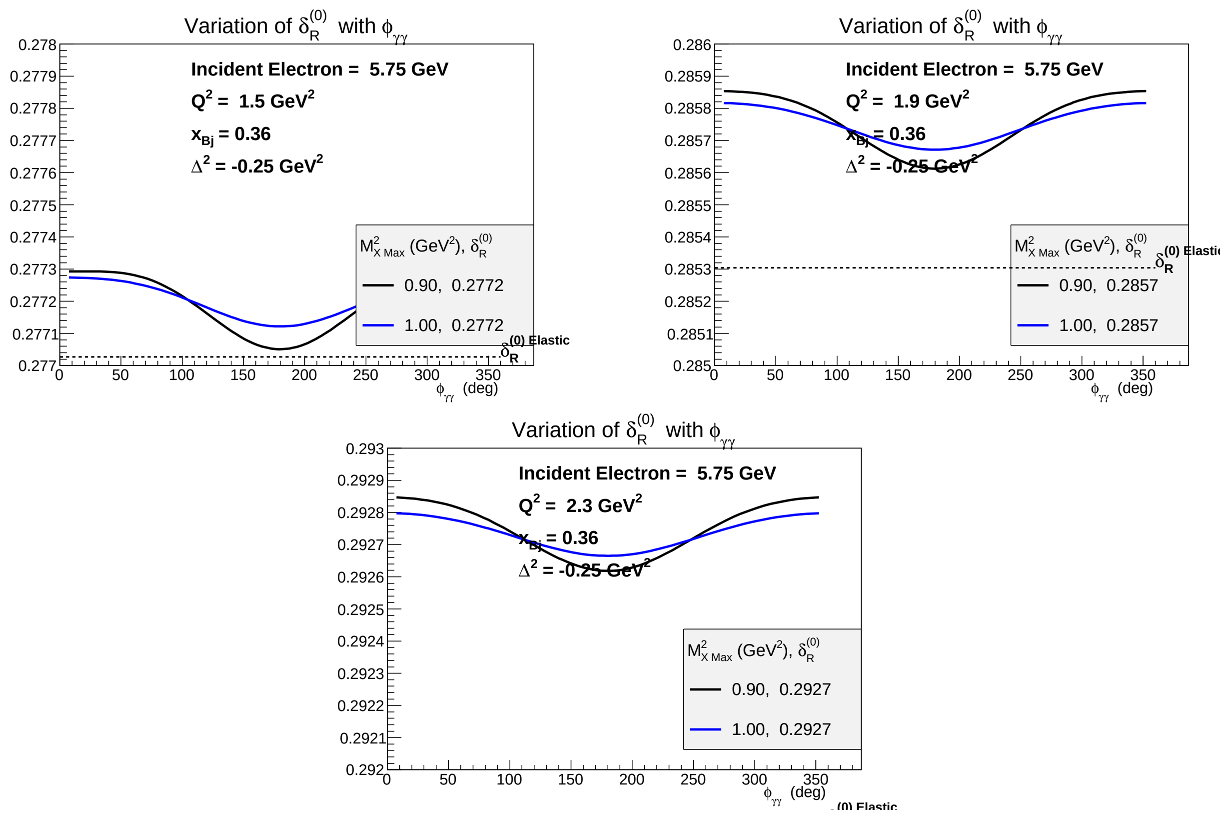

Figure 1. Radiative correction term for unresolved real radiative photons: $\delta_R^{(0)}$ from Eq. 17. Each solid curve is labeled by the missing mass squared cutoff its average value $\delta_T^{(0)}$. The dotted lines are the elastic approximation.

The real and virtual radiative correction factors for the kinematics of E00-110 are tabulated in Table I. These values in Columns 7 and 9 for the complete correction factors are larger than the value of 0.91 applied to these data in the PRL of Munoz Camacho *et al*, [2].

This is because we formerly just used a global correction factor equal to the analytic term for the VCS amplitude.

Table I. Radiative Correction Factors for the central kinematics of E00-110. Incident energy $k =$ 5.75 GeV. All factors are evaluated at $\Delta^2 = -0.25$ GeV^2. Column 6 is the ratio of the unpolarized cross section, including 1st order virtual radiative corrections, relative to the unradiated 'Born' cross section. Column 7 is the product of columns 5 and 6. This is the correction factor to apply to the experimental cross sections in order to obtain the best estimate of the unpolarized 'Born' cross section. Column 8 is the same as column 6, except now it is for the helicity correlated cross section. This factor is a few percent larger than Column 6, because this term is a pure BH∗DVCS interference term. Column 9 is the same as column 7, except this is now the complete correction to the helicity correlated cross section.

Kin	Q^2	x_B	$1/(1+\delta_R^{(0)})$	$\exp(-\delta_R^{(0)})$	$\frac{d\sigma^{\mathrm{Born}}}{d\sigma^{\mathrm{Rad,V}}}$	$\frac{d\sigma^{\mathrm{Born}}}{d\sigma^{\mathrm{Exp}}}$	$\frac{\Delta\sigma^{\mathrm{Born}}}{\Delta\sigma^{\mathrm{Rad,V}}}$	$\frac{\Delta\sigma^{\mathrm{Born}}}{\Delta\sigma^{\mathrm{Exp}}}$
1	1.5	0.36	0.7829	0.7579	1.255	0.9511	1.276	0.9671
2	1.9	0.36	0.7778	0.7515	1.262	0.9484	1.294	0.9724
3	2.3	0.36	0.7736	0.7462	1.267	0.9455	1.312	0.9790
X2	2.06	0.39	0.7760	0.7492	1.264	0.9470	1.302	0.9755
X3	2.17	0.34	0.7748	0.7478	1.265	0.9460	1.306	0.9766

[1] M. Vanderhaeghen, J. M. Friedrich, D. Lhuillier, D. Marchand, L. Van Hoorebeke and J. Van de Wiele, Phys. Rev. C **62**, 025501 (2000) [hep-ph/0001100].

[2] C. M. Camacho *et al.* [Jefferson Lab Hall A and Hall A DVCS Collaborations], Phys. Rev. Lett. **97**, 262002 (2006) [nucl-ex/0607029].