# SBS Optics and Spin Transport

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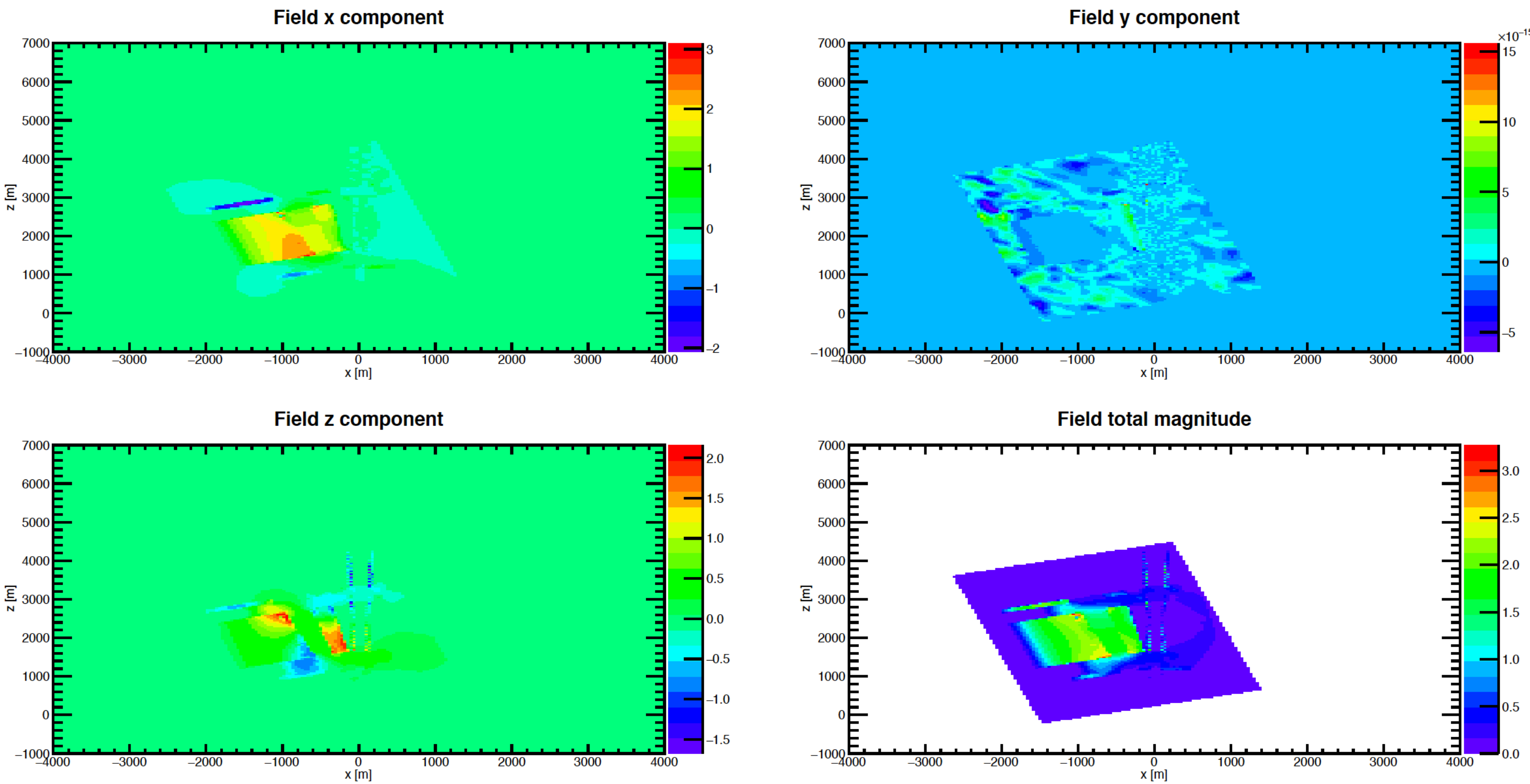
# Outline

- SBS/BigBite Optics studies from GEANT4
- Overview of previous studies in GEANT4
- Fitting SBS optics matrix—"reconstruction" coefficients
- SBS Spin Transport in GEANT4
  - Formalism—BMT equation
  - Ideal dipole approximation
  - Approach for fitting—added technical challenges compared to optics
- Current status and plans

# SBS magnetic field layout from TOSCA




- SBS field components calculated by TOSCA, projected on the xz plane, in Tesla.
    - y component in the xz plane (at y=0) is zero by symmetry
    - Includes field clamps and beamline magnetic shielding
- Field is predominantly horizontal and perpendicular to SBS axis in dipole gap
- Good approximation to a pure dipole field

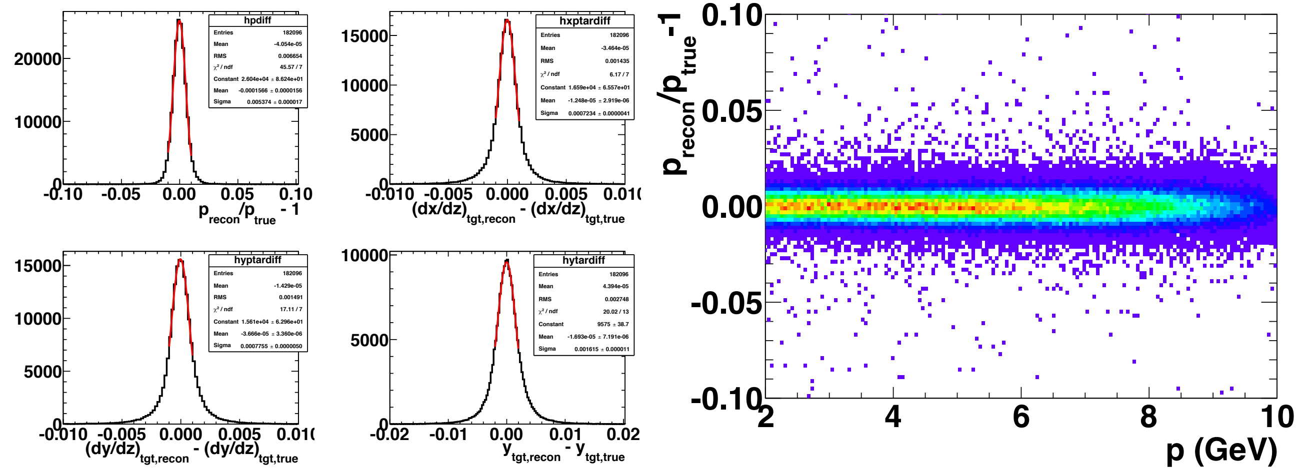
# BigBite and SBS optics from GEANT4

- Charged particles are traced through magnetic field layout of BigBite/SBS in GEANT4 using classical 4th-order Runge-Kutta numerical integration of the equation of motion.
- Charged-particles deposit energy in GEMs, making "hits"—GEM "hits" are then smeared by a Gaussian with a σ of 70 μm, representing the coordinate resolution of GEMs, and a straight-line track is fitted.
- In MC, we know the track parameters at the target and at the "focal plane" (GEM location).
- We expand the reverse transport matrix in a power series in the measured track parameters:

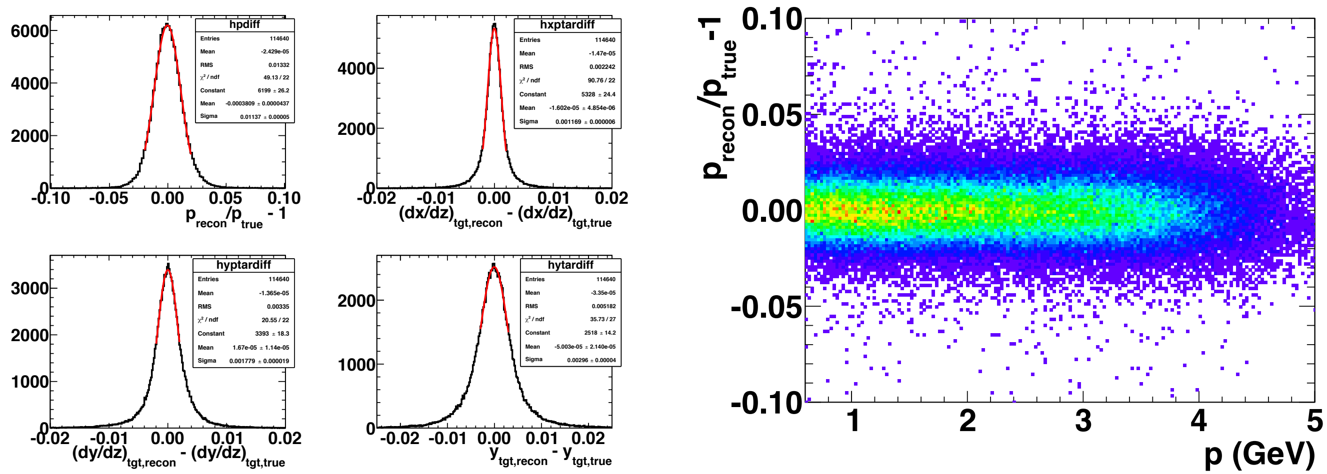
$$(x'_{tgt}, y'_{tgt}, y_{tgt}, 1/p) = \sum_{i+j+k+l+m\leq 6} C^{ijklm}_{x',y',y,1/p} x^i_{fp} y^j_{fp} x'^k_{fp} y'^l_{fp} x^m_{tgt}$$

- Track parameters at the target are ***linear*** functions of the expansion coefficients
  - Use standard linear algebra libraries, e.g., Singular Value Decomposition (SVD), to fit the coefficients, avoid pitfalls of nonlinear fitting/numerical minimization/Minuit
- **Why fit 1/p instead of p or the traditional $\delta = 100 \times (p/p_0 - 1)$ in the expansion?**
  - **SBS/BigBite are large-acceptance spectrometers—range of "delta" can equal or exceed ±100%, it is no longer a good expansion variable.**
  - **SBS/BigBite are non-focusing, dipole spectrometers→an expansion in 1/p converges very quickly—$x_{fp}$, $x'_{fp}$ are almost linear in 1/p for dipole magnets**

# SBS and BigBite Optics/Resolution from g4sbs—Old results

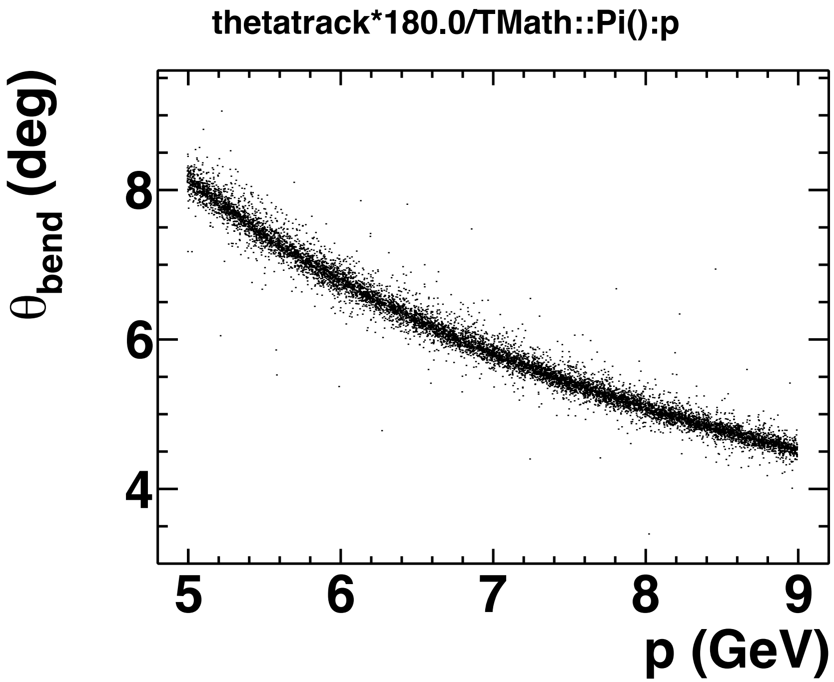



**SBS angle, vertex and momentum resolution for 1.4-Tesla uniform field, $\sigma_p/p \sim 0.5\%$ (average for 2-10 GeV pions)**

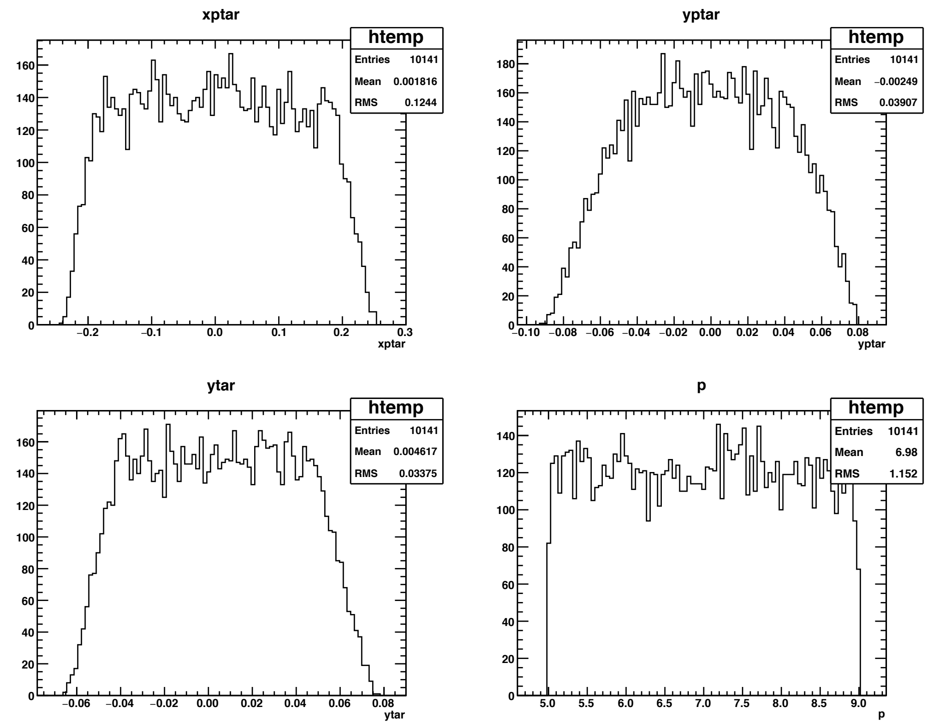



**BigBite angle, vertex and momentum resolution for "map_696A.dat", $\sigma_p/p \sim 1.1\%$**

# Optics and Spin Transport Studies for GEP, $Q^2$ = 12 $GeV^2$




**Track vertical bend angle vs. momentum**




**SBS angular and vertex acceptance for GEP highest $Q^2$**

- GEANT4 simulation for optics and spin transport:
  - Use "particle gun" generator with limits chosen wide enough to populate full acceptance of SBS (use 40 cm target)
  - Proton momenta generated in the range of 5-9 GeV (corresponding to highest $Q^2$ of GEP)
  - Generate 10,000 protons in three different starting spin orientations in the fixed TRANSPORT coordinate system:
    - Pure "X" (vertically down)
    - Pure "Y" (horizontal, toward small angle)
    - Pure "Z" (along SBS central ray)
  - Fit reconstruction coefficients and spin transport matrix elements

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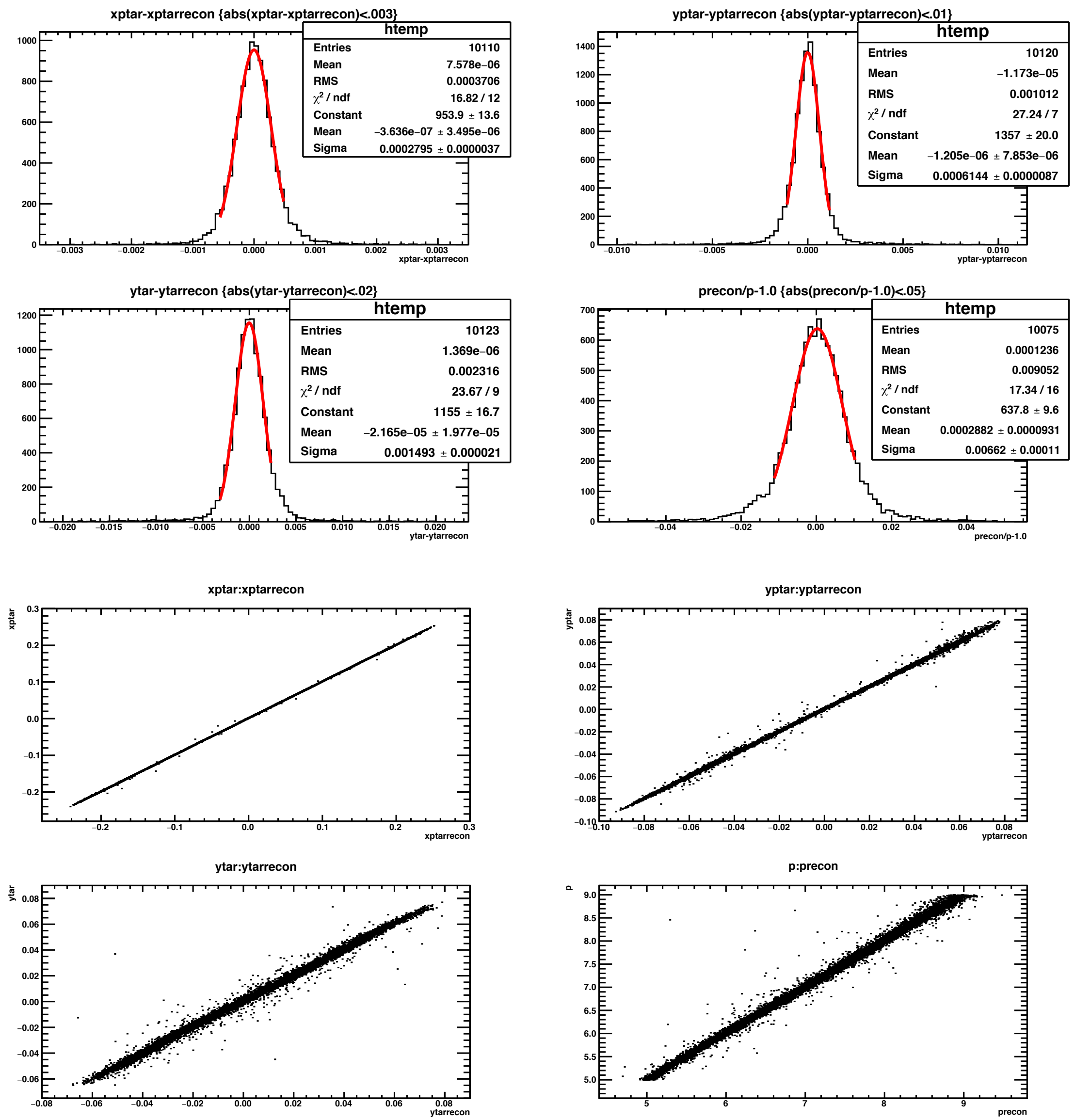
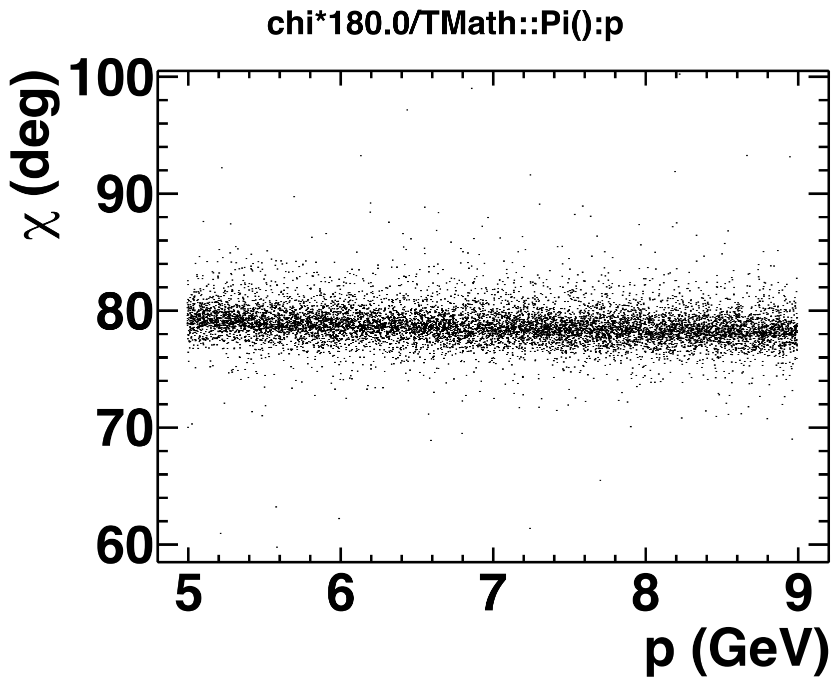
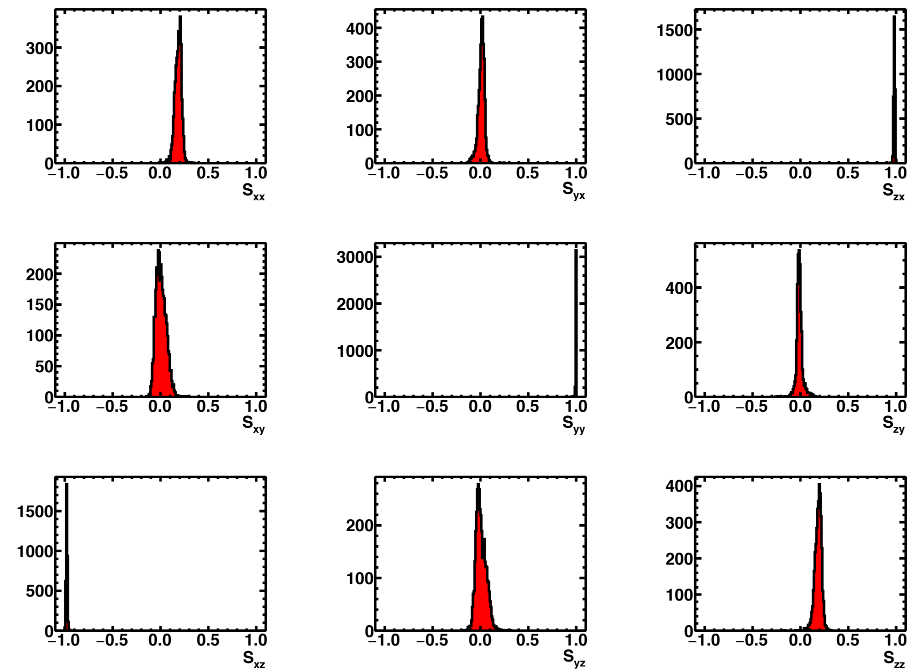
# SBS optics fitting from GEANT4

- SBS angle, vertex and momentum resolution for 5-9 GeV protons
- sigma(xptar) ~ 0.3 mrad
- sigma(yptar) ~ 0.6 mrad
- sigma(ytar) ~ 1.5 mm
- sigma(p)/p ~ 0.66%
- Improvement of the fit not significant beyond about 4[th]-order expansion of reconstruction coefficients



# Spin transport properties of SBS in GEP



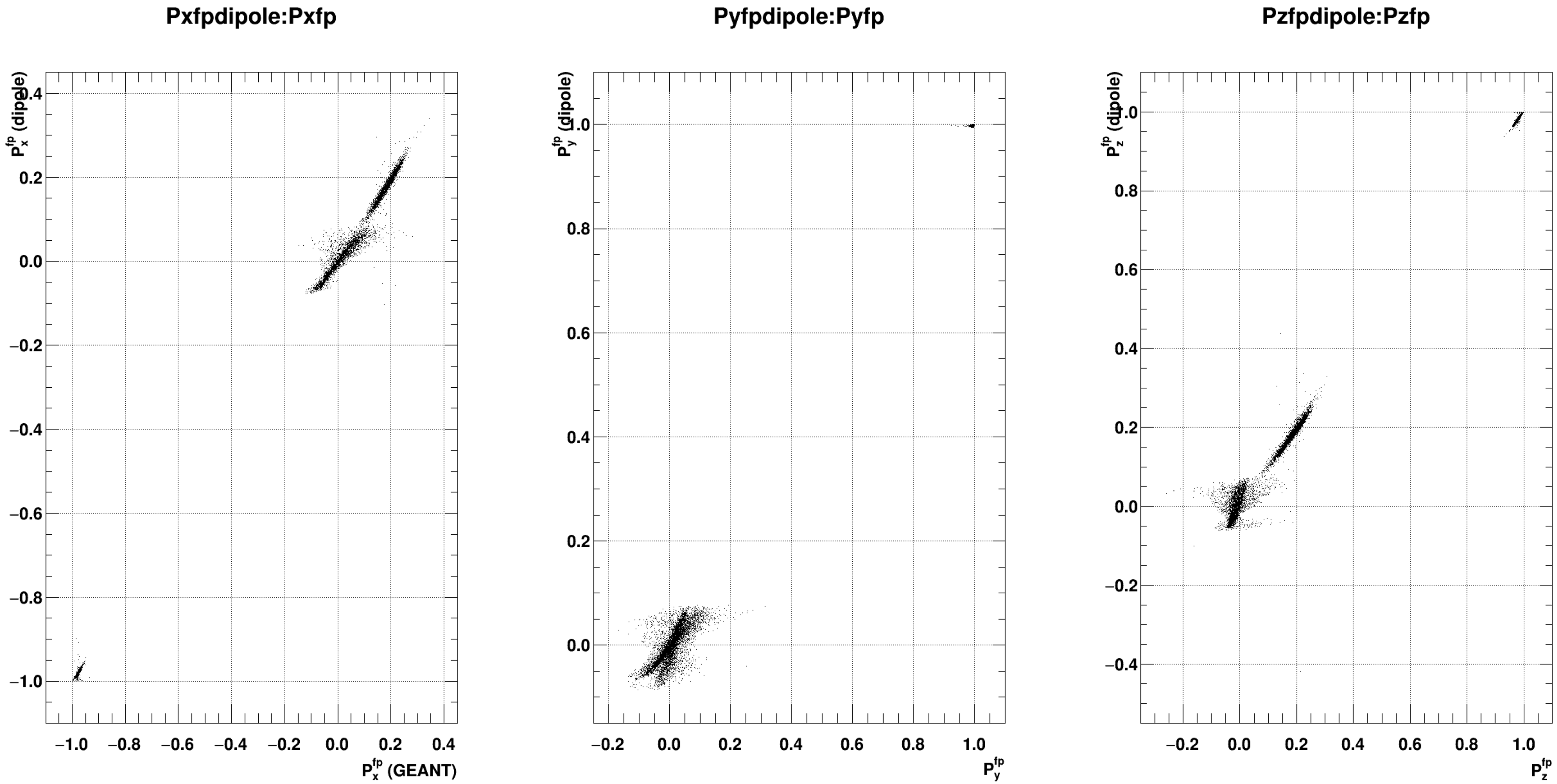




$$\frac{d\mathbf{S}}{dt} = \frac{e}{m\gamma}\mathbf{S} \times \left[\frac{g}{2}\mathbf{B}_{\parallel} + \left(1 + \gamma\left(\frac{g}{2} - 1\right)\right)\mathbf{B}_{\perp}\right]$$

**BMT equation for protons**

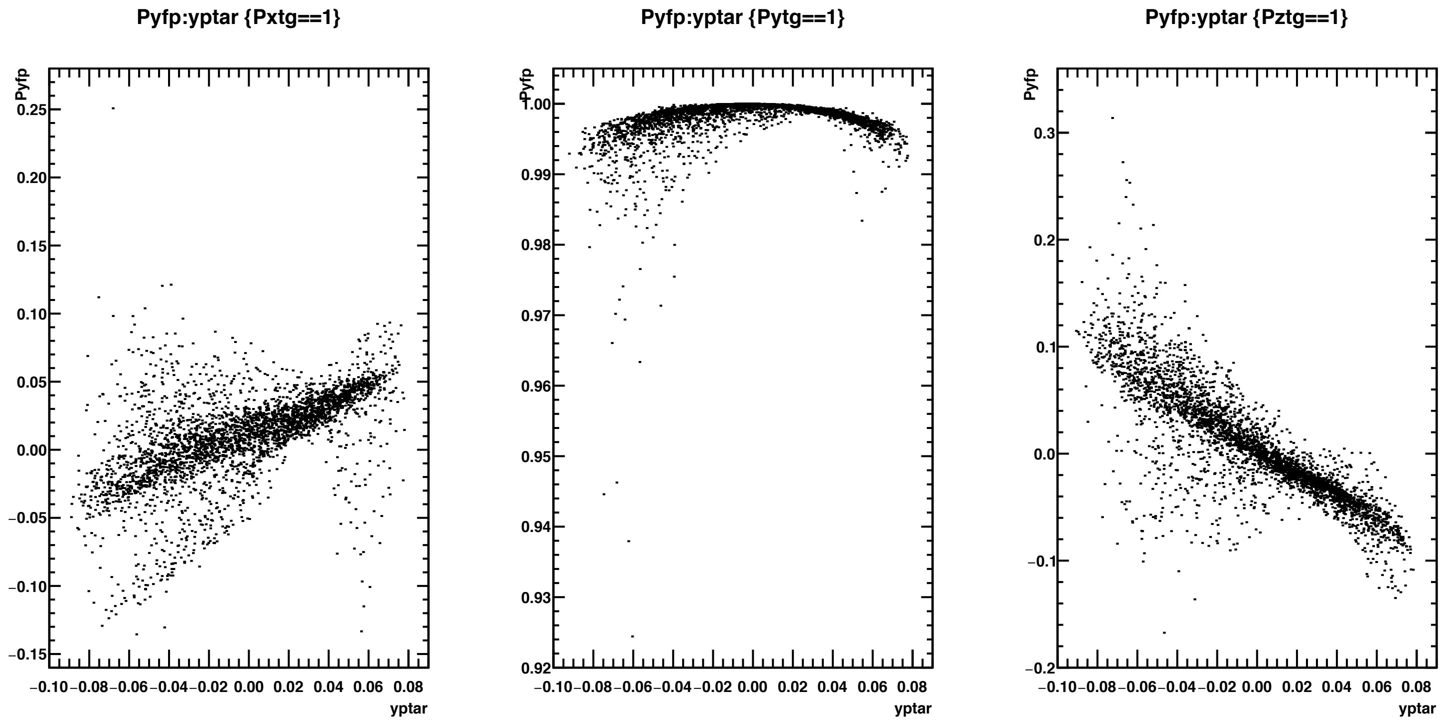
- Spin precession in a magnetic field for is governed by the BMT equation
- For an almost pure dipole field, as in SBS, the proton spin precesses relative to its trajectory by an angle: $\chi = \gamma \kappa_p \theta_{bend}$
- Precession angle is almost constant within useful acceptance of SBS for elastic ep events (cancellation between momentum dependence of gamma and thetabend)

# Differences between dipole and full calculation




- In dipole approximation, the spin component parallel to the field (Py) does not precess;
- Since SBS is non-focusing, the trajectory bend angle in the non-dispersive plane is close to zero for most trajectories
- Nevertheless, a small precession in the non-dispersive plane occurs for non-central trajectories
- This precession mixes $P_T$ and $P_L$ in extraction of GEp—important systematic uncertainty

# Non-dispersive precession

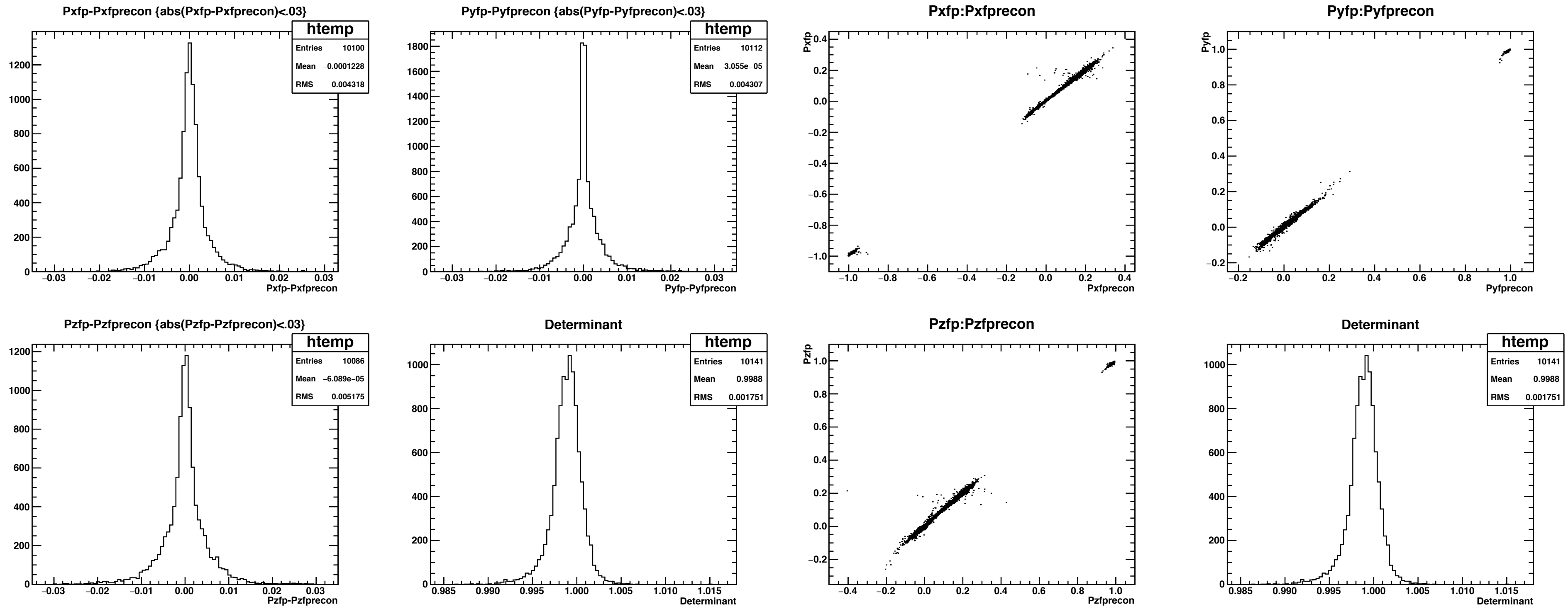



- Non-dispersive-plane precession is non-zero mainly for rays with "yptar" = dy/dz != 0 ("phitarget" in usual Hall A notation)
- $S_{yx}$ has a weak positive correlation with yptar
- $S_{yz}$ (which mixes $P_T$ and $P_L$), has a stronger negative correlation with yptar
- The slope of this correlation sets the scale for how accurately yptar needs to be determined to achieve a given accuracy on GEp/GMp (no, we haven't done the calculation yet)

# Formalism for fitting spin transport matrix elements

- In the usual Maximum-Likelihood analysis, the forward spin transport matrix elements are used. These in turn have to be computed from the reconstructed proton kinematics at the target.
- We expand the "small" deviations from the ideal dipole approximation as a power series in the proton trajectory parameters at the target:
  - xtar, ytar, xptar, yptar, 1/p
- We fit using the SVD as in the case of the optics
- Additional technical challenge:
  - There is no guarantee that if we fit the individual matrix elements without enforcing any constraints, that the 3x3 matrix computed from the resulting expansion coefficients will be a proper rotation for any given event
  - We can try to fit the Euler angle and/or the angle-axis decomposition of the total rotation for a given event, but the problem is that such a decomposition is not unique, and it is difficult to define "good" expansion parameters.
  - Would like to come up with some kind of constrained optimization procedure to guarantee that the fitted Spin transport coefficients are guaranteed to give a proper rotation in each event

# Spin fit results (5th-order)




- Fit deviations from the ideal dipole approximation up to 5th-order (still some room for improvement)
- Fit matrix elements directly
- Don't enforce any constraints
- Determinant results close to 1 in any case.

# Status and plans

- Figure out how to enforce orthogonality in every event
- Add sieve slit to MC, develop optics and spin transport calibration/optimization procedures
- Quantify systematics