Jefferson Lab
Thomas Jefferson National Accelerator Facility

# High $Q^2$ radiative corrections

Bogdan Wojtsekhowski

## Outline

- Theory papers from 1949 to 2024
- MC based on the PRC-2001 paper
- RC for electron-proton/neutron at 11 $GeV^2$
- Available code for RadCor calculation

# Theory papers

J. Schwinger, Phys. Rev. **76**, 790 (1949).

$$\delta(\vartheta,\Delta E\ll W)=\frac{4\alpha}{\pi}\left[\left(\log\frac{E}{\Delta E}-\frac{13}{12}\right)\left(\log\frac{2p_0}{K}\sin\frac{\vartheta}{2}-\frac{1}{2}\right)+\frac{17}{72}+\frac{1}{2}\sin^2\frac{\vartheta}{2}f(\vartheta)\right], \quad (2.105)$$

Landau book: $d\sigma_{\text{рад}} = -d\sigma^{(1)}\cdot\frac{2\alpha}{\pi}\ln\frac{\mathbf{q}^2}{m^2}\ln\frac{\varepsilon}{\omega_{\max}}, \quad \mathbf{q}^2 \gg m^2.$

L.M. Mo and Y.S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).

PHYSICAL REVIEW C, VOLUME 64, 054610

**Radiative corrections for $(e,e'p)$ reactions at GeV energies**

R. Ent,[1,2] B. W. Filippone,[3] N. C. R. Makins,[1,*] R. G. Milner,[1] T. G. O'Neill,[3,†] and D. A. Wasson[4,‡]
[1]*Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

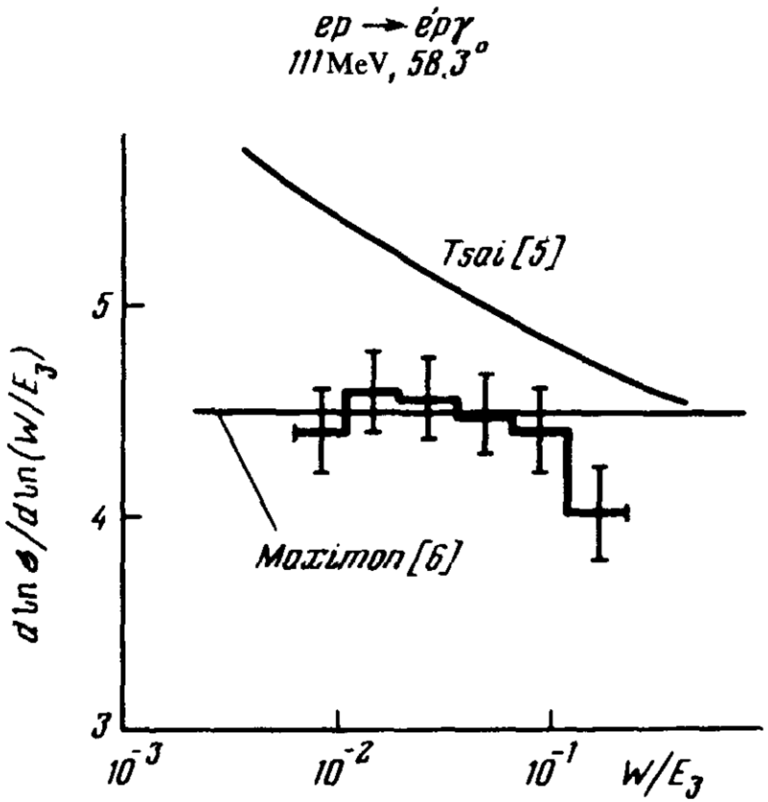
PHYSICAL REVIEW C, VOLUME 62, 054320

**Radiative corrections to electron-proton scattering**

L. C. Maximon
*Physics Department, The George Washington University, Washington, D.C. 20052*

J. A. Tjon

# In my first experiment




## Measurement of "radiation tail"—electron spectrum in the reaction $ep \to e'p\gamma$


B. B. Voitsekhovskiĭ, D. M. Nikolenko, S. G. Popov, and D. K. Toporkov

*Institute of Nuclear Physics, Siberian Branch, USSR Academy of Sciences*



(Submitted 14 November 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **29**, No. 1, 105–109 (5 January 1979)



A comparison is made of the spectrum of scattered electrons in the reaction $ep \to e'p\gamma$—measured at the initial energy of 111 MeV and scattering angle of 58.3°—with several theoretical calculations. A good agreement is observed, for the first time, over a wide spectral range between the experiment and calculations[6] which are based on the procedure of "exponentiation."

# Proton/Neutron difference

**Radiative corrections for $(e,e'p)$ reactions at GeV energies**

R. Ent,[1,2] B. W. Filippone,[3] N. C. R. Makins,[1,*] R. G. Milner,[1] T. G. O'Neill,[3,†] and D. A. Wasson[4,‡]

$$\frac{d\sigma}{d^3k'd^3\omega} \sim |\mathcal{M}_{\rm ei}+\mathcal{M}_{\rm ef}+\mathcal{M}_{\rm pi}+\mathcal{M}_{\rm pf}|^2.$$

R. ENT *et al.*

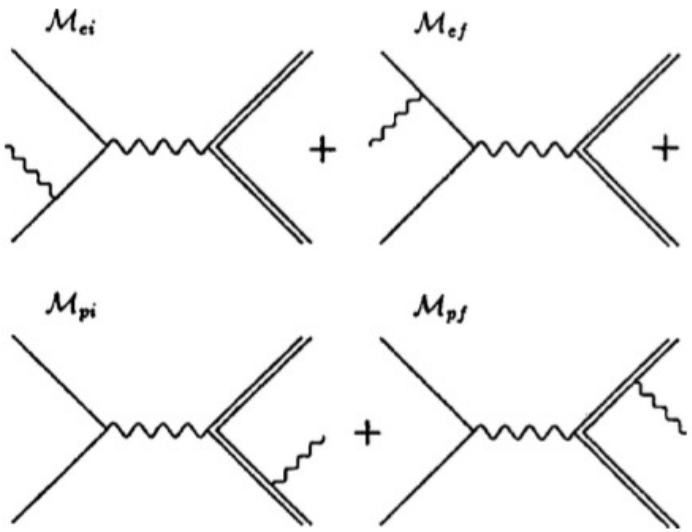



FIG. 2. Feynman diagrams contributing to first order bremsstrahlung radiation cross section.

# Proton/Neutron difference

$$J_e^{\mu}(q) = e\bar{u}_e(k+q)\gamma^{\mu}u_e(k).$$

$$J_p^{\mu}(q) = -e\bar{u}_p(p+q)\Gamma^{\mu}(q)u_p(p). \quad (10)$$

The deviation of the proton from a point particle is described by

$$\boxed{\Gamma^{\mu}(q) = F_1(q^2)\gamma^{\mu} + \frac{1}{2M}F_2(q^2)i\sigma^{\mu\nu}q_{\nu},} \quad (11)$$

# Proton/Neutron difference

$$\mathcal{M}_{\rm ef} = i\bar{u}_e(k')e\gamma^\nu\varepsilon_\nu\left[\frac{i\gamma^\nu(k'_\nu+\omega_\nu)+m}{(k'+\omega)^2-m^2}\right]$$

$$\times\gamma^\mu u_e(k)\frac{e^2}{q_p^2-\mu^2}\bar{u}_p(p')\boxed{\Gamma_\mu(q_p)}u_p(p),$$

$$\mathcal{M}_{\rm pi} = i\bar{u}_p(p')\boxed{\Gamma^\mu(q)}\left[\frac{i\gamma^\nu(p_\nu-\omega_\nu)+M}{(p-\omega)^2-M^2}\right]$$

$$\times(-e)\boxed{\Gamma^\nu(\omega)}\varepsilon_\nu u_p(p)\frac{e^2}{q^2-\mu^2}\bar{u}_e(k')\gamma_\mu u_e(k),$$

$$\mathcal{M}_{\rm pf} = i\bar{u}_p(p')(-e)\Gamma^\nu(\omega)\varepsilon_\nu\left[\frac{i\gamma^\nu(p'_\nu+\omega_\nu)+M}{(p'+\omega)^2-M^2}\right]$$

$$\times\Gamma^\mu(q)u_p(p)\frac{e^2}{q^2-\mu^2}\bar{u}_e(k')\gamma_\mu u_e(k). \qquad (12)$$

# Proton contribution to radiation

$$\lambda^{EQ} = \frac{\alpha}{\pi}\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right].$$

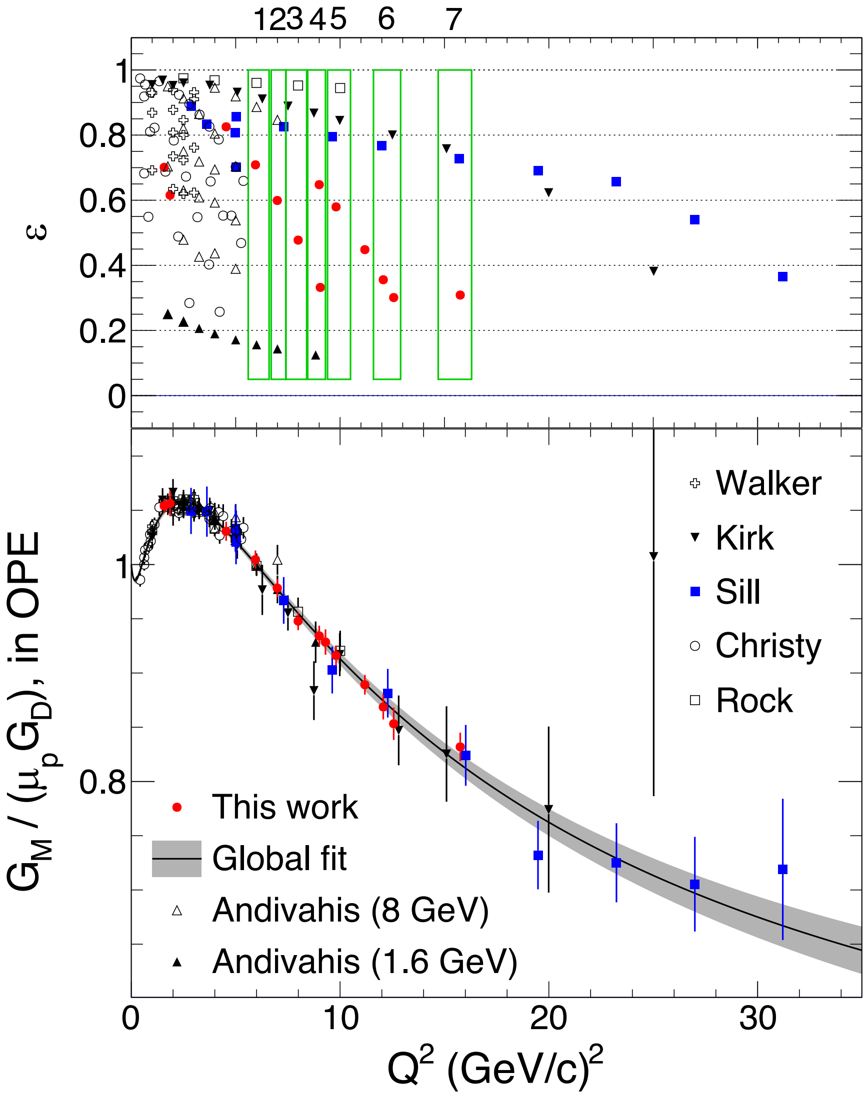
TABLE X. Comparison of the equivalent radiator thicknesses $\lambda$ for various kinematics. Values $\lambda^{EQ}$ are from Eq. (82), $\lambda$ are from Eqs. (58) and (59), and $\lambda^{mod}$ are from Eqs. (86)–(90).

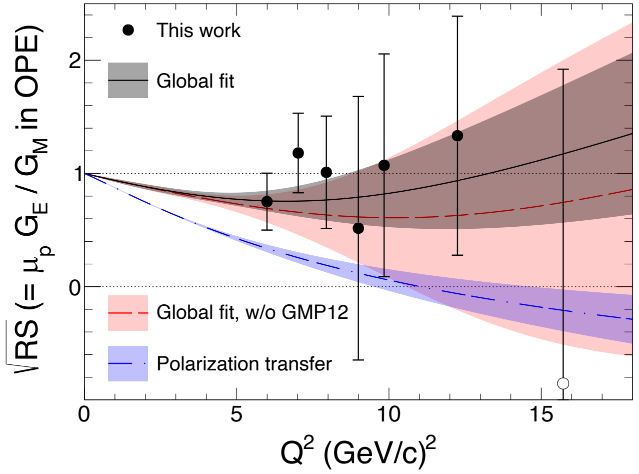
| $Q^2$ $(\mathrm{GeV}/c)^2$ | $\lambda^{EQ}$ (%) | $\lambda_e$ (%) | $\lambda_{e'}$ (%) | $\lambda_{p'}$ (%) | $\lambda_e^{mod}$ (%) | $\lambda_{e'}^{mod}$ (%) | $\lambda_{p'}^{mod}$ (%) |
|---|---|---|---|---|---|---|---|
| 1 | 3.322 | 3.936 | 3.767 | 0.042 | 3.502 | 3.614 | 0.037 |
| 3 | 3.561 | 4.149 | 3.790 | 0.326 | 3.652 | 4.282 | 0.287 |
| 5 | 3.669 | 4.279 | 3.790 | 0.485 | 3.786 | 4.619 | 0.429 |
| 7 | 3.736 | 4.369 | 3.790 | 0.590 | 3.883 | 4.836 | 0.524 |

$$\Gamma^{\mu}(q) = F_1(q^2)\gamma^{\mu} + \frac{1}{2M}F_2(q^2)i\sigma^{\mu\nu}q_{\nu}$$ => P vs. N in nucleon line

# The GMp12 experiment (E12-07-108)

Phys.Rev.Lett. 128 (2022) 10, 102002



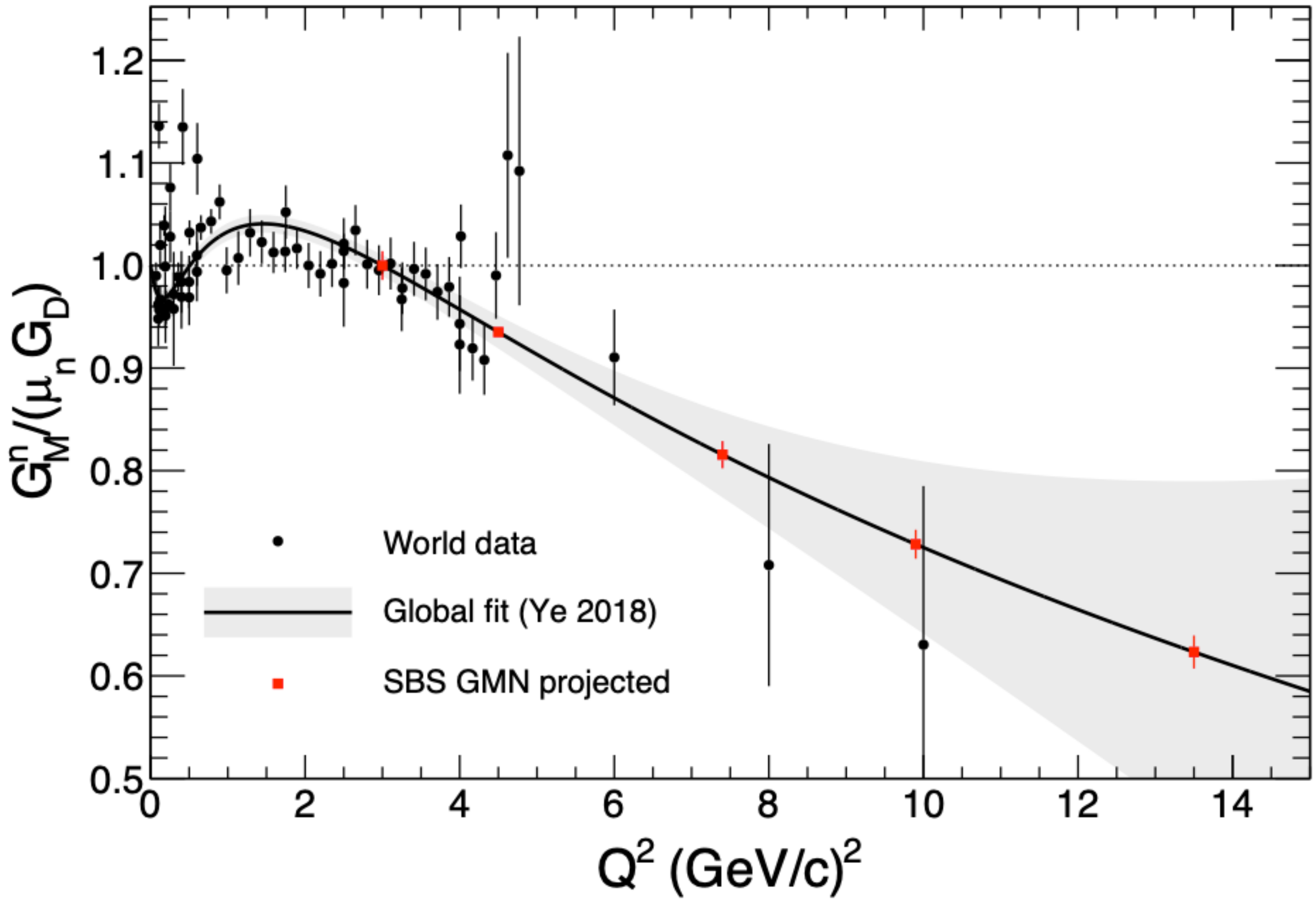

## GMp12 fit:

$$G_M = \mu_p (1 + a_1\tau)/(1 + b_1\tau + b_2\tau^2 + b_3\tau^3),$$
$$\mathrm{RS} = 1 + c_1\tau + c_2\tau^2.$$

| $a_1$ | $b_1$ | $b_2$ | $b_3$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|
| 0.072(22) | 10.73(11) | 19.81(17) | 4.75(65) | -0.46(12) | 0.12(10) |

courtesy of A. Gramolin and A. Puckett

# SBS GMn



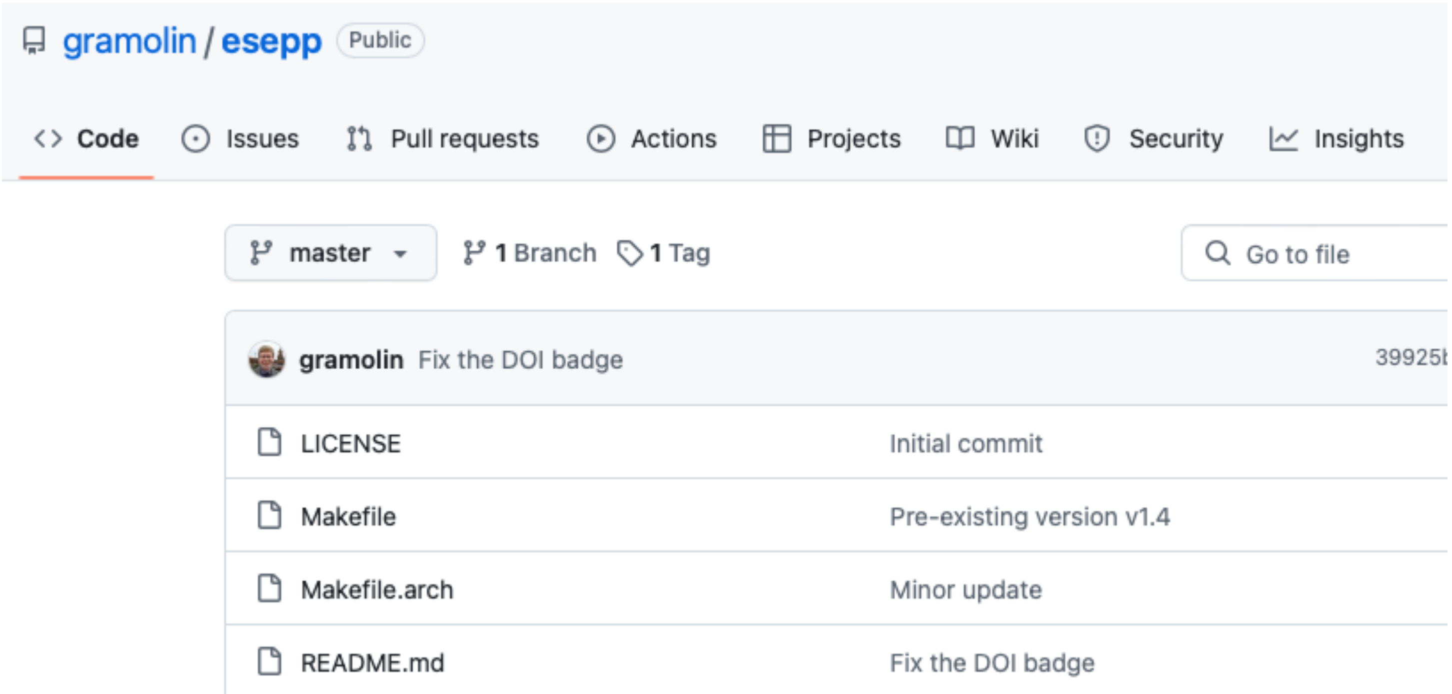
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
2
4
6
8
10
12
14
$G_M^n/(\mu_n G_D)$
$Q^2$ (GeV/c)$^2$
World data
Global fit (Ye 2018)
SBS GMN projected

# ESEPP

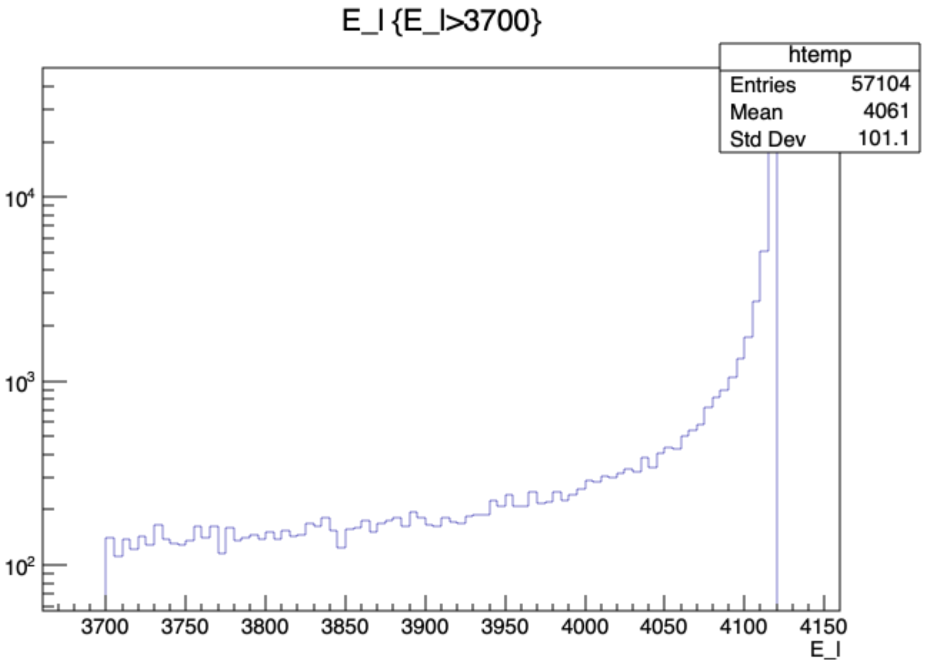
PHYSICAL REVIEW C **93**, 055201 (2016)

## Reanalysis of Rosenbluth measurements of the proton form factors

A. V. Gramolin* and D. M. Nikolenko
*Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia*
(Received 28 March 2016; published 10 May 2016)



# Result from ESEPP calculation




The example for:
$E_0$ = 10 GeV, 30 deg
$Q^2$ = 11 $GeV^2$

$E_{e'}$ > 90% of peak

$N_{obs}{}^{e\text{-}n} = (1 -- 0.162) * N_{FFn}$

$N_{obs}{}^{e\text{-}p} = (1 + 0.026) * N_{FFp}$

The GMn/GMp ratio is about
9% up from uncorrected values

# CLAS12 GMn https://www.osti.gov/biblio/2229724

NEUTRON MAGNETIC FORM FACTOR $G_M^N$ MEASUREMENT AT HIGH $Q^2$

NE

WITH CLAS12

A dissertation submitted in partial fulfillment of the

requirements for the degree of

DOCTOR OF PHILOSOPHY

in

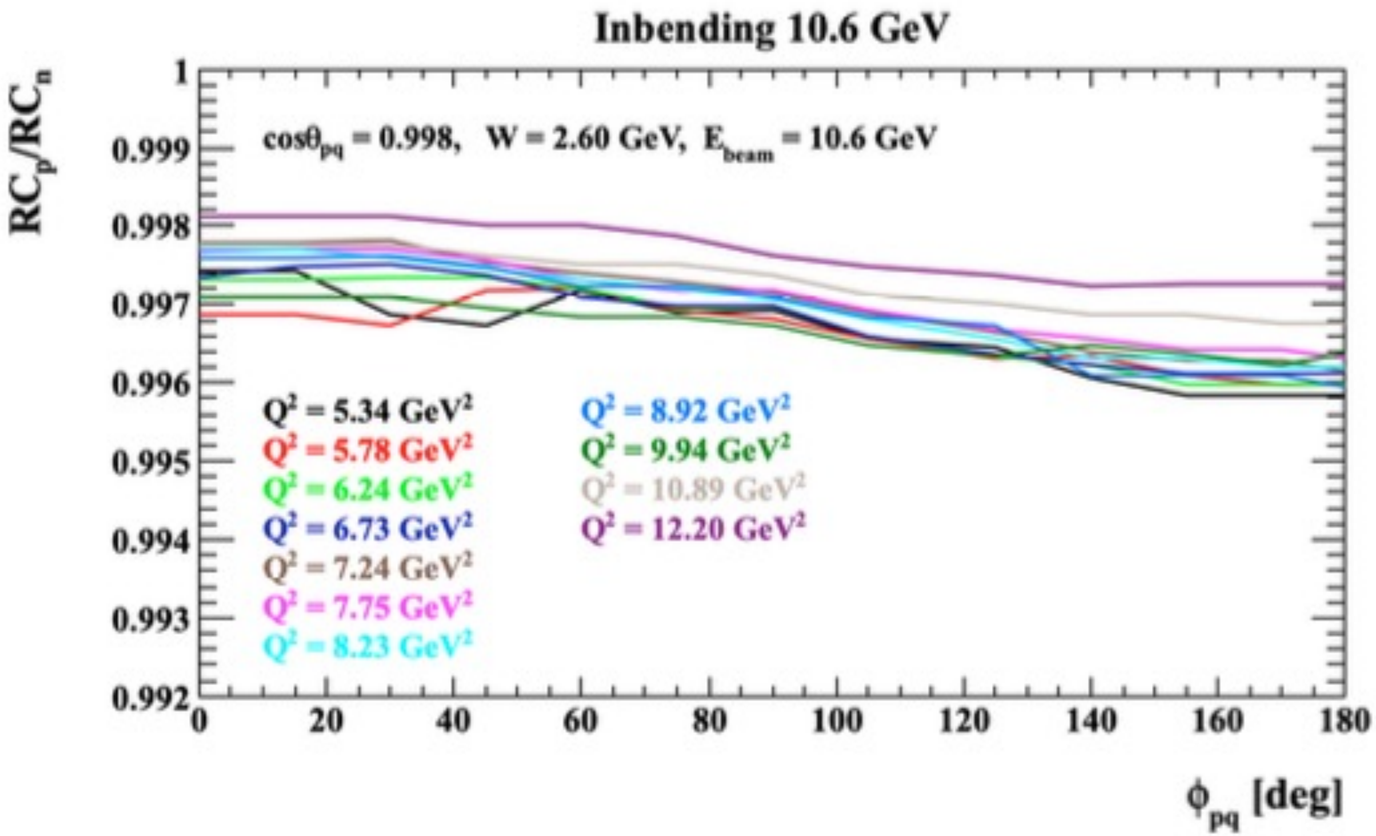
PHYSICS

by

Lamya Baashen

Lamya Baashen

# CLAS12 GMn https://www.osti.gov/biblio/2229724



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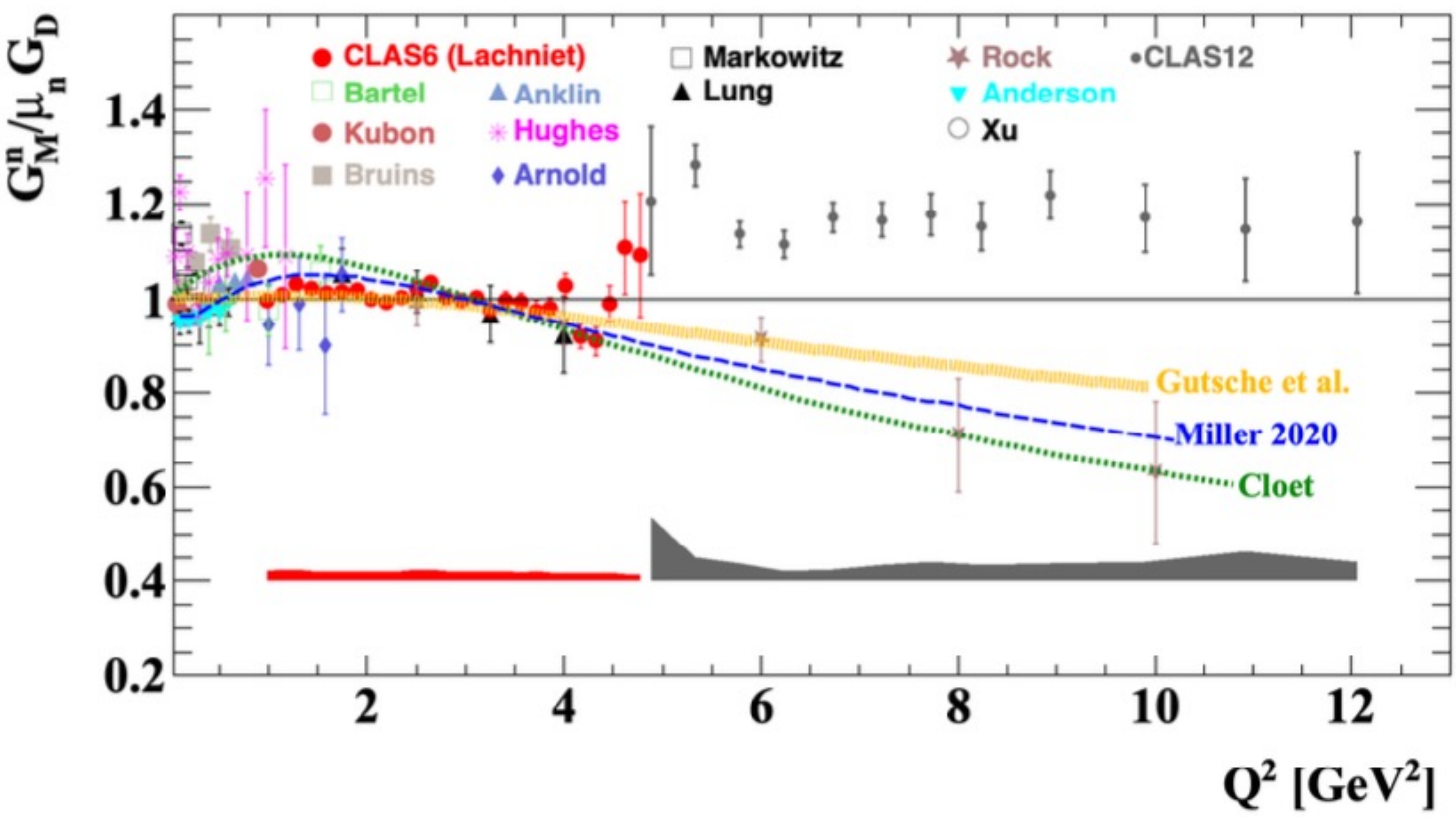

# Summary

- Difference of RadCor for e-p and e-n at high $Q^2$ could lead to change of the GMn/GMp up to 9%

- Accurate measurement of the cross section at high $Q^2$ requires an update of the radiative correction