

High Q^2 radiative corrections

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Outline

- Two-Photon-Exchange and GMn/GMp
- ESEPP with cut on electron/proton angle
- Blunden on TPE in $e, e'p$ and $e, e'n$

Proton/Neutron difference

Radiative corrections for $(e, e' p)$ reactions at GeV energies

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$$\frac{d\sigma}{d^3k' d^3\omega} \sim |\mathcal{M}_{ei} + \mathcal{M}_{ef} + \mathcal{M}_{pi} + \mathcal{M}_{pf}|^2.$$

R. ENT *et al.*

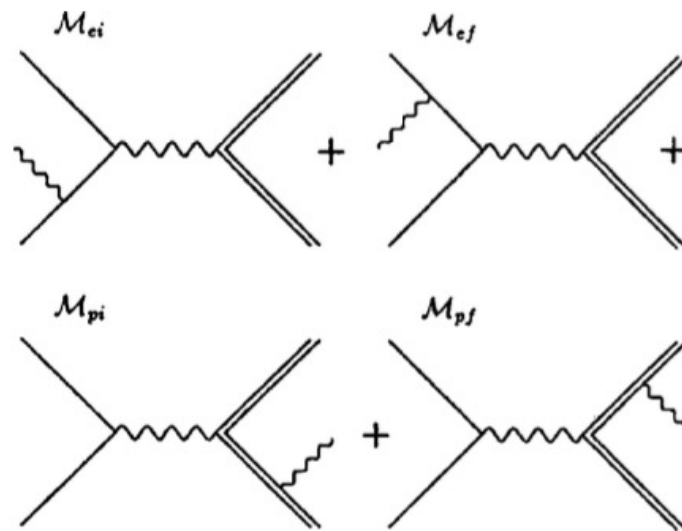


FIG. 2. Feynman diagrams contributing to first order bremsstrahlung radiation cross section.

Proton/Neutron difference

$$J_e^\mu(q) = e \bar{u}_e(k+q) \gamma^\mu u_e(k).$$

$$J_p^\mu(q) = -e \bar{u}_p(p+q) \Gamma^\mu(q) u_p(p). \quad (10)$$

The deviation of the proton from a point particle is described by

$$\Gamma^\mu(q) = F_1(q^2) \gamma^\mu + \frac{1}{2M} F_2(q^2) i \sigma^{\mu\nu} q_\nu, \quad (11)$$

Proton/Neutron difference

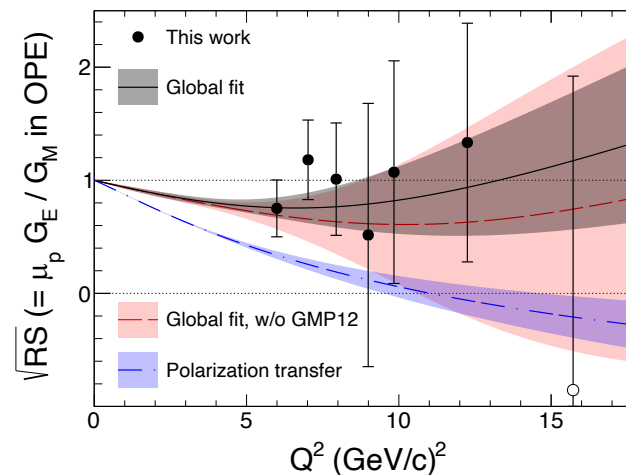
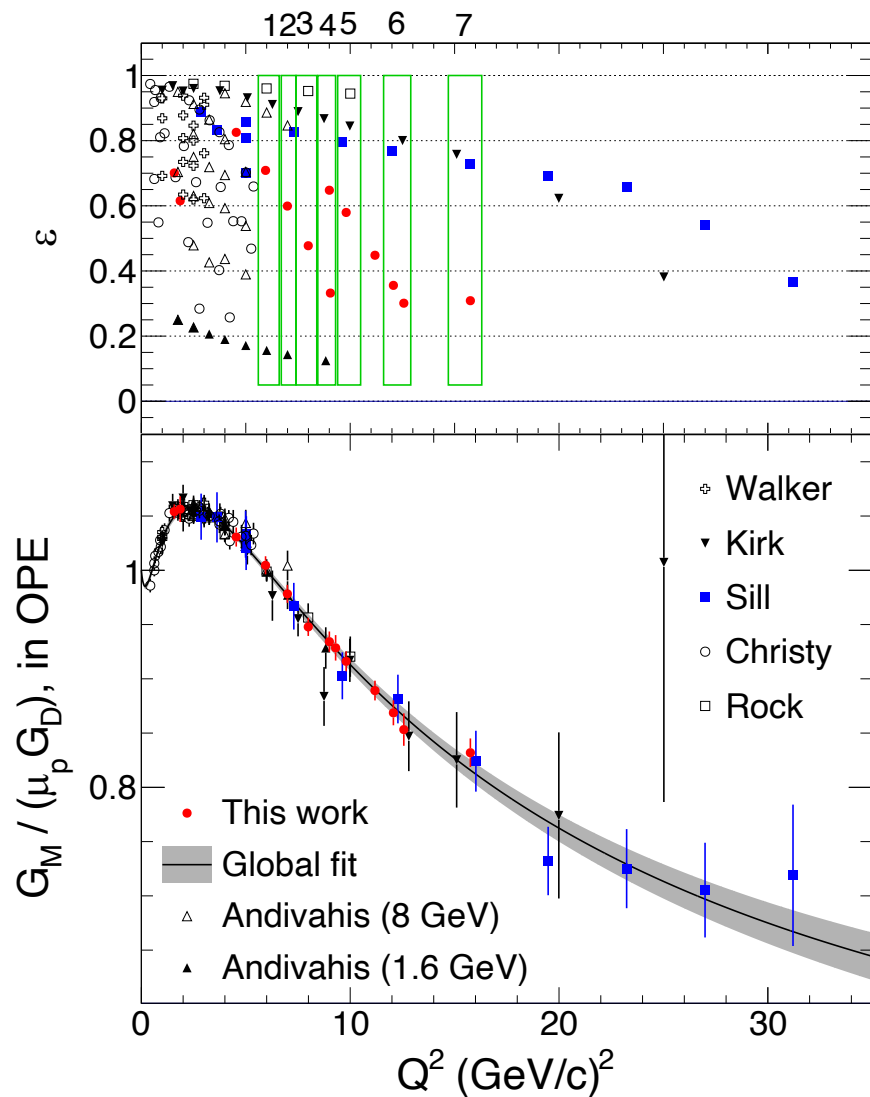
$$\mathcal{M}_{\text{ef}} = i\bar{u}_e(k') e \gamma^\nu \varepsilon_\nu \left[\frac{i\gamma^\nu(k'_\nu + \omega_\nu) + m}{(k' + \omega)^2 - m^2} \right] \\ \times \gamma^\mu u_e(k) \frac{e^2}{q_p^2 - \mu^2} \bar{u}_p(p') \Gamma_\mu(q_p) u_p(p),$$

$$\mathcal{M}_{\text{pi}} = i\bar{u}_p(p') \Gamma^\mu(q) \left[\frac{i\gamma^\nu(p_\nu - \omega_\nu) + M}{(p - \omega)^2 - M^2} \right] \\ \times (-e) \Gamma^\nu(\omega) \varepsilon_\nu u_p(p) \frac{e^2}{q^2 - \mu^2} \bar{u}_e(k') \gamma_\mu u_e(k),$$

$$\mathcal{M}_{\text{pf}} = i\bar{u}_p(p') (-e) \Gamma^\nu(\omega) \varepsilon_\nu \left[\frac{i\gamma^\nu(p'_\nu + \omega_\nu) + M}{(p' + \omega)^2 - M^2} \right] \\ \times \Gamma^\mu(q) u_p(p) \frac{e^2}{q^2 - \mu^2} \bar{u}_e(k') \gamma_\mu u_e(k). \quad (12)$$

The GMp12 experiment (E12-07-108)

Phys.Rev.Lett. 128 (2022) 10, 102002



GMp12 fit:

$$G_M = \mu_p (1 + a_1 \tau) / (1 + b_1 \tau + b_2 \tau^2 + b_3 \tau^3),$$

$$RS = 1 + c_1 \tau + c_2 \tau^2.$$

a_1	b_1	b_2	b_3	c_1	c_2
0.072(22)	10.73(11)	19.81(17)	4.75(65)	-0.46(12)	0.12(10)

courtesy of A. Gramolin and A. Puckett

ESEPP

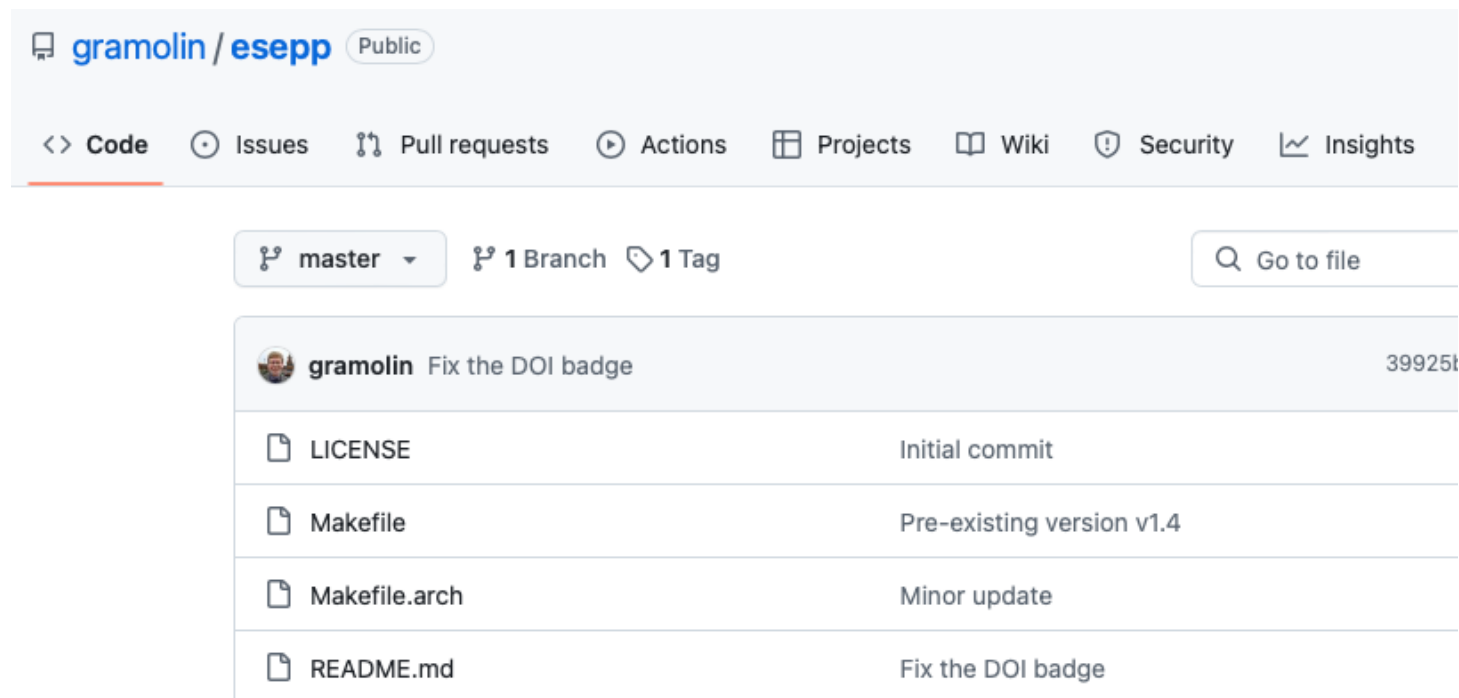
PHYSICAL REVIEW C **93**, 055201 (2016)

Reanalysis of Rosenbluth measurements of the proton form factors

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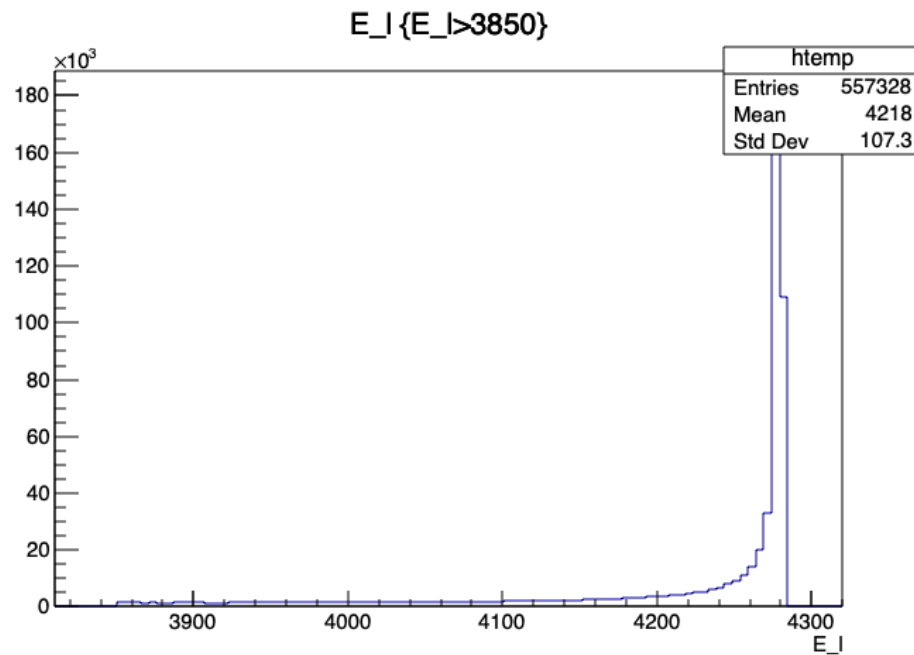
(Received 28 March 2016; published 10 May 2016)



The screenshot shows the GitHub interface for the repository 'gramolin / esepp'. The repository is public and has a 'Code' tab selected. The navigation bar includes links for Code, Issues, Pull requests, Actions, Projects, Wiki, Security, and Insights. Below the navigation bar, there are options for 'master' branch, '1 Branch', and '1 Tag', along with a 'Go to file' search box. The file list shows the following files and their commit messages:

File	Commit Message
LICENSE	Initial commit
Makefile	Pre-existing version v1.4
Makefile.arch	Minor update
README.md	Fix the DOI badge

Result from ESEPP calculation



The example for:
 $E_0 = 10$ GeV, 30 deg
 $Q^2 = 11$ GeV²

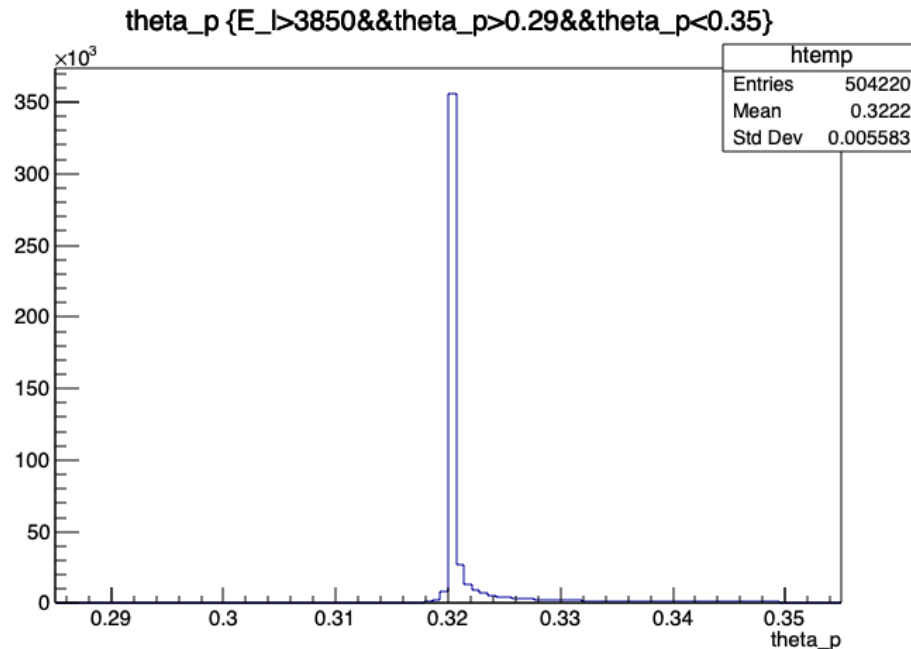
$E_{e'}$ > 90% of peak

$$N_{\text{obs}}^{e-n} = (1 - 0.088) * N_{\text{FFn}}$$

$$N_{\text{obs}}^{e-p} = (1 + 0.020) * N_{\text{FFp}}$$

The GMn/GMp ratio is about
6% up from uncorrected values

Result from ESEPP calculation



The example for:
 $E_0 = 10 \text{ GeV}$, 30 deg
 $Q^2 = 11 \text{ GeV}^2$

$E_{e'} > 90\%$ of peak

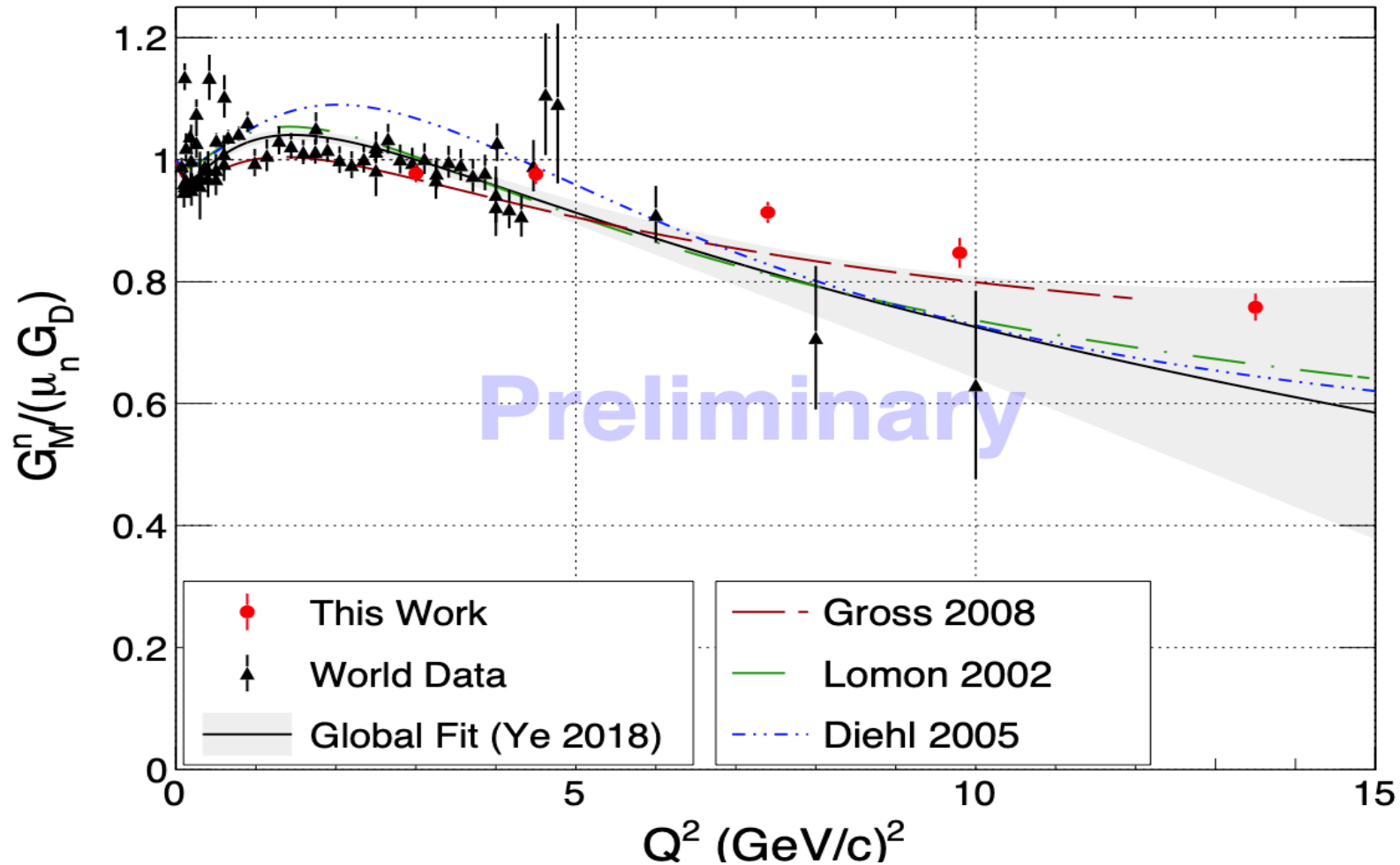
$\theta_p \text{ range} = \pm 2 \text{ deg}$

$$N_{\text{obs}}^{e-n} = (1 - 0.207) * N_{\text{FFn}}$$

$$N_{\text{obs}}^{e-p} = (1 - 0.069) * N_{\text{FFp}}$$

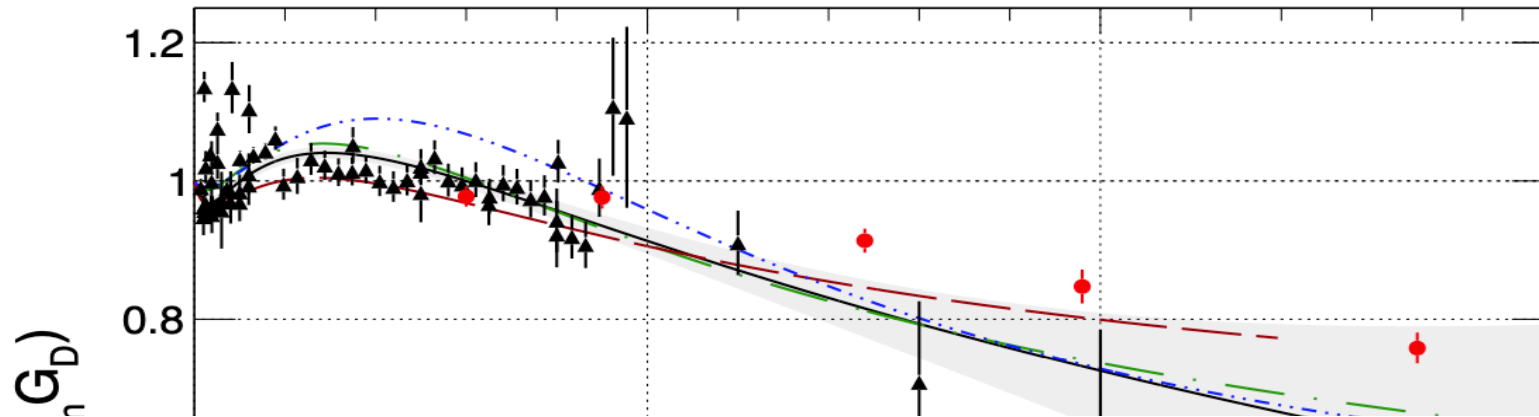
The GMn/GMp ratio is about
8% up from uncorrected values

SBS GMn in P.Datta thesis



interactions, meson exchange contributions, and radiative effects largely cancel out since they are nearly identical between neutron-tagged and proton-tagged events. This leaves the relative detec-

SBS GMn in P.Datta thesis



R , representing the en to ep Born cross-section ratio, is the most fundamental model-independent observable of the E12-09-019 experiment. It can be expressed using the Rosenbluth formula as follows:

$$R = \frac{\frac{\sigma_{\text{Mott}}}{\epsilon_n(1+\tau_n)} (\epsilon_n G_E^{n2} + \tau_n G_M^{n2})}{\frac{\sigma_{\text{Mott}}}{\epsilon_p(1+\tau_p)} (\epsilon_p G_E^{p2} + \tau_p G_M^{p2})} \quad (3.3)$$

This expression can be greatly simplified as the factors σ_{Mott} , ϵ , and τ are approximately the same for en and ep scattering.

Finally, by inverting Equation [3.3](#), G_M^n can be expressed in terms of R as:

$$G_M^n = - \left[\frac{1}{\tau_n} \frac{\epsilon_n(1+\tau_n)}{\epsilon_p(1+\tau_p)} \sigma_{\text{Red}}^p R - \frac{\epsilon_n}{\tau_n} G_E^{n2} \right]^{\frac{1}{2}}, \quad (3.4)$$

Blunden TPE, PHYSICAL REVIEW C 72, 034612 (2005)

$$\frac{d\sigma_0}{d\Omega} = \left(\frac{\alpha}{4Mq^2} \frac{E_3}{E_1} \right)^2 |\mathcal{M}_0|^2 = \sigma_{\text{Mott}} \frac{\tau}{\varepsilon(1 + \tau)} \sigma_R, \quad (7)$$

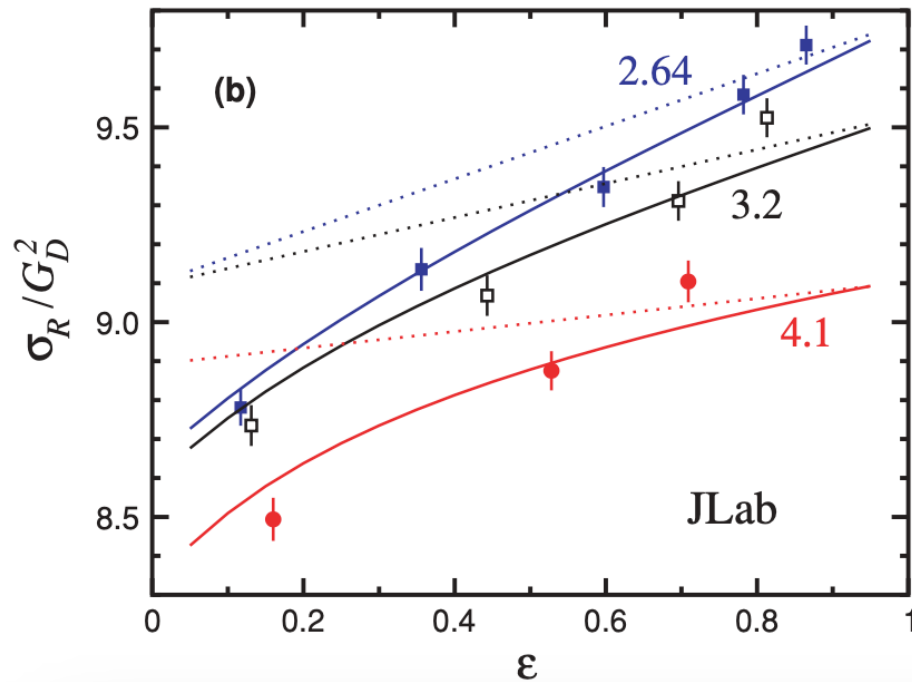
where σ_R is the reduced cross section given in Eq. (4), and the Mott cross section for the scattering from a point particle is

$$\sigma_{\text{Mott}} = \frac{\alpha^2 E_3 \cos^2 \frac{\theta}{2}}{4E_1^3 \sin^4 \frac{\theta}{2}}, \quad (8)$$

with E_1 and E_3 the initial and final electron energies and $\alpha = e^2/4\pi$ the electromagnetic fine structure constant. Including radiative corrections to order α , the elastic scattering cross section is modified as follows:

$$\frac{d\sigma_0}{d\Omega} \rightarrow \frac{d\sigma}{d\Omega} (1 + \delta), \quad (9)$$

Blunden TPE paper



The most notable difference with respect to the proton results is the sign and slope of the 2γ exchange correction. Namely, the magnitude of the correction $\delta^{\text{full}}(\epsilon, Q^2)$ for the neutron is ~ 3 times smaller than for the proton. The reason for

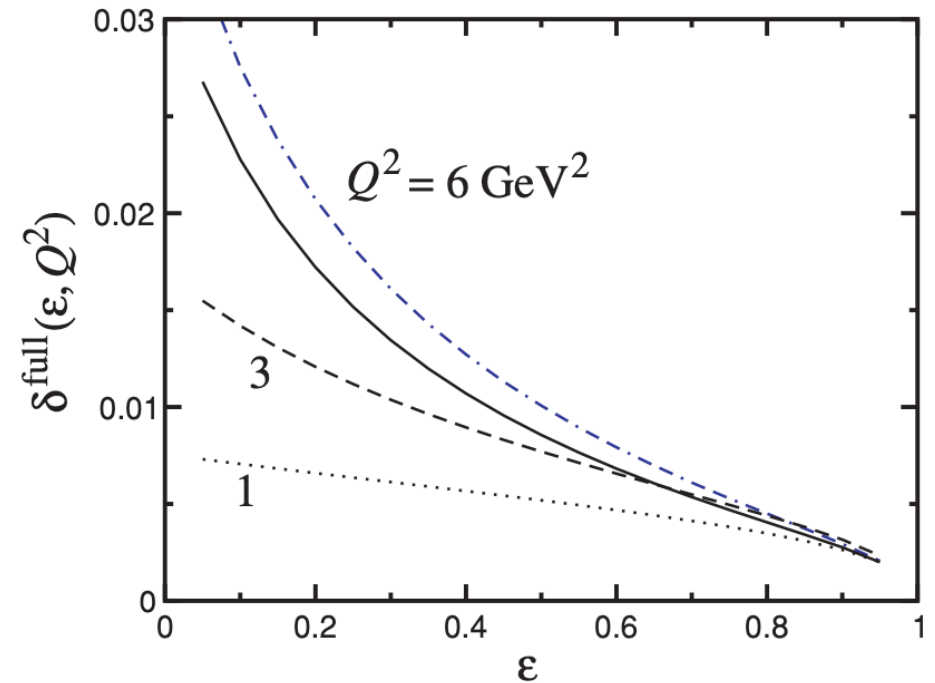
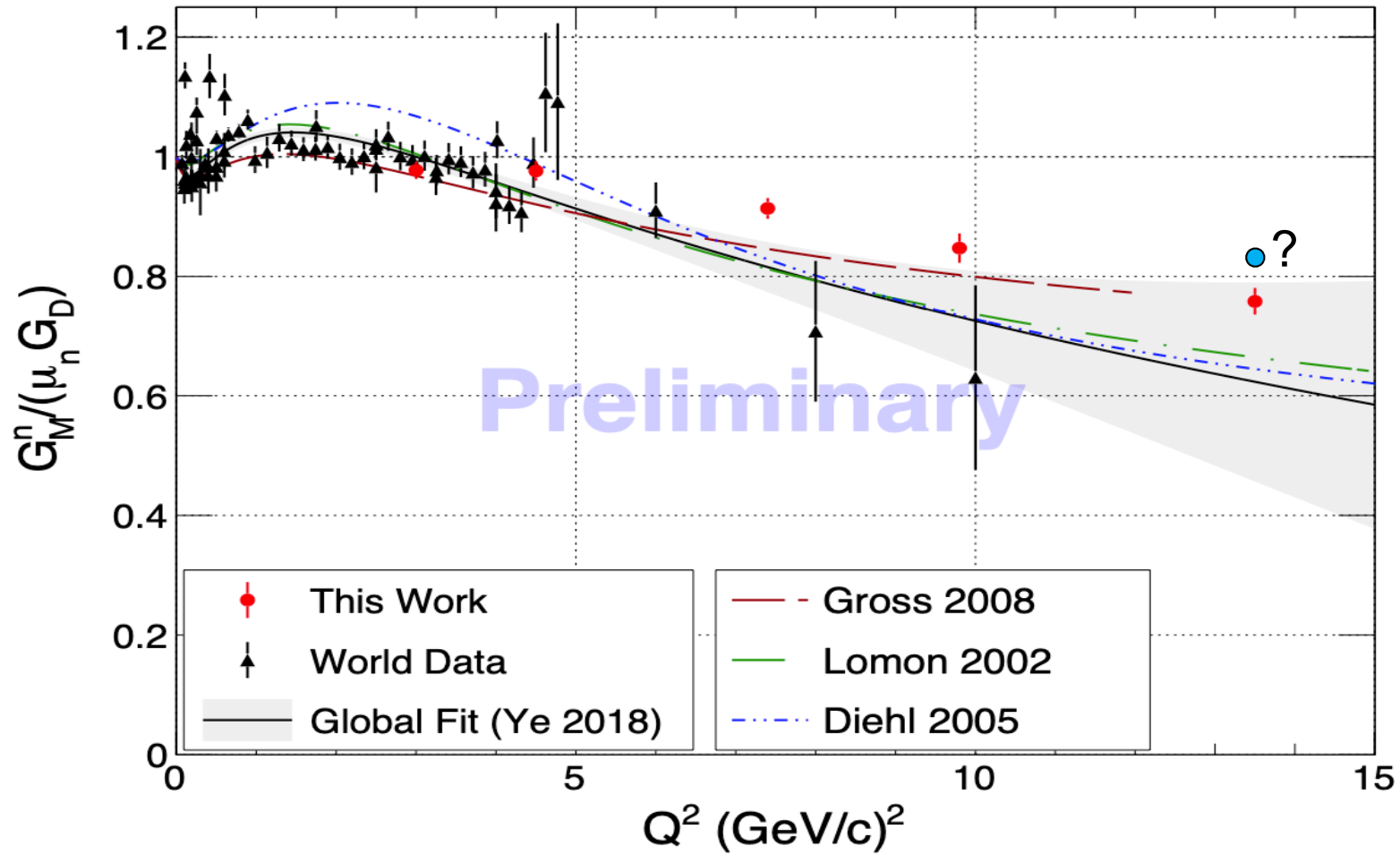


FIG. 12. (Color online) 2γ contribution to the unpolarized electron-neutron elastic scattering cross section, at $Q^2 = 1$ (dotted), 3 (dashed), and 6 GeV^2 (solid and dot-dashed). The dot-dashed curve corresponds to the form factor parametrization of Ref. [41], whereas the others are from Ref. [16] (as fitted by the parameters in Table I).

$\sigma_{e,e'n} / \sigma_{e,e'p}$ is down by $\sim 2\text{-}3\%$; We need calculation for 13.5 GeV^2

SBS GMn in P.Datta thesis



Summary

- ❖ Difference of RadCor for e-p and e-n at high Q^2 could lead to change of the GMn/GMp by +7%
- ❖ Accurate measurement of the cross section at high Q^2 requires an update of the radiative correction