Generalized parton distributions and form factors of the proton

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August 5, 2019
Generalized parton distributions and form factors

- **Generalized form factors** (GPDs) are three-dimensional descriptions of partonic structure.
- GPDs contain the elastic form factors.
- Two-dimensional slices of GPDs are seen in deeply virtual Compton scattering (DVCS).
- GPDs & DVCS offer additional insight to form factors.
- I will present model calculations/predictions for proton GPDs.
Form factors and proton structure

**Point particle**

\[ \langle p's' | j^\mu(0) | ps \rangle = \bar{u}'(p') \gamma^\mu u^s(p) \]

\[ \Delta = p' - p \quad t = \Delta^2 = -Q^2 \]

- Form factors appear in the electromagnetic current.
- They tell us about substructure—how is the proton different from a point particle?
- Seen in reactions where proton gets a momentum kick.

**General spin-half particle**

\[ \langle p's' | j^\mu(0) | ps \rangle = \bar{u}'(p') \left[ \gamma^\mu F_1(t) + \frac{i\sigma^\mu\Delta}{2m_p} F_2(t) \right] u^s(p) \]

- How kick is redistributed gives hint at structure—seen in \( t \) dependence.
- Stay together easily with large kick? *Slow \( t \) falloff*
- Trouble staying together? *Fast \( t \) falloff*
Electric and magnetic form factors

**Sachs form factors**

\[
G_E(t) = F_1(t) + \frac{t}{4m_p^2} F_2(t) \\
G_M(t) = F_1(t) + F_2(t)
\]

**Sum rules**

\[
G_E(0) = F_1(0) = 1 \\
G_M(0) = F_1(0) + F_2(0) \\
\mu_p = 1 + \kappa_p
\]

- 3D Fourier transforms of \(G_E(t)\) and \(G_M(t)\) characterize electric and magnetic response of charged objects (electrons) to proton; e.g.,

\[
\rho_E(\mathbf{r}) = \int \frac{d^3k}{2E_k(2\pi)^3} G_E(t = -k^2)e^{-i(k \cdot \mathbf{r})}
\]

- These are **not literal spatial densities** (see e.g., Miller, PRC99 (2019) 035202)

- **Charge and magnetization radii:**

\[
\langle r_E^2 \rangle = 6 \frac{dG_E(t)}{dt} \bigg|_{t=0} \\
\langle r_M^2 \rangle = 6 \frac{dG_M(t)}{dt} \bigg|_{t=0}
\]

...electrons respond as if proton had these radii.
Flavor-separated structure

- Proton & neutron have valence \( uud \) and \( udd \) structure.
- Assume **charge symmetry**: neutron is a proton with \( u \leftrightarrow d \) swap.

\[
F_{ip}(t) = \frac{2}{3} F_{ip}^u(t) - \frac{1}{3} F_{ip}^d(t)
\]

\[
F_{in}(t) = -\frac{1}{3} F_{ip}^u(t) + \frac{2}{3} F_{ip}^d(t)
\]

(where \( i = 1, 2 \)) and thus

\[
F_{ip}^u(t) = 2F_{ip}(t) + F_{in}(t)
\]

\[
F_{ip}^d(t) = F_{ip}(t) + 2F_{in}(t)
\]

- Neutron measurements can tell us more about the proton.
- Different forms for \( F_{ip}^u(t) \) and \( F_{ip}^d(t) \) hint at interesting isospin structure!
Diquark correlations

- One way to get behavior seen in Cates et al. is diquark correlations.
- Proton is made of three dressed quarks.
  - Dressed quarks are quasi-particles.
  - They are amalgamations of the true (current) quarks.
- Two of the three quarks are often bound together into a diquark.
- Proton then looks like a two-body state: quark and diquark.
The lightest diquarks are **scalar/isoscalar**.

- Isoscalarity ⇒ down quark is in diquark.
- Asymptotic form for two-body form factor \( \sim \frac{1}{Q^2} \) at large \( Q^2 \).
- Quark outside diquark (up quark): form factor goes as \( \frac{1}{Q^2} \).
- Quark within diquark: two layers of two-body behavior, \( \frac{1}{Q^4} \).
- Cates et al. result highly suggestive of isoscalar diquarks!

Figure from Cates et al., PRL106 (2011) 252003
Scalar & axial-vector diquarks

- More than just scalar diquarks may exist.
- **Axial-vector/isovector** diquarks are next-lightest diquark.
- Axial-vector diquarks needed to get large proton magnetic moment.
- **Question**: how could we see evidence of axial diquarks in observables? (a la Cates et al. for scalar diquarks).

Model calculations (on left):
- Nambu–Jona-Lasinio model
- No pion cloud included
- $Q^2$ falloff is slower than reality
- Introducing dipole with vector meson pole improves $Q^2$ falloff
- See arXiv:1907.08256 & Cloët et al. (2014) for details

<table>
<thead>
<tr>
<th></th>
<th>Scalar diquarks</th>
<th>Scalar+axial</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_p$</td>
<td>0.57</td>
<td>1.49</td>
<td>1.79</td>
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</tbody>
</table>

(Pion cloud improves model; Cloët et al., PRC90 (2014) 045202)
Multidimensional structure

More proton structure information if a new particle is created and detected in final state.

Two possible avenues for exploration:

1. **Hard exclusive reactions**: proton remains intact.
2. **Semi-inclusive deeply inelastic scattering (SIDIS)**: proton is (generally) destroyed, sum over all final states that include created/detected particle.

- **Option 1** is closest analogue to form factors (elastic scattering).
- **Deeply virtual Compton scattering (DVCS)**: the created/detected particle is a photon.
Deeply virtual Compton scattering

Deeply virtual Compton scattering (DVCS) is the hard electro-production of a photon.

Generalized parton distributions (GPDs) encode the QCD structure seen in DVCS.

Presence of a loop means one GPD variable is integrated over.

Integrated quantities seen in experiment: Compton form factors

\[ \mathcal{H}(\xi, t) = \int dx \left[ \frac{1}{\xi - x - i0} \mp \frac{1}{\xi + x - i0} \right] H(x, \xi, t) \]
The GPD/DVCS variables

\[ \frac{x + \xi}{1 + \xi} \] and \( \frac{x - \xi}{1 - \xi} \) are initial and final light cone momentum fractions of struck quark.

\( t \) is the invariant momentum transfer to the target.

\( Q^2 \) is the invariant momentum transfer from the electron.

\( t \neq -Q^2 \), in contrast to elastic scattering.

\( Q^2 \) acts as a resolution scale, like in deeply inelastic scattering (DIS).

(I will use \( Q^2 = 4 \text{ GeV}^2 \) for remainder of talk.)

\( t \) tells us about structure seen from redistribution of momentum kick.

\( t \) is the “form factor variable” rather than \( Q^2 \).
GPDs and form factors

- Helicity-independent GPDs defined using a non-local operator

\[ \frac{1}{2} \int \frac{dz}{2\pi} e^{-iP \cdot nz} \langle p' | \bar{q} \left( \frac{nz}{2} \right) \gamma_q \left( -\frac{nz}{2} \right) | p \rangle = \bar{u}(p') \left[ \gamma^q H^q(x, \xi, t) + \frac{i\sigma^q}{2m_N} E^q(x, \xi, t) \right] u(p) \]

...in the light cone gauge. Other gauges require a Wilson line.

- Breakdown analogous to form factors:

\[ \langle p' s' | j^\mu(0) | ps \rangle = \bar{u} s'(p') \left[ \gamma^\mu F_1(t) + \frac{i\sigma^\mu}{2m_p} F_2(t) \right] u^s(p) \]

- These are even related through integration

\[ \int dx H^q(x, \xi, t; Q^2) = F_1^q(t) \quad \int dx E^q(x, \xi, t; Q^2) = F_2^q(t) \]

- GPDs are, in a sense, extensions of form factors
Non-skewed proton GPD: $H^q(x, \xi = 0, t)$

Orange is up; blue is down.

- **Forward limit:** $H^q(x, 0, 0) = q(x)$
- Related to (flavor-separated) **Dirac form factor:**

\[
\int dx \, H^q(x, 0, t) = F_1^q(t)
\]

- Additionally related to **gravitational form factor:**

\[
\int dx \, x H^q(x, 0, t) = A^q(t)
\]

Describing distribution of energy

- Show hybrid of behaviors of form factors and parton distribution functions (PDFs)
Non-skewed proton GPD: $E^q(x, \xi = 0, t)$

Orange is up; blue is down.

- No forward limit.
- Related to (flavor-separated) **Pauli form factor**:
  $$\int dx\ E^q(x, 0, t) = F^q_2(t)$$

- Up and down quarks have opposite signs—contribute constructively to $\kappa_p$
- Additionally related to **gravitational form factor**:
  $$\frac{1}{2} \int dx\ x\ (H^q(x, 0, t) + E^q(x, 0, t)) = J^q(t)$$

describing distribution of *total* angular momentum
A normalized, flavor-separated super-ratio

- How much faster is down quark falloff for different $x$ slices?
- Normalize out the $t = 0$ value:

$$S = \frac{H^d(x, 0, t)}{H^d(x, 0, 0)} / \frac{H^u(x, 0, t)}{H^u(x, 0, 0)}$$

- In scalar diquark model, down quarks are within diquark.
  - The $d/u$ ratio is small to begin with at large $x$
  - At small $x$, $d/u$ ratio decreases as diquark dies off with $t$
- We see new behavior offered by this multidimensional description!

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Scalar diquarks only
Super-ratio: a model comparison

Scalar diquarks only

- Noticeable qualitative difference!
- This is because down quark can be outside an axial diquark.
Skewed GPDs
Skewed proton GPD: $H^q(x, \xi = 0.5, t)$

Orange is up; blue is down.

- Still related to (flavor-separated) **Dirac form factor**:
  \[
  \int dx \, H^q(x, \xi, t) = F_1^q(t)
  \]
  (integration kills $\xi$ dependence!)

- Related to **two gravitational form factors**:
  \[
  \int dx \, xH^q(x, \xi, t) = A^q(t) + \xi^2 C^q(t)
  \]

- $C^q(t)$ describes how **shear and pressure** are distributed within the proton (see Polyakov & Schweitzer).

- Being able to study $\xi$ dependence offers an exciting new window into probing forces!

A. Freese (ANL)
Flavor-separated gravitational form factors?

- **Sum rules:** $A(0) = 1$, $B(0) = 0$, $C(0) < 0$, after sum over quarks & gluons
  
  ... hence why $A^u(0) + A^d(0) < 1$ in plot above

- $A^u(t) > A^d(t)$ since there are two up quarks, but ...

- ... down quarks contribute more to shear/pressure?? (We need to understand this better within the model)

- Flavor-separated gravitational form factors could tell us so much more about the proton.

- But GPDs & GFFs are not directly measured—long term project.

- **What could we do more directly?**
DVCS and Bethe-Heitler processes

- DVCS interferes with Bethe-Heitler process: photon emission by electron
- This is a boon rather than a bane: interference between diagrams seen in beam-spin asymmetry!
- Interference term seen in sin φ modulation of beam spin asymmetry:

\[
A_{LU}(\phi) = \frac{\sigma^+(\phi) - \sigma^-(\phi)}{\sigma^+(\phi) + \sigma^-(\phi)} \propto \text{Im} \left[ F_1 \mathcal{H} - \frac{t}{4m_p^2} F_2 \mathcal{E} + \xi (F_1 + F_2) \tilde{\mathcal{H}} \right] \sin \phi + \cos \phi \text{ modulations} + \ldots
\]
Compton form factors

\[ A_{LU}(\phi) \propto \text{Im} \left[ F_1 \mathcal{H} - \frac{t}{4m_p^2} F_2 \mathcal{E} + \xi (F_1 + F_2) \tilde{\mathcal{H}} \right] \sin \phi + \ldots \]

(diagram taken from Pisano et al., PRD91 (2015) 052014)

- \( \tilde{\mathcal{H}} \) is a helicity-dependent Compton form factor.
- The imaginary parts of Compton form factors appear as:

\[ \mathcal{H}(\xi, t) = -\pi \sum_q e_q[H^q(\xi, \xi, t) - H^q(-\xi, \xi, t)] \]
\[ \mathcal{E}(\xi, t) = -\pi \sum_q e_q[E^q(\xi, \xi, t) - E^q(-\xi, \xi, t)] \]
\[ \tilde{\mathcal{H}}(\xi, t) = -\pi \sum_q e_q[\tilde{H}^q(\xi, \xi, t) + \tilde{H}^q(-\xi, \xi, t)] \]
\[ \tilde{\mathcal{E}}(\xi, t) = -\pi \sum_q e_q[\tilde{E}^q(\xi, \xi, t) + \tilde{E}^q(-\xi, \xi, t)] \]

- CFFs and beam spin asymmetry give two-dimensional slices of GPDs.
- Need precision determinations of form factors in order to extract CFFs.
- Question: is there anything about diquarks we can see in just imaginary CFFs?
Model calculation: flavor-separated Compton form factors
Super-ratio for Compton form factors

- Let’s see if this works for CFFs, which are measured.
- How much faster is down quark falloff for different $\xi$ slices?
- This is a new lever-arm offered by virtual Compton scattering!
- Normalize out the $t = 0$ value:

$$ S = \frac{\mathcal{H}^d(\xi, t)}{\mathcal{H}^d(\xi, 0)} / \frac{\mathcal{H}^u(\xi, t)}{\mathcal{H}^u(\xi, 0)} $$

- High $x \Rightarrow$ high $\xi$ in imaginary CFFs
- In scalar diquark model, down quarks are within diquark.
  - The $d/u$ ratio is small to begin with at large $\xi$
  - At small $\xi$, $d/u$ ratio decreases as diquark dies off with $t$

![Graph showing scalar diquarks only](image-url)
Super-ratio: Compton form factor case

Scalar diquarks only

- Noticeable *qualitative* difference, still!
- Again because down quark can be *outside* an axial diquark.

Scalar and axial diquarks
Conclusions and summary

- **Generalized parton distributions** (GPDs) are multi-dimensional descriptions of proton structure which contain the elastic form factors.
- GPDs can also be used to determine **gravitational form factors**.
- **Compton form factors** are two-dimensional slices of GPDs, which are measured in **deeply virtual Compton scattering** (DVCS).
- Compton form factors contain the potential to elaborate upon the quark-diquark structure of the proton.

Thank you for your time and attention!