Electron Ionization Energy Loss

Implementation in Hall A Analyzer:

$$\frac{dE}{dx} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln\left(\frac{\tau^2(2+\tau)}{2(I/m_e)^2} \right) + (1-\beta^2) + \left(\frac{\frac{\tau^2}{8} - (2\tau+1)\ln 2}{(\tau^2+1)^2} \right) - \delta \right]$$

$K = 0.1536 \ \frac{\text{GeV cm}^2}{g}$

$\tau = \gamma - 1$ (Kinetic energy in terms of mass)

~~I is taken~~

I (excitation energy) is taken from table

$$\delta = 2\left[\ln\left(\frac{\hbar\omega}{I}\right) + \ln(\beta\gamma) - \frac{1}{2} \right]$$

$\hbar\omega = 28.816 \sqrt{\frac{\rho Z}{A}}$ eV

☆ So, we are calculating mean Energy Loss in the Hall A Analyzer. The correct formula for electrons is used.

Implementation in SIMC:

$$\text{Eloss} = K\frac{Z}{A}\frac{1}{\beta^2}T\left[\ln\left(\frac{m_e\beta^2\gamma^2}{\underset{①\nearrow}{I^2}}\right)-\beta^2-\frac{\delta}{2}+\underset{②\rightarrow}{1.063}+\ln\left(\frac{KZ}{A}T\frac{1}{\beta^2}\right)\right]$$

T = length x density

$$I = \begin{cases} 21.8\times10^{-6}\,eV, & \text{if } Z=1 \\ 16\times Z^{0.9}\times10^{-6}\,eV, & \text{if } Z\neq1 \end{cases} \quad ③$$

$$\frac{\delta}{2} = \begin{cases} 0, & \text{if } \log_{10}\beta\gamma<0 \\ C_0+\ln(\beta\gamma)+\overset{④}{\left|\frac{C_0}{27}\right|}(3-\log_{10}(\beta\gamma))^3, & \text{if } \log_{10}\beta\gamma<3 \\ C_0+\ln(\beta\gamma), & \text{if } \log_{10}\beta\gamma<4.7 \\ C_0+4.7\ln 10, & \text{if } \log_{10}\beta\gamma>4.7 \end{cases}$$

$$C_0 = \ln\left(\frac{\hbar\omega}{I}\right)+0.5 \quad ⑤$$

Comments:

① why is I squared?

② I suppose this is because we calculate most probable loss

③ Better Parameterization $I = \begin{cases} 12Z+7, & 1\leq Z<13 \\ 9.76Z+58.8Z^{-0.19}, & Z\geq13 \end{cases}$

④ This term should be divided by Z?

⑤ This should be $C_0 = -\ln\left(\frac{\hbar\omega}{I}\right)+0.5$?

⑥ Applicability to electrons (c.f. O'Brien

$E_{MP} = AT\left(\tilde{K}+\ln\left(\frac{T}{\rho}\right)\right)$

$A = K\,Z/A$, $\tilde{K} = 19.26$)

Additional comments, September 2016:

Looked at some documents:

- Eloss.ps document for HallC (in SIMC documents folder)
- "Calculations of electron energy loss straggling", NIMB, Mejaddem, et. al.
- Information included in documents attached here

- From looking at these documents, we can say that what SIMC is doing currently on comments (1), (2), and (6) is fine. It is using most-probable energy loss, and sampling from a landau distribution (or at least a reasonable approximation of it).
- For comment (3), SIMC will be updated with the better parameterization for the excitation energy.

For comments (4) and (5), first note that the term in the brackets should be δ (not $\delta/2$). See the discussion on the next two pages for the parameterization of δ

For (2), 1.063 should be $1 + \ln(2) - \underset{\text{Euler's constant}}{\underset{\parallel}{\gamma}} = 1.116$.

- In PDG, $1-\gamma$ is replaced by 0.2. This number is not too important.

$$\delta_e = 2\left[\ln\frac{\hbar\omega}{I} + \ln\beta\gamma - \frac{1}{2}\right]$$

From 'SIMC' Note:

$$\delta = 2(\ln 10)(\log_{10}\beta\gamma) + 2\ln\frac{I}{\hbar\omega} + 1, \quad \{ \text{ if } \log_{10}\beta\gamma \geq 3$$

$$= 2\left[\ln\beta\gamma + \ln\frac{I}{\hbar\omega} + \frac{1}{2}\right]$$

$$2(\ln 10)(\log_{10}\beta\gamma) + 2\ln\frac{I}{\hbar\omega} + 1 + \frac{2\ln\frac{I}{\hbar\omega}+1}{27}(3-\log_{10}\beta\gamma)^3,$$

$$\{ \text{ if } 0 \leq \log_{10}\beta\gamma \leq 3$$

$$0, \quad \{ \text{ if } \log_{10}\beta\gamma < 0$$

In SIMC:

$$\delta = 2(\ln 10)(\ln 10) + \ln\frac{\hbar\omega}{I} + \frac{1}{2}, \quad \{ \text{ if } \log_{10}\beta\gamma \geq 4.7$$

$$\ln(10)\log_{10}\beta\gamma + \ln\frac{\hbar\omega}{I} + \frac{1}{2}$$

$$= \ln\beta\gamma + \ln\frac{\hbar\omega}{I} + \frac{1}{2}, \quad \{ \text{ if } \cancel{\log_{10}\beta\gamma} \; 3 \leq \log_{10}\beta\gamma \leq 4.7$$

$$(\ln 10)(\log_{10}\beta\gamma) + \ln\frac{\hbar\omega}{I} + \frac{1}{2} + \left|\frac{\ln\frac{\hbar\omega}{I}+\frac{1}{2}}{27}\right|(3-\log_{10}\beta\gamma)^3,$$

$$\{ \text{ if } 0 \leq \log_{10}\beta\gamma \leq 4.7$$

$$0, \quad \{ \text{ if } \log_{10}\beta\gamma < 0$$

In Sternheimer:

$$\delta = 2(\ln 10)(\log_{10}\beta\gamma) - 2\ln\left(\frac{I}{\hbar\omega}\right) - 1 \quad , \left\{ \cancel{x \geq} \; \log_{10}\beta\gamma > x_1 \right.$$

$$= 2\left[\ln\frac{\hbar\omega}{I} + \ln\beta\gamma - \frac{1}{2}\right]$$

$$2(\ln 10)(\log_{10}\beta\gamma) - 2\ln\left(\frac{I}{\hbar\omega}\right) - 1 + a(x_1 - \log_{10}\beta\gamma)^m ,$$

$$\left\{ x_0 < \log_{10}\beta\gamma < x_1 \right.$$

$$0 \quad , \left\{ \cancel{x} \; \log_{10}\beta\gamma < x_0 \right.$$

o update SIMC, use the x_0, x_1, a in the note. But ~~the~~ orrect for sign errors. So, we have the following:

$$\left\{ \begin{array}{l} x = \log_{10}\beta\gamma \\ C_0 = 2\ln\frac{\hbar\omega}{I} - 1 \end{array} \right.$$

$$\delta = 2(\ln 10)x + 2\ln\frac{\hbar\omega}{I} - 1 \quad , \left\{ \text{if } x \geq 3 \right.$$

$$= 2\left[\ln\beta\gamma + \ln\frac{\hbar\omega}{I} - \frac{1}{2}\right]$$

$$2(\ln 10)x + 2\ln\frac{\hbar\omega}{I} - 1 + \left|\frac{2\ln\frac{\hbar\omega}{I} - 1}{27}\right| (3 - x)^3 ,$$

$$\left\{ \text{if } 0 \leq x < 3 \right.$$

$$0 \quad , \left\{ \text{if } x < 0 \right.$$

utiLandau PHENIX utility

Purpose

Energy loss by charged particles in thin media follows Landau distribution.

$$f(\lambda) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(\lambda + e^{-\lambda})}$$

where

$$\lambda = \frac{\Delta E - (\Delta E)_{most\ probable}}{\xi}$$

$$\xi = \frac{2\pi N_A\, z^2 e^4}{m_e c^2} \frac{Z}{A} \frac{\rho}{\beta^2} X$$

utiLandau utility generates random numbers according to this distribution.

Input Variables

randseed = The current random number seed (long)

Output Variables

xlandau = output random value (float)

Function Prototype

void void utiLandau(long *randseed, float *xlandau)

Special Instructions

This utility utilizes the utiRandom utility for its random number generation.

Algorithm

Since

$$\int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\lambda+e^{-\lambda})} d\lambda = 2 \cdot \int_{0}^{+\infty} e^{\frac{-x^2}{2}} dx$$

where

$$x = e^{-\frac{\lambda}{2}}$$

it is enough to generate positive Gaussian-distributed random number X, and calculate Landau-distributed random number as

$$\lambda = -2.0 \cdot log(x)$$

utiLandau.c

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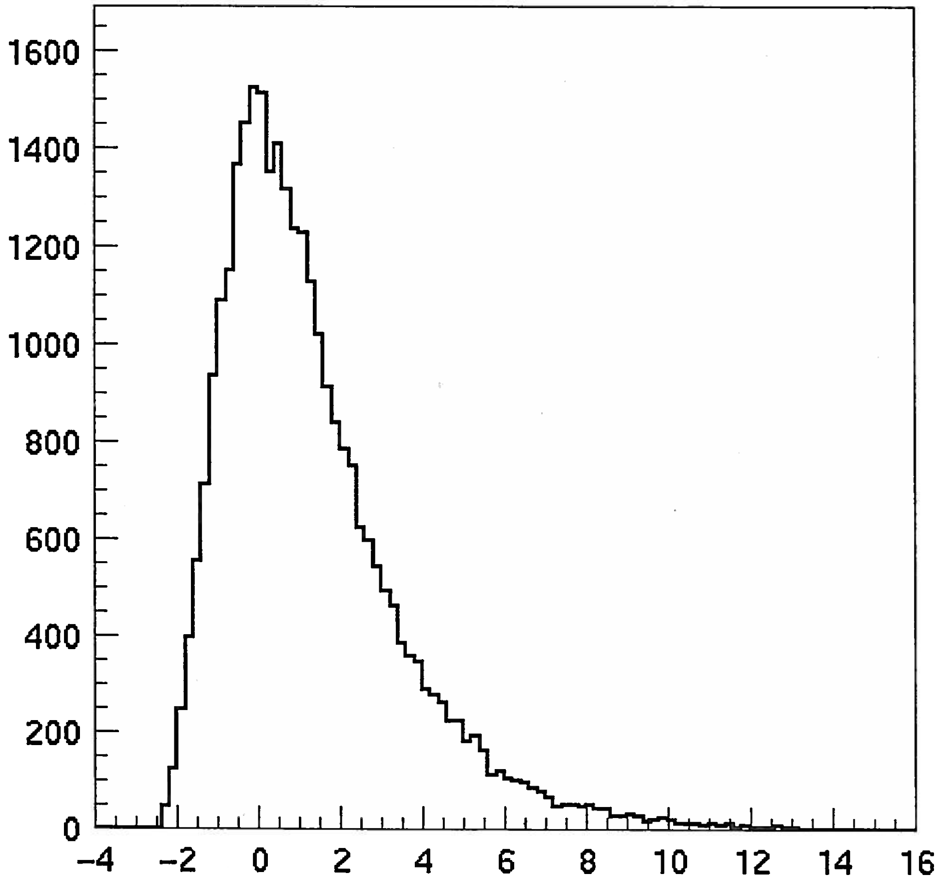
utiLandau Results

The plot below was generated from samples of the output of the utiLandau utility.



Last updated 04/06/98 by Sasha Lebedev

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kost@
Wed May 11 15:13:03 PDT 1994

```
/***************************************************************************
 ***************************************************************************

 utiLandau
 -----------

 DESCRIPTION:   Generates random value according to Landau distribution.
                Needs uniform random number generator (utiRandom).

 ALGORITHM:     Integral of the Landau distribution is
                equal to the integral of a Gaussian distribution
                after following substitution: x = exp(-lambda/2)
                Thus, it is enough to generate Gaussian random value x,
                and Landau distributed random number will be
                lambda = -2.0*log((fabs)(x))

 AUTHOR/CONTACT:  Sasha Lebedev, ISU, lebedev@iastate.edu

 REVISIONS:
       Date            Author          Description

       04/13/98        Sasha Lebedev   Original

 INPUT VARIABLES: randseed = Current random number seed (long)

 OUTPUT VARIABLES: xlandau =  Output random value (float)

 ***************************************************************************
 ***************************************************************************/

#include <stdio.h>
#include <math.h>
#include <stdlib.h>

extern float utiRandom(long *randseed);

void utiLandau(long *randseed, float *xlandau)
{

  long rseed;
  float lambda,xgauss,X,Y;
  float S = 0.449871;
  float T = -0.386595;
  float A = 0.19600;
  float B = 0.25472;
  float R1 = 0.27597;
  float R2 = 0.27846;
  float Q = 1.0;
  float U = 0.1;
  float V = 0.0;

  /* EXECUTABLE STATEMENTS */

  rseed = *randseed;   /* copy random seed for internal use */

/* Generate Gaussian random value. RNORML cernlib function,
   algorithm by J.L. Leva, ACM Trans. Math. Softw., v.18(1992) p. 454 */

  while((Q > R2) || (V*V > -4.0*log(U)*U*U))
  {
    U = utiRandom(&rseed);
```

```
    V = utiRandom(&rseed);
      V = 1.7156 * (V - 0.5);
        X = U - S;
        Y = (fabs)(V) - T;
          Q = X*X + Y*(A*Y - B*X);

            xgauss = V/U;
  }

/* Generate Landau distributed random number */

  lambda = -2.0*log((fabs)(xgauss));

  *xlandau = lambda;   /* return Landau distributed random value */

  *randseed = rseed;   /* return the modified seed */

}   /* end utiLandau */
```

Energy loss

Charged particles traversing material lose energy through inelastic collisions with the atomic electrons of the material. Because of the statistical nature of the ionization process, the energy lost E will have a spread δE about the most probable energy loss E_p. Let W_m be the maximum energy transfer to an atomic electron in a single collision. Then if $\delta E \gg W_m$, the distribution of energy loss is Gaussian and E_p is equal to the average energy loss of the particles(10). If however, $\delta E \ll W_m$, the distribution of energy losses is that calculated by Landau(11). This distribution is asymmetric, with a long tail on the high energy side that falls off as E^{-2}.

The program decides which type of distribution is applicable for each region in turn. If δE is approximately equal to W_m and neither distribution is applicable, a message is printed. The value of E_p is calculated from the expression(12)

$$\begin{aligned} E_p &= \frac{At}{\beta^2}\left[B + \overset{\cancel{1.075}\ 1.116}{\cancel{1.06}} + 2\ln\frac{p}{mc} + \ln\frac{At}{\beta^2} - \beta^2 - d - U\right] \\ &= \frac{At}{\beta^2}\left[34.92 + 2\ln\frac{p}{mc} - 2\ln(I) + \ln\frac{At}{\beta^2} - \beta^2 - d - U\right] \end{aligned}$$

with p = the momentum of particle,
m = the mass of the particle,
I = the ionization potential of material in eV,
U = the shell correction term,
t = the thickness of material in g/cm^2,
$A = 0.1536\text{x}10^{-9}\text{x}\ \frac{\text{Atomic number}}{\text{Atomic weight}}$,
$B = \ln\frac{m_e c^2 \cdot 10^6}{I^2} = 33.86 - 2\ln(I)$,
m_e = mass of the electron,
$\beta = v/c$,
v = velocity of the particle,
c = the speed of light,
d = the density effect correction by Sternheimer(13)

The parameter d is given by

$$\begin{aligned} d &= 0 & x &< x_0 \\ d &= 4.606\,x + C + a(x_1 - x)^m & x_0 < x &< x_1 \\ d &= 4.606\,x + C & x &> x_1 \end{aligned}$$

with

$$x = \log_{10}\frac{p}{mc}$$

and a, C, m, x_0 and x_1 constants that depend on the material.

It is assumed that the shell correction term U is negligible. This assumption is good when the velocity of the particle is much greater than the velocities of the atomic electrons. The expression for E_p is valid for both electrons and heavy particles. The program also calculates the width of the distribution that typically is about 20 percent of E_p. Where necessary, E_p and δE are averaged over the atomic species in the region using Bragg's Law.

As particles are traced through the system, the ionization energy loss E for each region is selected randomly from the relevant distribution using the values of E_p and δE previously calculated for that region. Because the distribution is obtained from a table, linear interpolation is used to get values intermediate between those given in the table, except where the value of $E > 8\delta E$, when the distribution is assumed to go as E^{-2} for the Landau curve. The accuracy of these calculations is optimized for 600MeV protons and 200MeV pions,for a wide range of materials.

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