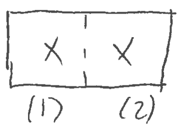
$$\log(a) + \log(b) = \log(ab)$$

$$a\log(b) = \log(b^a)$$

$$\frac{E_{MP}}{X} \propto a\log(X) + b$$



(1) ~~&~~ (2) : $E_{MP} = 2X\,(a\log(2X) + b) = 2X(a\log(x) + b) + 2X(a\log(2))$

(1) + (2) : $E_{MP} = 2\,[X(a\log(x) + b)] = 2X(a\log(x) + b)$

- -

For Electrons, $\mathcal{E}_{max} = \dfrac{M_e c^2(\gamma - 1)}{2}$ (half of KE)

$\mathcal{E} = 0.1536\, \frac{Z}{A}\, \frac{1}{\beta^2}\, X$ $\quad \beta \approx 1$ for GeV electrons

$x = \mathcal{E}/\mathcal{E}_{max} = \dfrac{0.307075\, Z}{M_e c^2\; A(\gamma - 1)}\, X$ $\quad M_e c^2 = 0.511$ MeV, X in g/cm^2

For incoming electron, $E \approx 2 - 10$ GeV

$\Rightarrow \gamma \approx 4000 - 20000$

For scattered electron, $E \approx 0.5 - 4$ GeV

$\Rightarrow \gamma \approx 1000 - 8000$

~~For~~ Hydrogen target: Max length = 15 cm $\quad \rho = 0.0723\, g/cc$

$Z = 1,\ A \approx 1$

$$x_{max} = \frac{(0.307075)}{(0.511)(1000)}\,(15)(0.0723) \approx 0.0007$$

Aluminum ; Max length ≈ 0.04 $\quad \rho \approx 2.8\, g/cc$

$Z = 13,\ A = 26.98$

$$x_{max} = \frac{(0.307075)(13)}{(0.511)(26.98)(1000)}\,(0.04)(2.8) \approx 3.2 \times 10^{-5}$$

Air: Max length $\approx$ 40 cm $\rho = 0.001\,g/cc$

$Z = 7.2$, ~~A=13.66~~ $A = 14.4$

$$\chi_{max} = \frac{(0.307075)(7.2)}{(0.511)(14.4)(1000)}(40)(0.001) \approx 1.2\times10^{-5}$$

Kapton (Mylar): Max length $\approx$ 0.03 cm $\rho = 1.39\,g/cc$

$Z = 4.5$ $A = 8.7$

$$\chi_{max} = \frac{(0.307075)(4.5)}{(0.511)(8.7)(1000)}(0.03)(1.39) \approx 1.3\times10^{-5}$$

Eric Christy's code:

- only for hadrons
- calculates mean energy loss for given thickness ($\Delta\bar{E}$). (See eq. 33.5 in PDG review). Code does not include density correction.
- χ calculated as above.

(i) For $0.01 \le \chi \le 12$,

sample Vavilov (λ_V)

$$\Delta E = \Delta\bar{E} + \varepsilon\left(1 - C + \beta^2 + \frac{\lambda_V}{\chi}\right) \quad \{C = \text{Euler's constant}$$

$$\Rightarrow \lambda_V = \frac{\Delta E - \Delta\bar{E}}{\varepsilon_{max}} - \chi(1 - C + \beta^2)$$ (eq. in Meshkov, ...)

(ii) For $\chi > 12$,

sample Gaus (f_g)

$$f_2 = \frac{\varepsilon}{\sqrt{\chi}}\sqrt{1 - \beta^2/2}$$

$$\Delta E = \Delta\bar{E} + f_2 f_g$$

$$f(x,\Delta E) \simeq \frac{1}{\sqrt{2\pi\gamma_2}} e^{-(\Delta E - \bar{\Delta E})^2 / 2\chi\gamma_2}$$ (eq. 15 in Mejadden, Y...)

$$\gamma_2 = \frac{\varepsilon^2}{\chi x}\left(1 - \beta^2/2\right)$$

$$f_2 = \sqrt{\chi\gamma_2}$$ (sigma of distribution)

(iii) For $\chi < 0.001$,

sample Landau λ

$$\Delta E = \Delta\bar{E} + \varepsilon(\lambda + 1 - C + \beta^2 + \ln(\chi))$$

$$\lambda = \lambda_{v}/\chi - \ln\chi$$ (eq. 16 in Mejadden, Y...)

$$\Rightarrow (\lambda + \ln\chi)\chi = \frac{\Delta E - \Delta\bar{E}}{\varepsilon_{max}} - \chi(1 - C + \beta^2)$$ (eq. 17)

$$\frac{\Delta E - \Delta\bar{E}}{\varepsilon_{max}} = \chi(\lambda + 1 - C + \beta^2 + \ln\chi)$$

In Majadden, Y. ..., the various ranges are given as follows:

$0.05 \le \chi \le 10$: Eric's (i)

$\chi > 10$: Eric's (ii)

$\chi < 0.05$: Eric's (iii) (for $\chi < 0.01$ transforms to Landau dist.)

Because Landau width is too small for thin absorbers, they suggest the following:

$\chi < 0.01$ and $b^2 \le 1 \equiv$ Landau $\qquad b^2 = 2\times10^{-5} Z^{4/3} \frac{\Delta E}{\varepsilon^2}$

$\chi < 0.01$ and $b^2 > 1 \equiv$ Blunck-Leisegang

For the hydrogen target, we can have $\overline{\Delta E}$ up to 10 MeV in 15cm
So, $b^2 \equiv 2\times10^{-5}\,(1)^{4/3}\,\frac{10}{[(0.1536)(15)(0.0723)]^2}$

$= 0.007$

... or ~2 MeV in ~2cm $\Rightarrow b^2 = 0.08$

A few other comments:

(i) For the forward-going particle, the energy used in every material is the energy at the vertex. Probably fine.

(ii) For the reconstructed particle, the energy used in every material is the reconstructed momentum at the spectrometer. Again, this is probably fine. The bigger issue is getting the most-probable loss when passing through multiple materials.

(iii) The reconstructed pathlength depends on the reconstructed vertex and angle. So, there is a resolution effect for the energy loss correction.

What is done in geant4?
- Physics reference manual, chapter 7

A cut value T_{cut} is set on the kinetic energy of the secondary particle. Below the cut, the soft secondaries are treated as continuous energy loss. Above the threshold, the secondary is generated according to the cross section.

· For energy loss fluctuations from a thick target (Kappa > 10), a gaussian distribution is used. A suitable modification is made because of T_{cut}.

- For thin absorbers, they model as two possible excitation values (E_1 and E_2) and a continuous ionization distribution ($\propto 1/E^2$). The number of each type is sampled from a Poisson dist. They say this gets the M.P. value right, but leaves the width too small. So, a width correction algorithm is applied.