Proton Magnetic Form Factor from Existing Elastic e-p Cross Section Data

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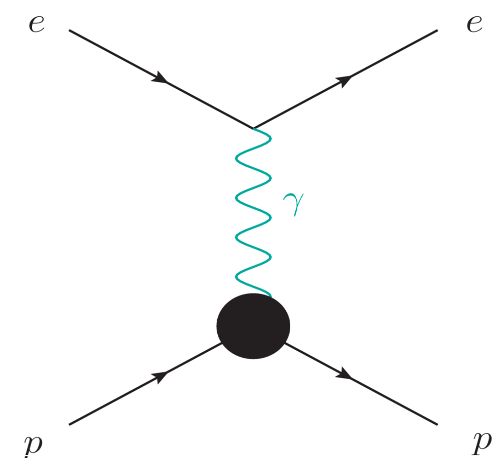
Nucleon Electromagnetic Form Factors

- Fundamental quantities defined in the context of one-photon exchange
  - Characterization of spatial distributions of charge and magnetization in the nucleon
  - Rigorous test of nucleon models
  - Input to nuclear structure and parity violation experiments
  - New information on basic hadron structure as first moments of GPD
- Experimental measurement has been conducted for several decades $\rightarrow$ not completely understood yet

Measurement of Proton Form Factor



Rosenbluth separation:

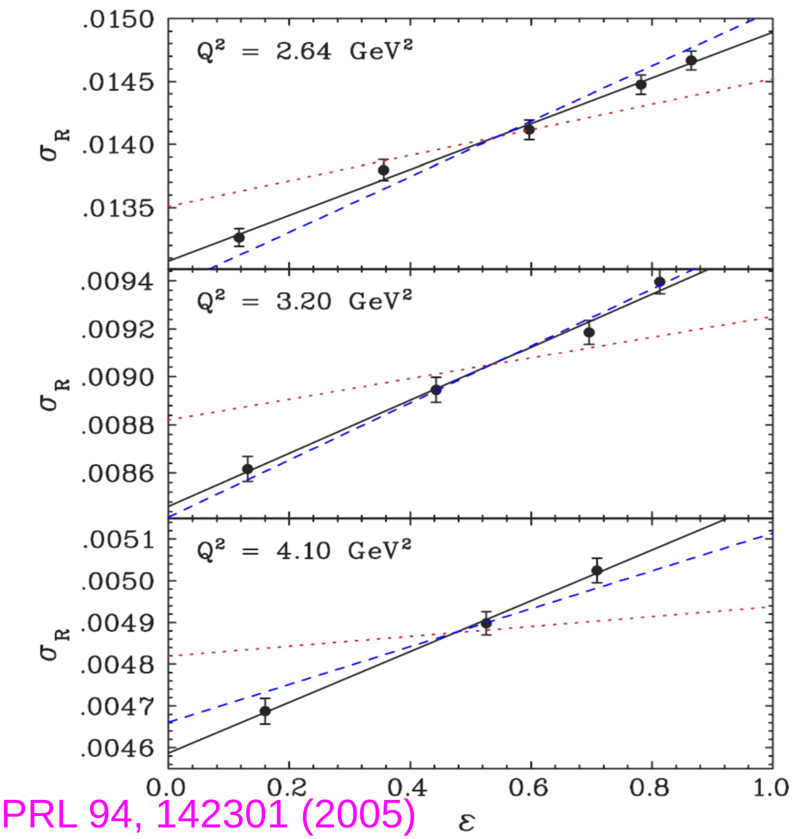
In one-photon exchange approximation, the e-p elastic scattering cross section can be expressed in terms of two form factors:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\frac{\epsilon\left(G_E^p\right)^2+\tau\left(G_M^p\right)^2}{\epsilon\left(1+\tau\right)}$$

$$\sigma_R\equiv\epsilon\left(1+\tau\right)\frac{d\sigma}{d\Omega}/\left(\frac{d\sigma}{d\Omega}\right)_{Mott}=\epsilon\left(G_E^p\right)^2+\tau\left(G_M^p\right)^2$$

$$\tau=Q^2/4M_p^2 \qquad \epsilon=\left[1+2\left(1+\tau\right)\tan^2\left(\theta/2\right)\right]^{-1}$$

- Measure angular dependence of *ep* elastic cross section at fixed Q^2
- Extract electric and magnetic form factor from linear dependence of reduced cross section on ε



Qattan et al.PRL 94, 142301 (2005)

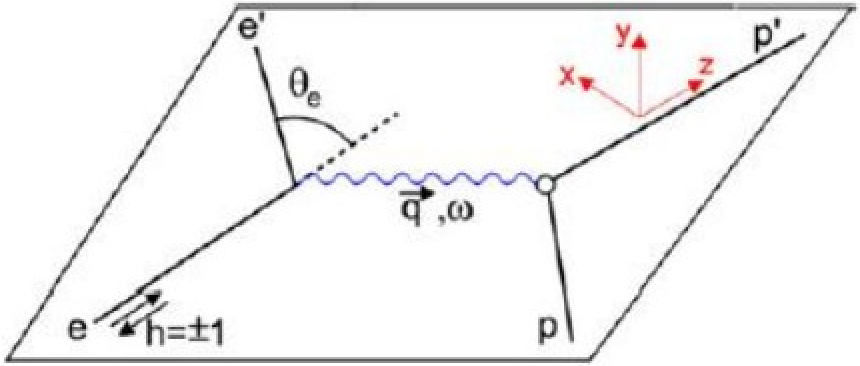
Measurement of Proton Form Factor

Polarization transfer:

Longitudinally polarized electrons scatters elastically from unpolarized proton target

The ratio of elastic to magnetic form factors are determined by transverse (P_t) and longitudinal (P_l) polarization components of the recoil proton

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{(E+E')}{2M_p}\tan\left(\frac{\theta}{2}\right)$$



Pros:
1 Not suffer from dramatically reduced sensitivity to G_E at large Q^2
2 Relatively small two-photon exchange correction
Cons:
Only measure ratio, need cross section measurement to derive G_E and G_M

Discrepancy Between Two Methods

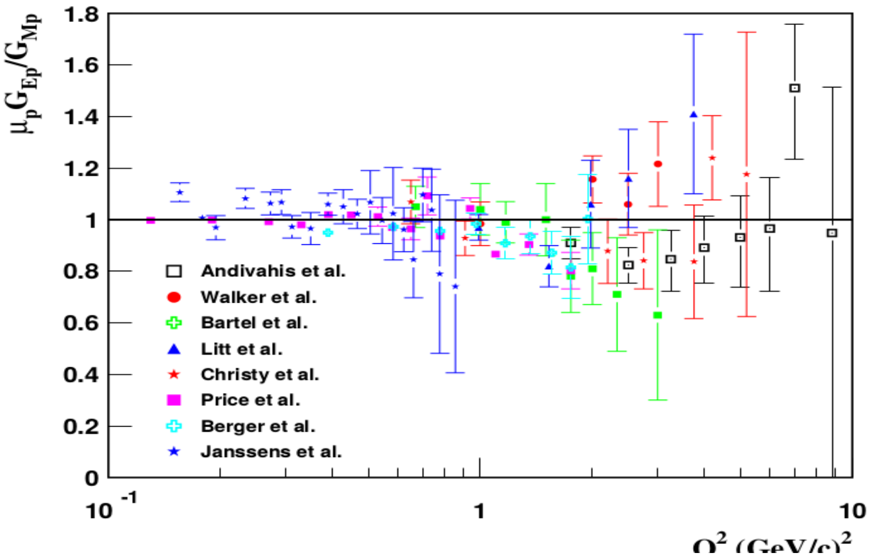
- Two methods yield significantly different results in form factor ratio in the region $Q^2 \geqslant 1.0 (GeV/c)^2$
  - Rosenbluth separation results show approximate scaling behavior of G_E and G_M with large uncertainties at high Q^2 values:

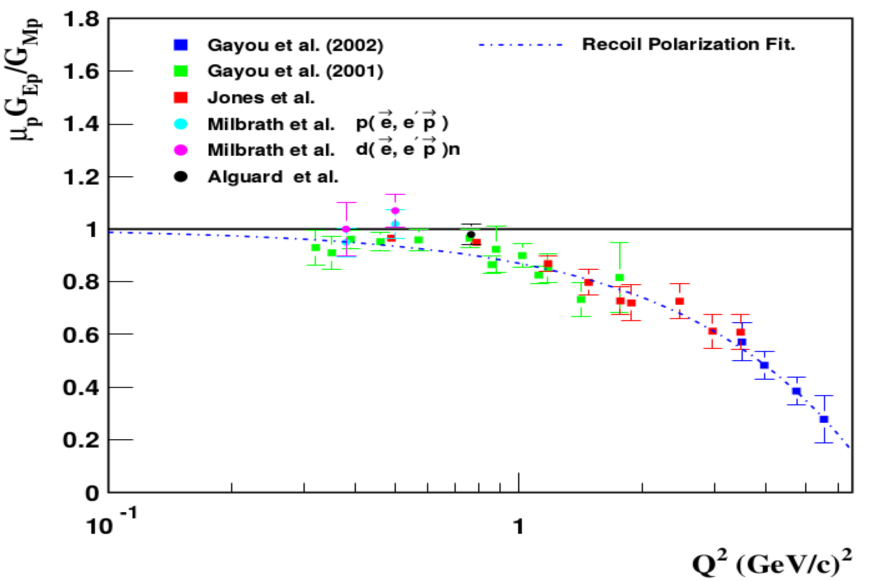
$$G_E^p \approx G_M^p / \mu_p \approx G_D = (1 + Q^2 / 0.71 GeV^2)^{-2}$$

  - Recoil polarization method see the ratio decreases with increasing Q^2 and is well parametrized as:

$$\frac{\mu_p G_E^p}{G_M^p} = \mu_p R = 1 - 0.13(Q^2 - 0.04)$$

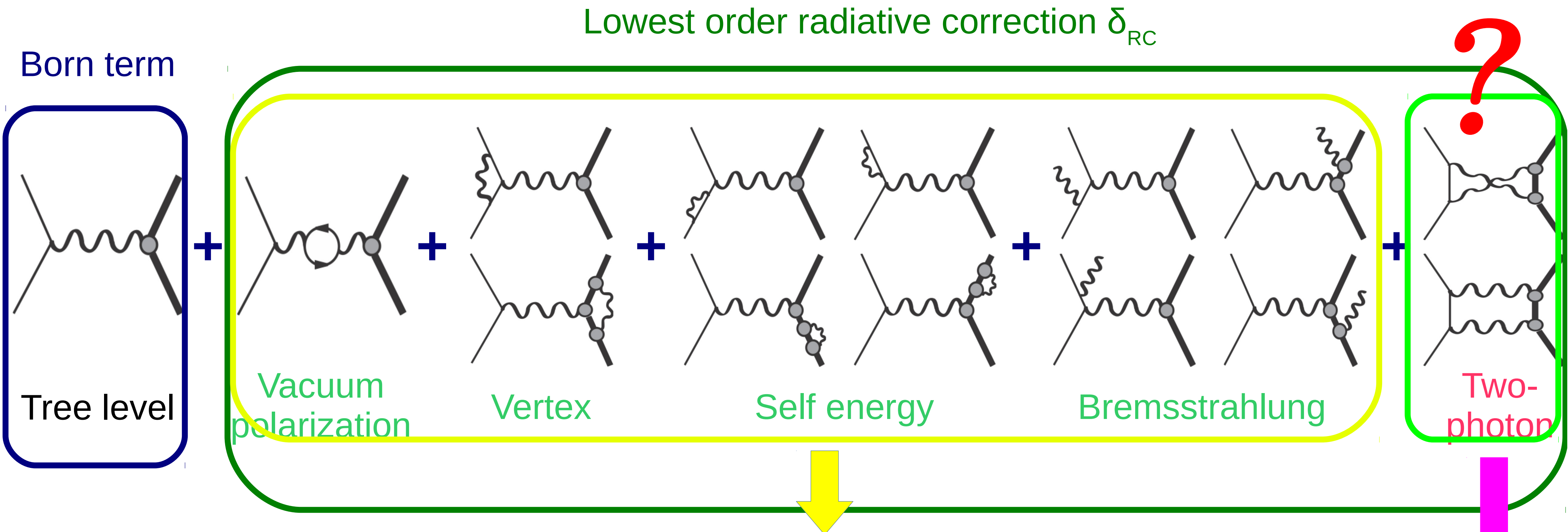
- Possible explanation: Two-photon exchange (TPE) effects





Exchange of Two Hard Photons

Radiative corrections for ep scattering: $\left(\frac{d\sigma}{d\Omega}\right)_{obs} = \left(\frac{d\sigma}{d\Omega}\right)_{Born} (1+\delta_{RC})$



Included in standard analysis of radiative correction
(Mo&Tsai, Rev. Mod. Phys. 41, 205(1969))

Full theoretical calculation is complicated

Not fully implemented in Mo&Tsai's formalism

ε-dependent correction

Exchange of Two Hard Photons

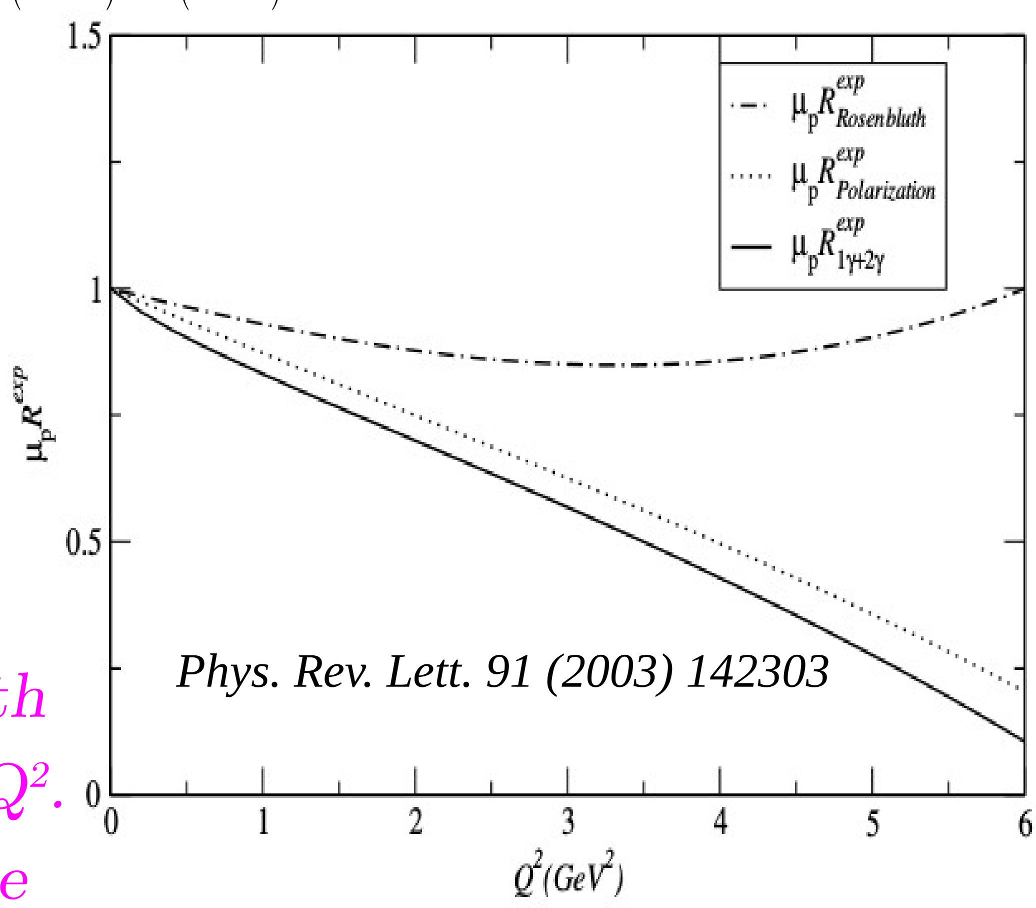
TPE effects affect the ratio of form factors obtained from Rosenbulth separation method and polarization transfer technique in different ways (*Guichon & Vanderhaeghen, Phys. Rev. Lett. 91 (2003) 142303*)

$$Y_{2\gamma}(\epsilon, Q^2) = \Re\left(\frac{\nu \tilde{F}_3}{M^2 |\tilde{G}_M|}\right) \sim \alpha \simeq 1/137 \qquad \nu = M^2 \sqrt{(1+\epsilon)/(1-\epsilon)}\sqrt{\tau(1+\tau)}$$

$$R^{\exp}_{polarization} = \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\epsilon}{1+\epsilon}\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma}$$

$$\left(R^{\exp}_{Rosenbluth}\right)^2 = \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma}$$

TPE correction impact Rosenbluth ratios more significantly at high Q^2. Polarization result is more reliable



Cross Section with TPE Correction

No non-linearity has been observed so far in the ε-dependence of the cross section

Theoretical calculations show that TPE contribution to the cross section Should be zero at ε=1 ***(JHEP04(2013)029)***

Our phenomenological model to include TPE effect:

TPE correction

$$\sigma_R=\tau\left[\left(G_M^p\right)^2+\frac{\epsilon}{\tau}\left(G_E^p\right)^2+\left(G_M^p\right)^2 a\left(Q^2\right)\left(1-\epsilon\right)\right]$$

$$\sigma_R=\tau\left(G_M^p\right)^2\left[1+\frac{\epsilon}{\tau}R^2+a\left(Q^2\right)\left(1-\epsilon\right)\right]$$

$R=G_E^p/G_M^p$

$$\boxed{\sigma_{fit}=\sigma_{Mott}\frac{\tau}{\epsilon\left(1+\tau\right)}\left(G_M^p\right)^2\left[1+\frac{\epsilon}{\tau}R^2+a\left(Q^2\right)\left(1-\epsilon\right)\right]}$$

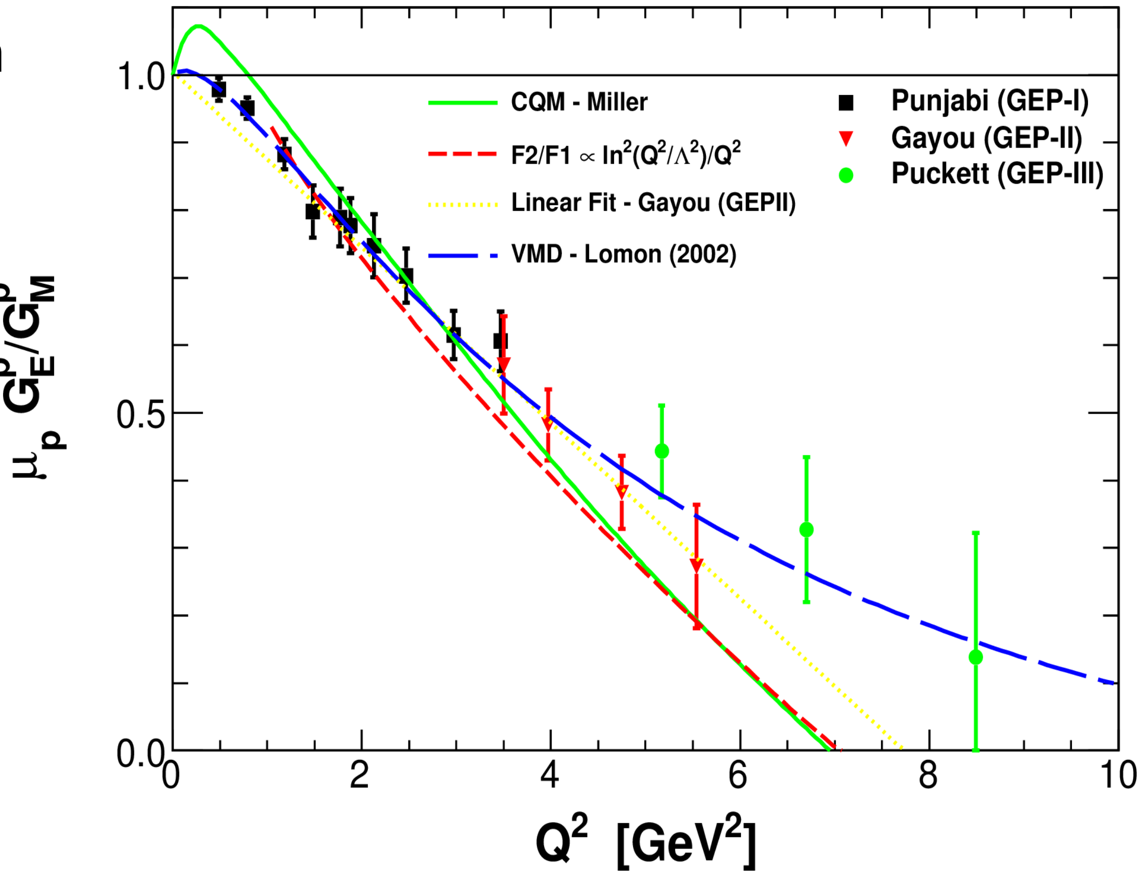
Parametrization for G_M and $a(Q^2)$: $\frac{G_M^p(Q^2)}{\mu_p}=\frac{1+b_0\tau}{1+b_1\tau+b_2\tau^2+b_3\tau^3}$ $\tau=\frac{Q^2}{4M_p^2}$

$$a\left(Q^2\right)=p_0+p_1Q^2+p_2Q^4+p_3Q^6$$

Model for the Ratio G_E/G_M

$$\sigma_{fit} = \sigma_{Mott} \frac{\tau}{\epsilon(1+\tau)} (G_M^p)^2 \left[1 + \frac{\epsilon}{\tau} R^2 + a(Q^2)(1-\epsilon)\right]$$

- Ratio of electric to magnetic form factors are well measured by polarization transfer experiments at Q^2 below 8 GeV^2
- A model describing the evolution of R is needed to extend the analysis to high Q^2 region
  - Linear fit of R below 7.7 GeV^2 and assume R=0 above that
  - Lomon's VMD fit (*Phys. Rev. C 66, 045501 (2002)*)



Fitting Procedure

Parameters to be fit:

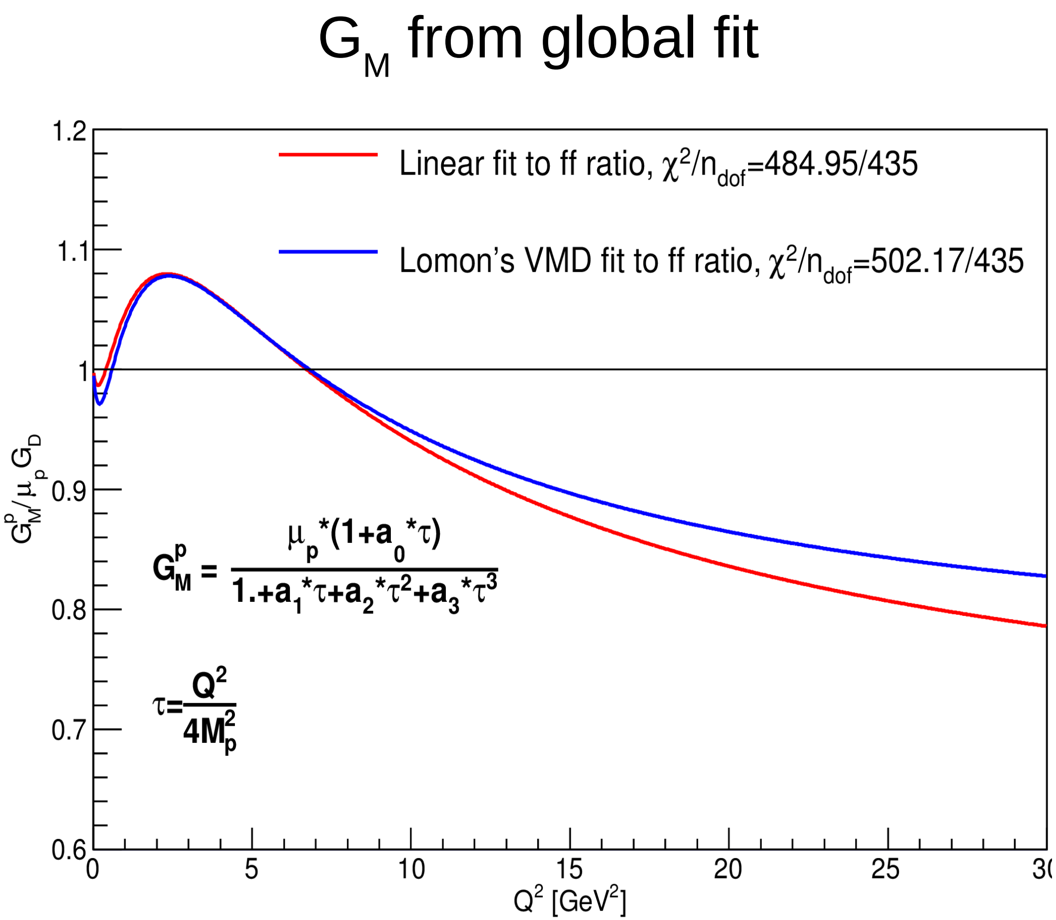
- Parameters describing the form factor G_M
- Parameters characterizing the TPE corrections
- Normalization factor η for each dataset

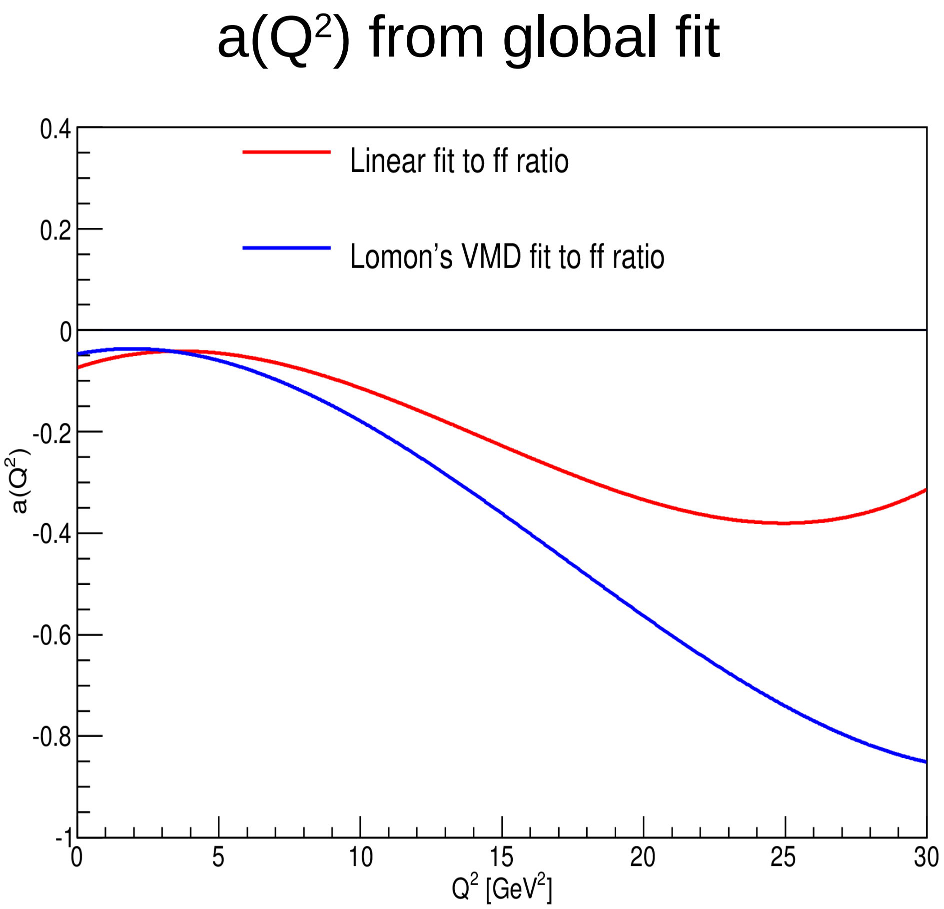
These parameters are determined by minimizing the following χ^2*:*

$$\chi^2 = \sum_{j=1}^{N_{expt}} \sum_{i=1}^{N_{data\ point}} \left[\frac{(\sigma_i - \sigma_{fit}/\eta_j)^2}{(d\sigma_i)^2} + \frac{(\eta_j - 1)^2}{(d\eta_j)^2} \right]$$

Ratio R of electric to magnetic FFs are taken from experimental data and theoretical models

Fitting Results





Summary and Future Plans

Summary:

- A phenomenological model based on recent theoretical investigation is built to describe TPE correction
- The parameters in the model are fit to *ep* elastic cross section data

Future plans:

- Global analysis of cross section with other models describing the ratio of form factors
- Analysis of proton form factor in time-like region