Bethe-Bloch Formula

Mean rate of energy loss (Stopping power) for a charge particle is:

$$\frac{-dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right],$$

Where,

A: *atomic mass of the absorber*

$$\frac{K}{A} = 4\pi N_A r_e^2 m_e c^2 / A$$

$$= 0.307075\ MeV g^{-1} cm^2,\ for\ A = 1\, g\, mol^{-1}$$

z: *atomic number of incident particle*

Z: *atomic number of absorber*

T_{max}: *max. transferable energy*

I: *characterstic ionization constant material dependent*

$\delta(\beta\gamma)$: *density effect correction*

$x = \rho s$, *mass thicknes, where, s is the length*

Max. transferable energy

$$Inelastic\ collision - energy\ loss$$

$$Maximum\ transferable\ kinetic\ energy:\ T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{m_o} + \left(\frac{m_e}{m_o}\right)^2}$$

$Where,$

$$\beta^2 = \frac{v^2}{c^2} = \frac{P^2}{E^2} = \frac{1}{1 + \frac{m_e^2}{p^2}}$$

$$since,\ m_e \ll P,\ \beta \approx 1.$$

$$For\ m_e = m_o$$

$$\gamma\beta \approx 1$$

$$m_e c^2 \approx 0.55\ MeV$$

$$T_{max} \approx \frac{1}{1+2+1} = 0.25\ MeV$$

Ionization energy Loss For Hydrogen

$$Ionization\ constant:$$

$$\frac{I}{Z} = \left(12 + \frac{7}{Z}\right) eV.$$

$$For\ H\ atom\ Z = 1,$$

$$So,\ I = 19\ eV.$$

$$So,\ the\ mean\ rate\ of\ energy\ loss\ due\ \text{to}\ ionization:$$

$$-\frac{dE}{dx} = 0.307075\left[\frac{1}{2}\ln\left(\frac{0.25\times10^{3}}{19^{2}} - 1 - \frac{1}{2}\right)\right]$$

$$\approx -\,0.517\ MeV/cm.$$

$$\left(\frac{dE}{dx}\right)_{H} = 0.517\ MeV/cm.$$

$$Average\ ionization\ energy\ loss = -\frac{dE}{dx}\frac{\Delta x}{2}$$

$$= 0.517\times7.5$$

$$= 3.8777\ MeV$$

For Aluminum

$Ionization\ constant:$

$$\frac{I}{Z} = \left(12 + \frac{7}{Z}\right) eV.$$

$For\ Al\ atom\ Z = 13,$

$$So,\ I = 163\ eV.$$

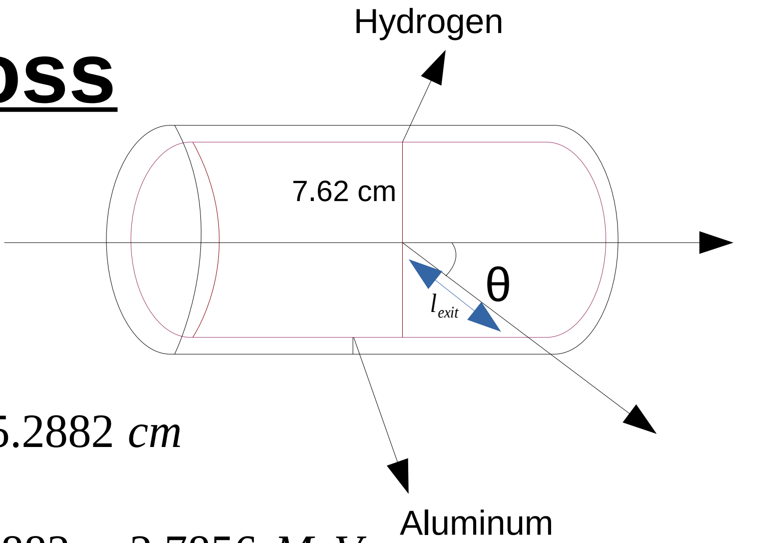
$So,\ the\ mean\ rate\ of\ energy\ loss\ for\ electron\ is:$

$$-\frac{dE}{dx} = \frac{0.307075}{26.98}\left[\frac{1}{2}\ln\left(\frac{0.25\times 10^{3}}{163^{2}} - 1 - \frac{1}{2}\right)\right]$$

$$\approx -0.04362\ MeV/cm.$$

$$\left(\frac{dE}{dx}\right)_{Al} = 0.04362\ MeV/cm$$

Event by Event Loss



$For\ Hydrogen,$

$$The\ amount\ of\ material\ exit\ (l_{exit}) = \frac{d}{2\ \sin 45} = \frac{7.62}{\sqrt{2}} = 5.2882\ cm$$

$$Ionization\ energy\ loss\ (\Delta E)_H = \frac{dE}{dx} \times l_{exit} = 0.517 \times 5.2882 = 2.7856\ MeV$$

$For\ Aluminum,$

$$Thickness = 0.44\ cm$$

$$l_{exit} = \frac{0.44}{2\ \sin 45} = 0.3111\ cm$$

$$(\Delta E)_{Al} = 0.3111 \times 0.04362$$
$$= 0.01357\ MeV$$

$$So,\ the\ total\ ionization\ energy\ loss\ (\Delta E) = (\Delta E)_H + (\Delta E)_{Al}$$
$$= 2.7856 + 0.01357$$
$$= 2.7992\ MeV$$