Bethe-Bloch Formula

Mean rate of energy loss (Stopping power)for a charge particle is:

$$\frac{-dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right],$$

Where,

A: atomic mass of the absorber

$$\frac{K}{A} = 4\pi N_A r_e^2 m_e c^2 / A$$

$$= 0.307075\ MeV g^{-1} cm^2,\ for\ A = 1\, g\, mol^{-1}$$

z: atomic number of incident particle

Z: atomic number of absorber

T_{max}: max. transferable energy

I: characterstic ionization constant material dependent

$\delta(\beta\gamma)$: density effect correction

$x = \rho s$, mass thicknes, where, s is the length

Max. transferable energy

→ Inelastic collision -energy loss

Maximum transferable kinetic energy: $T_{max} = \dfrac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \dfrac{m_e}{m_o} + \left(\dfrac{m_e}{m_o}\right)^2}$

Where,

$$\beta^2 = \frac{v^2}{c^2} = \frac{P^2}{E^2} = \frac{1}{1 + \dfrac{m_e^2}{p^2}}$$

And,

Since, $m_e \ll P$, $\beta \approx 1$.

For $m_e = m_o$

$m_e c^2 \approx 0.511\ MeV$

For incoming electron $T_{max} \approx 2012.22\ MeV$

For scattered electron $T_{max} \approx 1224.49\ MeV$

For incoming electron

$$\gamma = \frac{E}{E_o}$$

$$\gamma = \frac{2.057 \times 10^9}{0.511 \times 10^6}$$

$$= 4025.4403$$

For scattered electron

$$\gamma = \frac{P}{E_o}$$

$$= \frac{1.225 \times 10^9}{0.511 \times 10^6}$$

$$= 2397.2603$$

Average Ionization energy Loss For H

$X = \log_{10}(\gamma\beta) = 3.6048$

For H, $X_o = 0.4759$, $X_1 = 1.9215$,

$-C = 3.2632$

$\delta = 4.6052\, X + C \quad \text{As, } X > X_o$

$= 4.6052 \times 3.6048 - 3.2632$

$= 13.338$

Ionization constant:

$$\frac{I}{Z} = \left(12 + \frac{7}{Z}\right) eV.$$

For Hydrogen Z = 1,

$$So,\ I = 19\ eV.$$

The mean rate of energy loss due to ionization:

$$-\frac{dE}{dx} = 0.307075\left[\frac{1}{2}\ln\left(\frac{4025.4403^2 \times 2012.22 \times 10^{12}}{19^2}\right) - 1 - \frac{13.338}{2}\right]$$

$$\approx 4.7\ MeV\, cm^2/g.$$

$$\left(\frac{dE}{dx}\right)_H = -4.7\ MeV\, cm^2/g.$$

$$\text{Average ionization energy loss} = \frac{dE}{dx}\frac{\Delta x}{2}$$

$$= -4.7 \times 7.5 \times 0.0723$$

$$= -2.5486\ MeV$$

Mean rate for scattered electron for H

$$X = \log_{10}(\gamma\beta)$$
$$= 3.37972$$
$$\delta = 4.6052\,X + C \quad \text{As, } X > X_o$$
$$= 4.6052 \times 3.37972 - 3.2632$$
$$= 12.301$$

Ionization constant $(I) = 19\ eV$.

So, the mean of rate loss due to ionization for scattered electron:

$$-\frac{dE}{dx} = 0.307075\left[\frac{1}{2}\ln\left(\frac{2397.2603^2 \times 1224.49 \times 10^{12}}{19^2}\right) - 1 - \frac{12.301}{2}\right]$$

$$\approx 4.624\ MeV\,cm^2/g.$$

$$\left(\frac{dE}{dx}\right)_H = -4.624\ MeV\,cm^2/g.$$

For Aluminum

$X = \log_{10}(\gamma\beta) = 3.37972$

For Al, $X_o = 0.1708$, $X_1 = 3.0127$,

$-C = 4.24$

$\delta = 4.6052\, X + C$ As, $X > X_o$

$= 4.6052 \times 3.37972 - 4.24$

$= 11.342$

Ionization constant:

$$\frac{I}{Z} = \left(9.76 + 58.8 \times Z^{-1.19}\right) eV\,.$$

For Aluminium Z =13; $I = 166\ eV$.

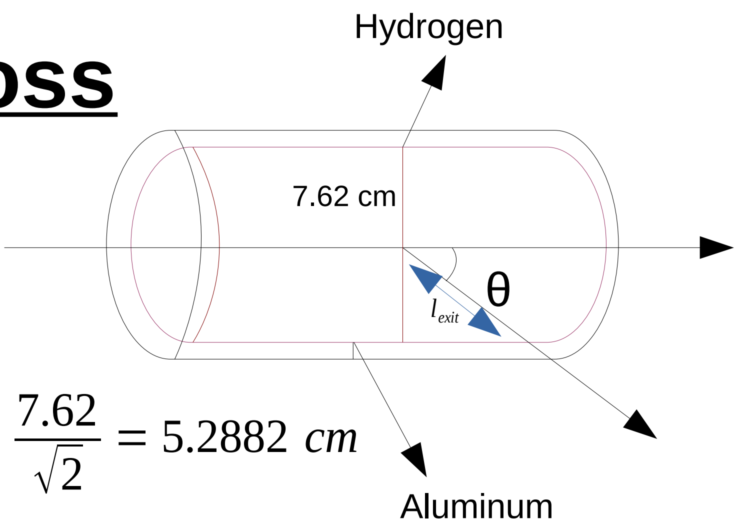
So, the mean rate of energy loss for electron is:

$$-\frac{dE}{dx} = \frac{13 \times 0.307075}{26.98}\left[\frac{1}{2}\ln\left(\frac{2397.2603^2 \times 1224.49 \times 10^{12}}{166^2}\right) - 1 - \frac{11.3424}{2}\right]$$

$$\approx 2.0543\ MeV\ cm^2/g\,.$$

$$\left(\frac{dE}{dx}\right)_{Al} = -2.0543\ MeV\ cm^2/g$$

Event by Event Loss



For Hydrogen,

$$\text{The amount of material exit} \left(l_{exit}\right) = \frac{d}{2\ \sin 45} = \frac{7.62}{\sqrt{2}} = 5.2882\ cm$$

$$\text{Ionization energy loss} \left(\Delta E\right)_H = \frac{dE}{dx} \times l_{exit} \times \rho_H = -4.624 \times 5.2882 \times 0.0733 = -1.7924\ MeV$$

For Aluminum,

$$\text{Thickness} = 0.173\ \text{mm}$$

$$l_{exit} = \frac{0.0173}{\sin 45} = 0.02447\ cm$$

$$\left(\Delta E\right)_{Al} = -2.0543 \times 0.02447 \times 2.7$$
$$= -0.1357\ MeV$$

$$\text{So, the total ionization energy loss} \left(\Delta E\right) = \left(\Delta E\right)_H + \left(\Delta E\right)_{Al}$$
$$= -1.7924 - 0.1357$$
$$= -1.928\ MeV$$