Goal: Extraction of GMp and TPE contribution from a combined analysis of LT and PT measurements

Assumptions in our model:

- The correction from two-photon exchange should go to zero at ε=1
  - ✔ TPE calculations
  - ✔ Comparisons of positron to electron scattering at small angles
- The TPE contribution in the reduced cross section is linear in ε
  - ✔ Deviation from linear behavior in Rosenbluth plot was not observed at Q^2 less than 5 $(GeV/c)^2$

Data set used in our fit:

We include information from both cross section measurements and polarization experiments

- Most of the cross section data are taken from J. Arrington's paper Phys. Rev. C 69 022201(R) (2004) (https://hallcweb.jlab.org/resdata/database/resdata_protonff.txt)
- Recent cross section measurements from Qattan and Christy are also included
- World data on G_E/G_M up to 8.5 $(GeV/c)^2$ are used in our fit

Parametrization:

- The proton magnetic form factor G_M and two-photon exchange contribution δ_{TPE} are parametrized, while G_E is *fixed* by G_M and form factor ratios
  - G_M is described by Kelly's parametrization:

$$\frac{G_M}{\mu_p} = \frac{1 + a_1\tau}{1 + b_1\tau + b_2\tau^2 + b_3\tau^3}$$

  - δ_{TPE} is parametrized to be linear in ε (also employed by Borisyuk and Kobushkin in Phys Rev D 83 057501 (2011)):

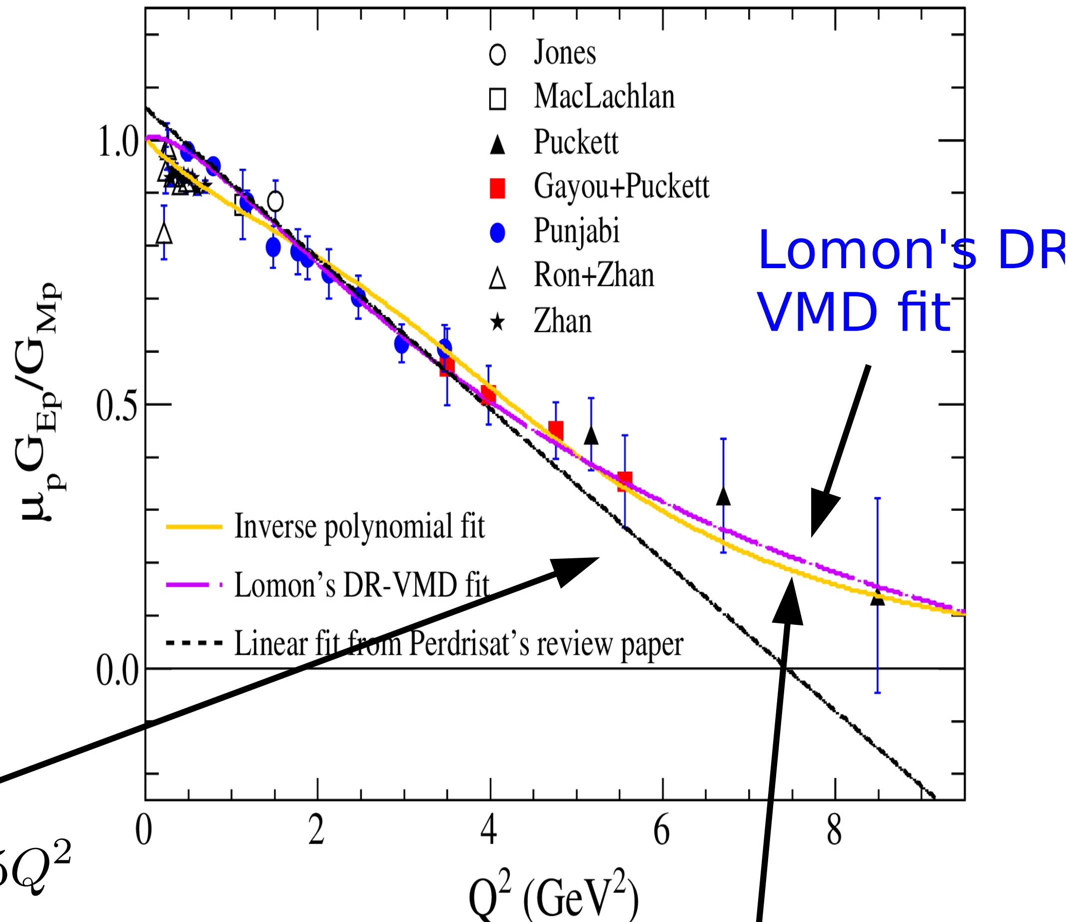
$$\delta_{TPE} = f(Q^2)(1-\varepsilon)G_M^2$$

  - How $f(Q^2)$ depends on Q^2 is not clear for us, and a polynomial representation is employed for simplicity:

$$f(Q^2) = \sum_{i=0}^{3} p_i Q^{2i}$$

Form-factors ratios:

- Results on proton form-factors ratios are available up to Q^2 = 8.5 $(GeV/c)^2$
- To extend the ratios to high Q^2 regime (>8.5 $(GeV/c)^2$), the data are fitted to an empirical model and a DR-VMD model (Lomon's GKex model, arxiv:nucl-th/0609020v2)



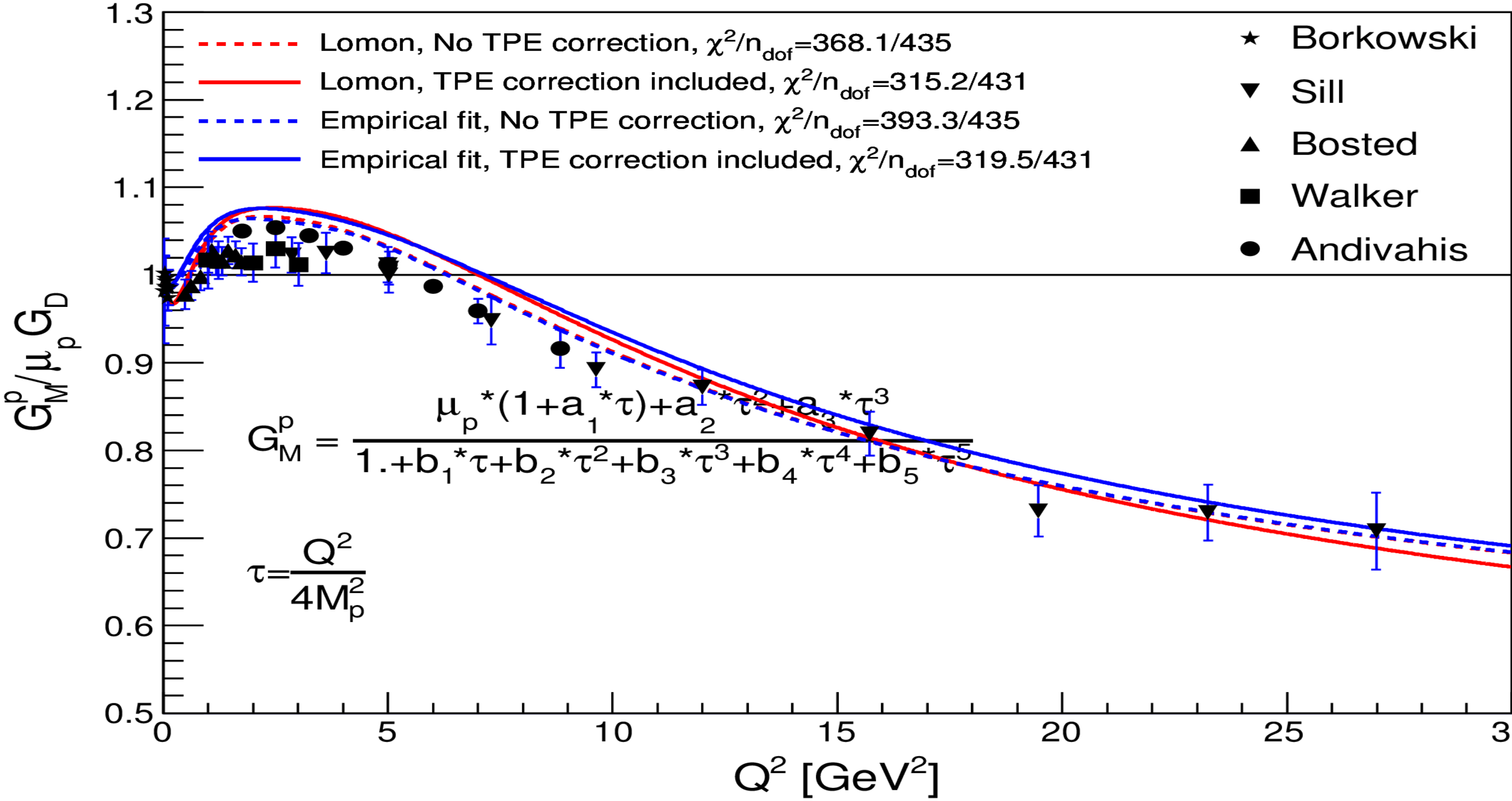
$$\mu_p G_E/G_M = 1.0587 - 0.14265 Q^2$$

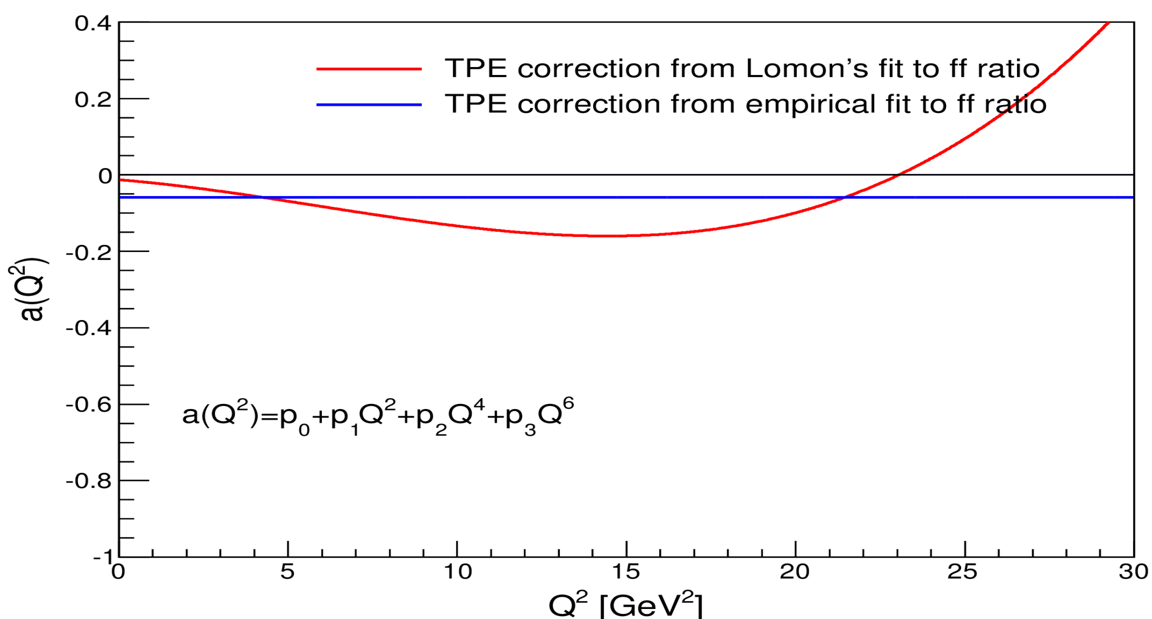
Empirical fit: $$\mu_p G_E/G_M = \frac{1}{1 + 0.1669 Q^2 - 0.0359 Q^4 + 0.0124 Q^6}$$

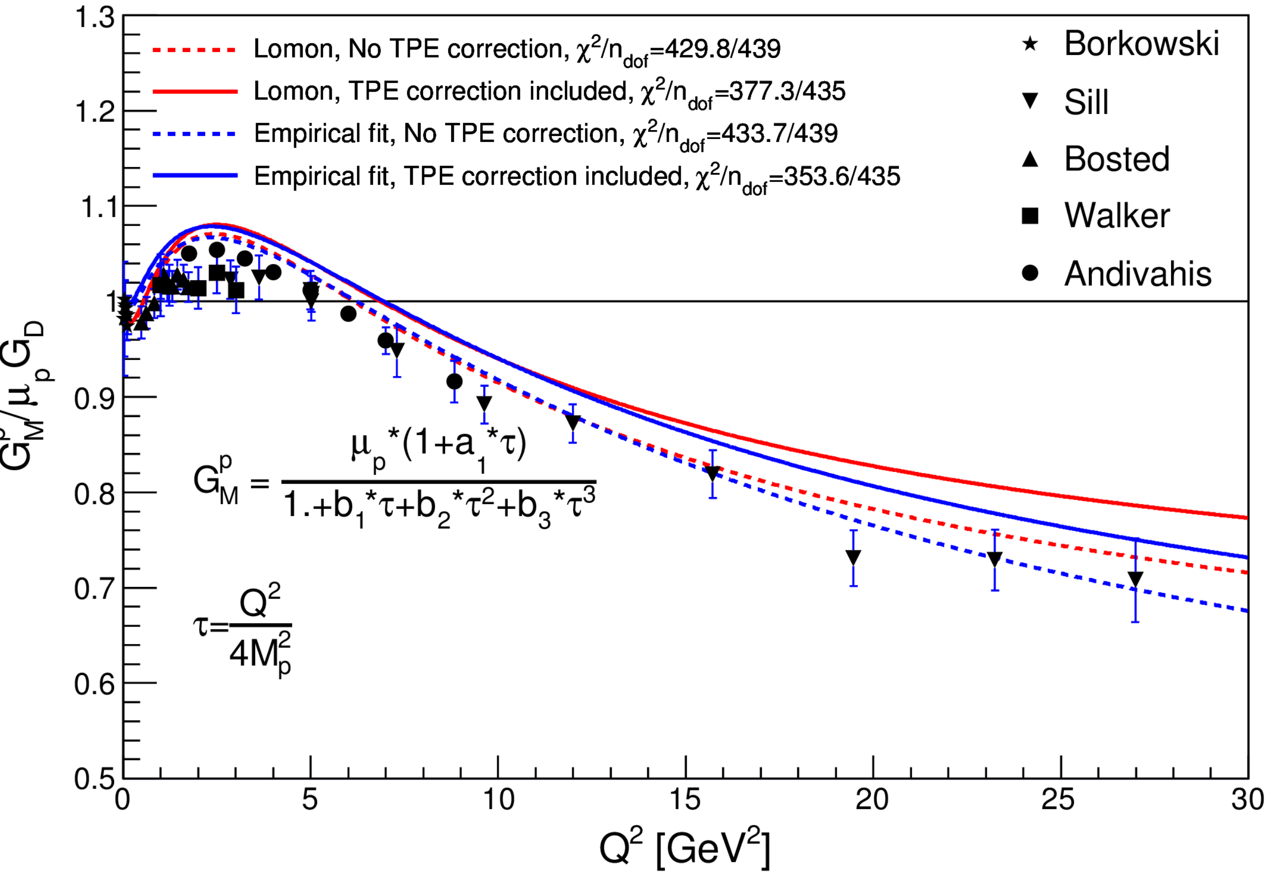
Fitting procedure:

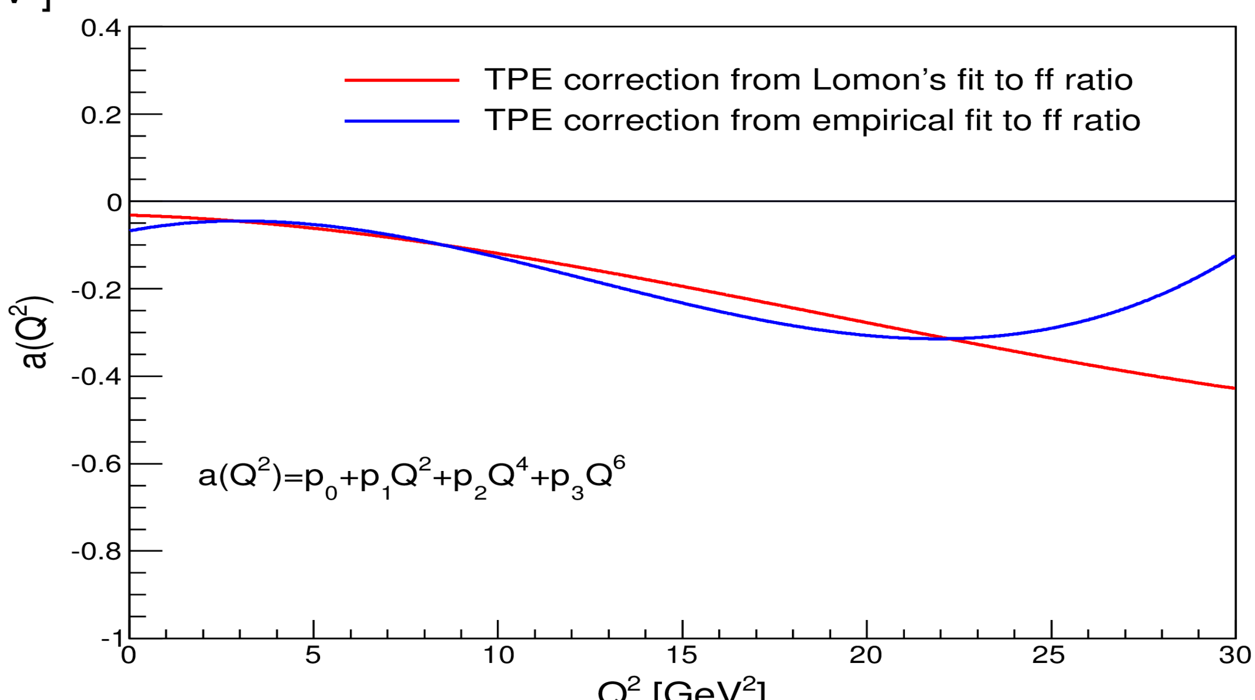
- The parameters for describing the magnetic form factor G_M and two-photon exchange contribution δ_{TPE} are determined by minimizing the χ^2:

$$\chi^2 = \sum_{i=1}^{N_{\rm expt}} \left\{ \sum_{j=1}^{N_i^{\rm cs}} \left[\frac{\sigma_{\rm red}^{\rm exp}(i,j) - \sigma_{\rm red}^{\rm fit}/\eta_i}{\Delta\sigma_{\rm red}^{\rm exp}(i,j)} \right]^2 + \left(\frac{1-\eta_i}{\Delta\eta_i} \right)^2 \right\}$$


Lomon, No TPE correction, χ²/n_dof=368.1/435
Lomon, TPE correction included, χ²/n_dof=315.2/431
Empirical fit, No TPE correction, χ²/n_dof=393.3/435
Empirical fit, TPE correction included, χ²/n_dof=319.5/431
Borkowski
Sill
Bosted
Walker
Andivahis
G^p_M/μ_p G_D
G^p_M = μ_p*(1+a_1*τ)+a_2*τ^2+a_3*τ^3 / 1.+b_1*τ+b_2*τ^2+b_3*τ^3+b_4*τ^4+b_5*τ^5
τ=Q^2/4M_p^2
Q^2 [GeV^2]


TPE correction from Lomon's fit to ff ratio
TPE correction from empirical fit to ff ratio
a(Q^2)
a(Q^2)=p_0+p_1Q^2+p_2Q^4+p_3Q^6
Q^2 [GeV^2]


Lomon, No TPE correction, χ2/ndof=429.8/439
Lomon, TPE correction included, χ2/ndof=377.3/435
Empirical fit, No TPE correction, χ2/ndof=433.7/439
Empirical fit, TPE correction included, χ2/ndof=353.6/435
Borkowski
Sill
Bosted
Walker
Andivahis
$G_M^p/\mu_p G_D$
$G_M^p = \frac{\mu_p*(1+a_1*\tau)}{1.+b_1*\tau+b_2*\tau^2+b_3*\tau^3}$
$\tau=\frac{Q^2}{4M_p^2}$
Q^2 [GeV^2]


TPE correction from Lomon's fit to ff ratio
TPE correction from empirical fit to ff ratio
$a(Q^2)$
$a(Q^2)=p_0+p_1Q^2+p_2Q^4+p_3Q^6$
Q^2 [GeV^2]

Some considerations or questions in our analysis:

The linearity of the Rosenbluth plot is verified experimentally up to 5 GeV^2 <--> Our assumption is TPE correction is linear in all the Q^2 range

Polarization experiments measured the form factor ratios up to 8.5 GeV^2 <--> We fit the ratio to a phenomenological model and extrapolate it to high 30 GeV^2. Extrapolation can cause large uncertainty

How to parametrize TPE? Currently polynomial form is chosen for simplicity. More reasonable function should be used to describe TPE

How to parametrize G_M. Kelly's parametrization has proper behavior at low and high Q^2, but doesn't reflect the physics inside and may not describe the change of form factor at moderate Q^2

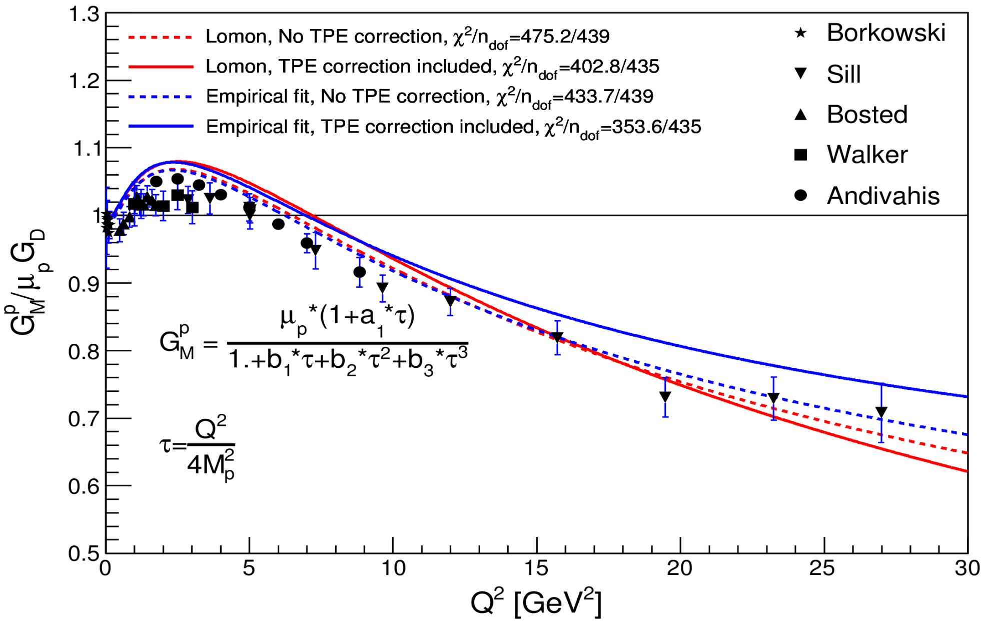
Some considerations or questions in our analysis:

How to use Lomon's parametrization of the form factors?

Currently we just use the latest fit from 2006 paper which does not include GEPIII data and updated GEPII data

In Lomon's original fit, he employed information from both polarization measurements and Rosenbluth measurement (he used the form factor instead of cross section values from Rosenbluth separation experiments). If we use his value of parameters to fit the cross section, is it a loop in logic?

I tried to fit Lomon's parametrization to polarization data alone and apply these parameters to our cross section fit, but the result is not ideal.



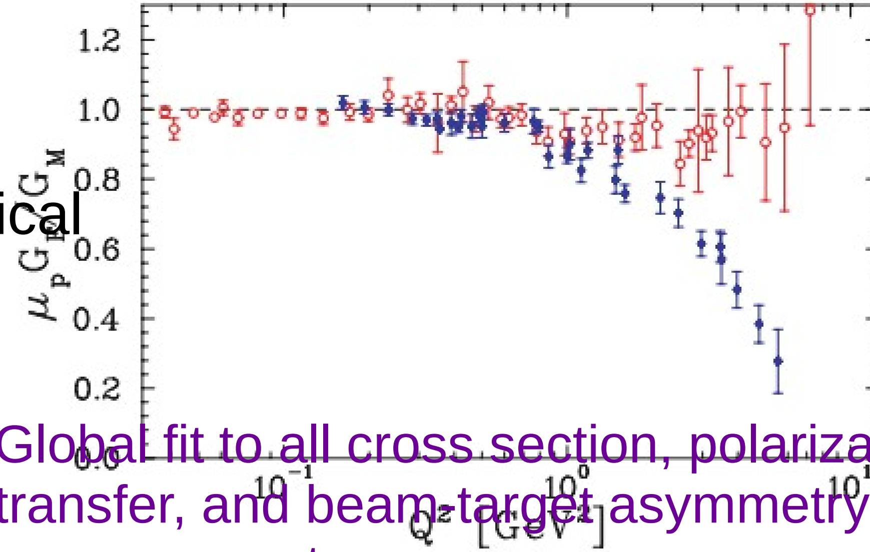
Comparison with previous analysis:

Arrington, Melnitchouk, Tjon *Phys Rev C 76 035205 (2007)*

TPE corrections based on the calculation of BMT (Phys Rev C 72 034612 (2005)) are applied to the raw cross section data

Above Q^2=1GeV^2, an extra phenomenological correction is added

$$\delta^*_{2\gamma} = 0.01[\varepsilon - 1]\frac{\ln Q^2}{\ln 2.2}$$

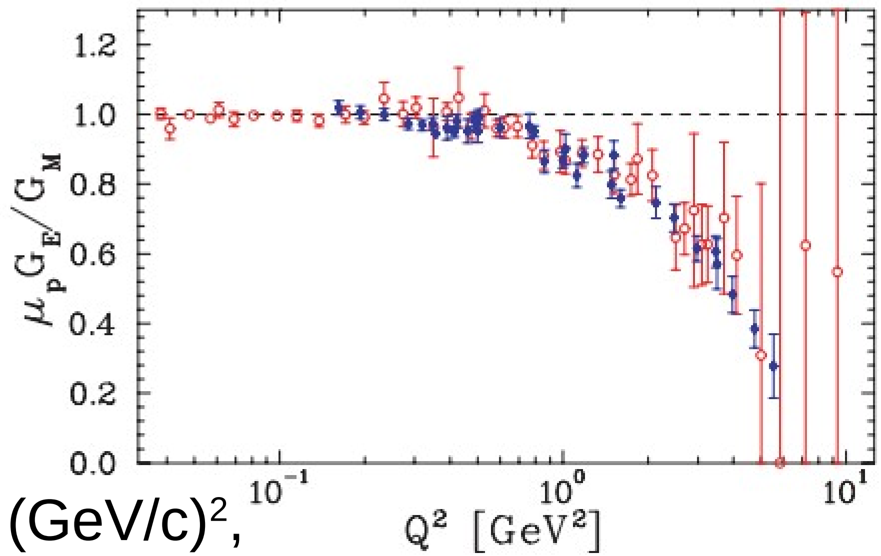


Global fit to all cross section, polarization transfer, and beam-target asymmetry measurements

Additional high-Q^2 "data" points are added at 50, 100, 200, 400 GeV^2 for G_M and 10, 15, 20, 25 GeV^2 for G_E

$$G_M = 0.7 G_D = 0.7[1 + Q^2/(0.71\,\mathrm{GeV}^2)]^{-2}$$

$$G_E = 0$$



G_M and G_E are respectively parametrized using Kelly's fitting function

G_E, G_M and $\mu_p G_E/G_M$ are extracted up to 6 (GeV/c)^2, G_M is also determined for Q^2 up to 30 (GeV/c)^2

Comparison with previous analysis:

Arrington *Phys Rev C 69 022201(R) (2004)*
Phys Rev C 68 034325 (2003)

Parametrized the G_E and G_M separately as the inverse of a polynomial in Q^2

The ratios of the form factors G_E/G_M were fixed for Q^2 values above 6 GeV^2 “to avoid unreasonable behavior in the region where G_E is unconstrained by data”

A combined analysis of cross section measurement and polarization results was made but the resulting form factor ratio is systematically higher than the PT data set

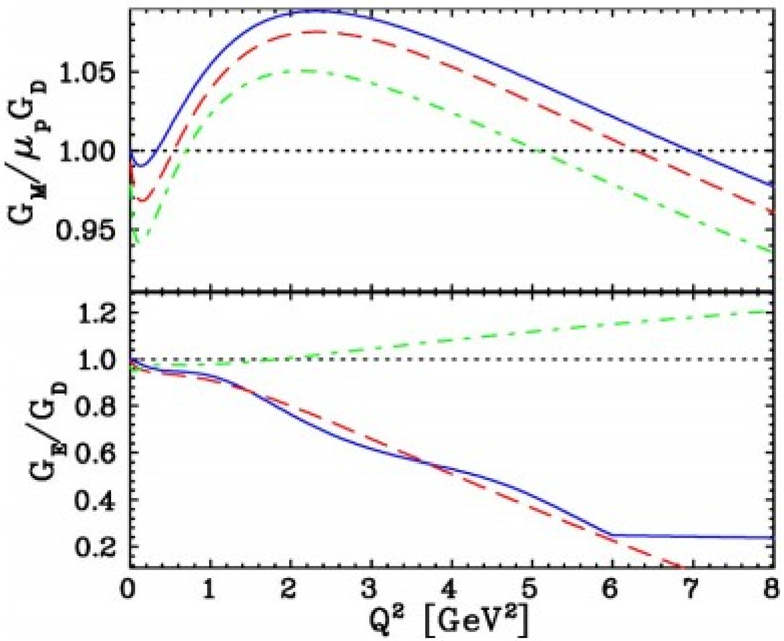


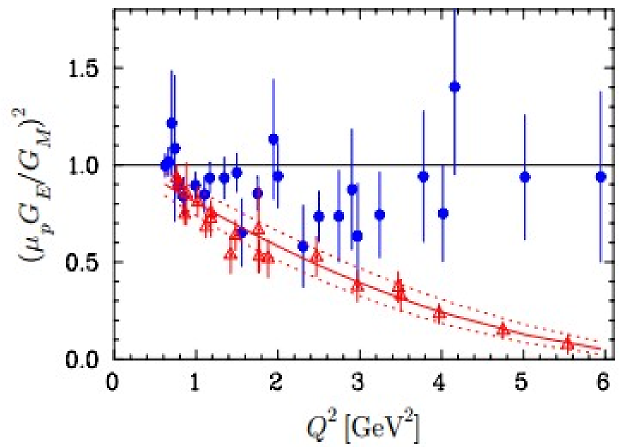
FIG. 3. (Color online) The “polarization form factors” (solid line) for G_E and G_M, relative to the dipole form. The dot-dashed line is the previous fit to Rosenbluth extracted form factors from Ref. [20], and dashed curve is the fit to G_M from Ref. [19], with the form factor ratio constrained to give $\mu_p G_E/G_M = 1-0.13(Q^2-0.04)$.

A 6% linear correction was applied to cross section data to reconcile the discrepancy between the results of the LT and PT techniques

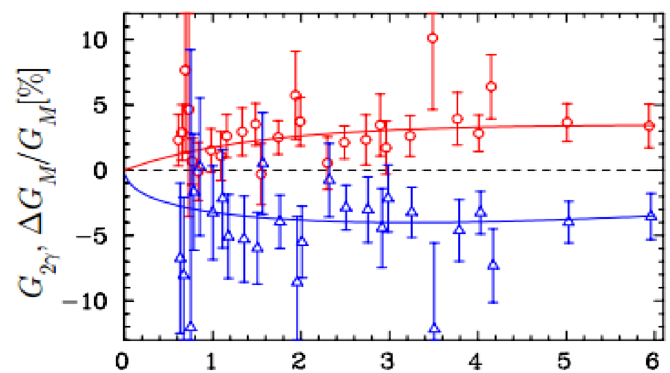
Comparison with previous analysis:

Arrington *Phys Rev C 71 015202 (2005)*

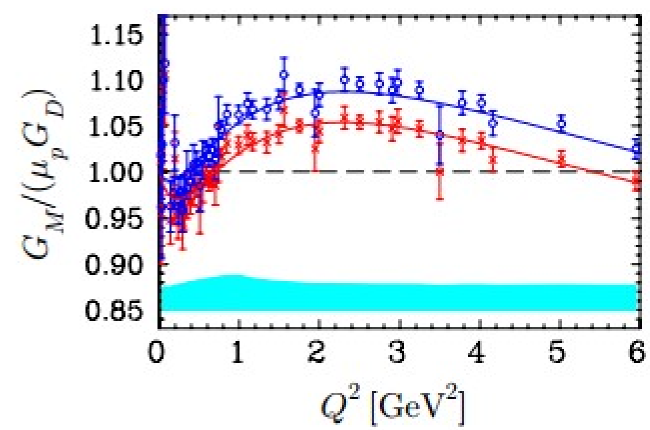
This paper employed Vandehaeghen's formalism to extract TPE correction and form factors with some assumptions



The difference between the ratios measured by two techniques were used to extract the $Y_{2\gamma}$ term (The PT ff ratio is described by a parametrization)



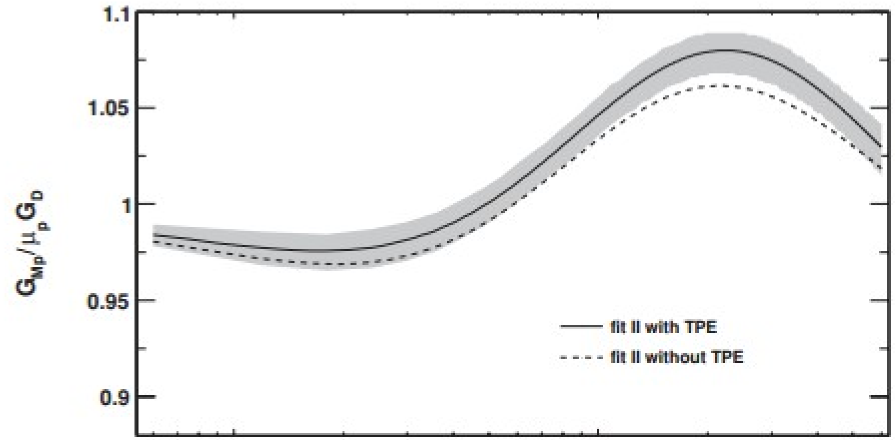
By requiring the TPE contribution to the cross section from $Y_{2\gamma}$ be cancelled by the contribution from ΔG_M, ΔG_M can be extracted



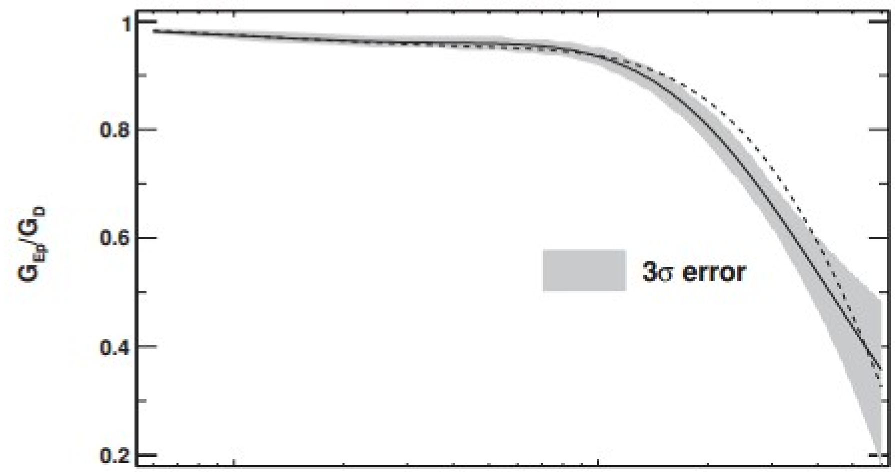
Comparison with previous analysis:

Alberico *et al*. *Phys Rev C 79 065204 (2009)*

Two fits were performed: one parametrized G_M while keep G_E consrained by a linear description of form factor ratios, the other parametrize both G_E and G_M



Kelly's prescription for form factors were used



A TPE correction was included with the parametrization:

$$F(Q^2, y) = \alpha G_D^2(Q^2)y +_D^2 (Q^2)y^3$$

$$y = \sqrt{\frac{1-\varepsilon}{1+\varepsilon}}$$

An error analysis was performed for the two fits

Only considered data below 6 GeV^2

Comparison with previous analysis:

Qattan, Alsaad, and Arrington *Phys Rev C 84 054317 (2011)*

Two parametrizations of TPE correction were used:

$$\sigma_R = G_{Mp}^2(1+\frac{\epsilon}{\tau}R^2)+\varepsilon f(Q^2)$$

$$\sigma_R = G_{Mp}^2(1+\frac{\varepsilon}{\tau}R^2)+2a(1-\varepsilon)G_{Mp}^2$$

The ratios of form factors were constrained by a linear fit to the PT results

The fit was performed to fixed-Q^2 data set, i.e., the relevant function $f(Q^2)$ and $a(Q^2)$ is just a constant in each fit

The data points considered are all below 6 GeV2