GamPP - Radiator In/Out run time division

We'd like to calculate the ratio between T_i/T_o that minimizes the relative statistical error: $\frac{\Delta N_\gamma}{N_\gamma}$

$R_i = R_e + R_\gamma$
$R_o = R_e$
$N_i = R_i(1-\epsilon)T$
$N_o = R_o\epsilon T$

$\Delta R_i = \frac{R_i}{\sqrt{N_i}}$

$\Delta R_o = \frac{R_o}{\sqrt{N_o}}$

$\Delta N_\gamma = R_\gamma(1-\epsilon)T$

Minimize:

$\frac{\Delta N_\gamma}{N_\gamma} = \frac{\Delta R_\gamma}{R_\gamma} = \frac{\Delta(R_i - R_e)}{R_i - R_e} = \frac{\sqrt{\Delta R_i^2 + \Delta R_e^2}}{R_i - R_e} = \frac{\sqrt{\frac{R_i^2}{N_i} + \frac{R_o^2}{N_o}}}{R_i - R_o} = \frac{\sqrt{\frac{R_i}{(1-\epsilon)T} + \frac{R_o}{\epsilon T}}}{R_i - R_o}$

$\frac{d(\frac{\Delta N_\gamma}{N_\gamma})}{d\epsilon} \sim \frac{\frac{R_i}{(1-\epsilon)^2} - \frac{R_o}{\epsilon^2}}{\sqrt{\frac{R_i}{(1-\epsilon)} + \frac{R_o}{\epsilon}}} = 0$

$\Rightarrow \frac{R_o}{\epsilon^2} = \frac{R_i}{(1-\epsilon)^2} \Rightarrow \epsilon = \frac{\sqrt{\frac{R_o}{R_i}}}{1+\sqrt{\frac{R_o}{R_i}}}$