Measurement of the Coulomb quadrupole amplitude in the $\gamma^{*}p \rightarrow \Delta(1232)$ reaction in the low momentum transfer region

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Introduction

- Take measurements of the $p(e,e'p)\pi^0$ (pion electroproduction) reaction
  - At energies in the area of the Δ resonance
  - With low momentum transfer (Q^2) between the electron and proton
  - To better understand the Coulomb quadrupole transition amplitude behavior in this region and how it affects nucleon deformation

Overview

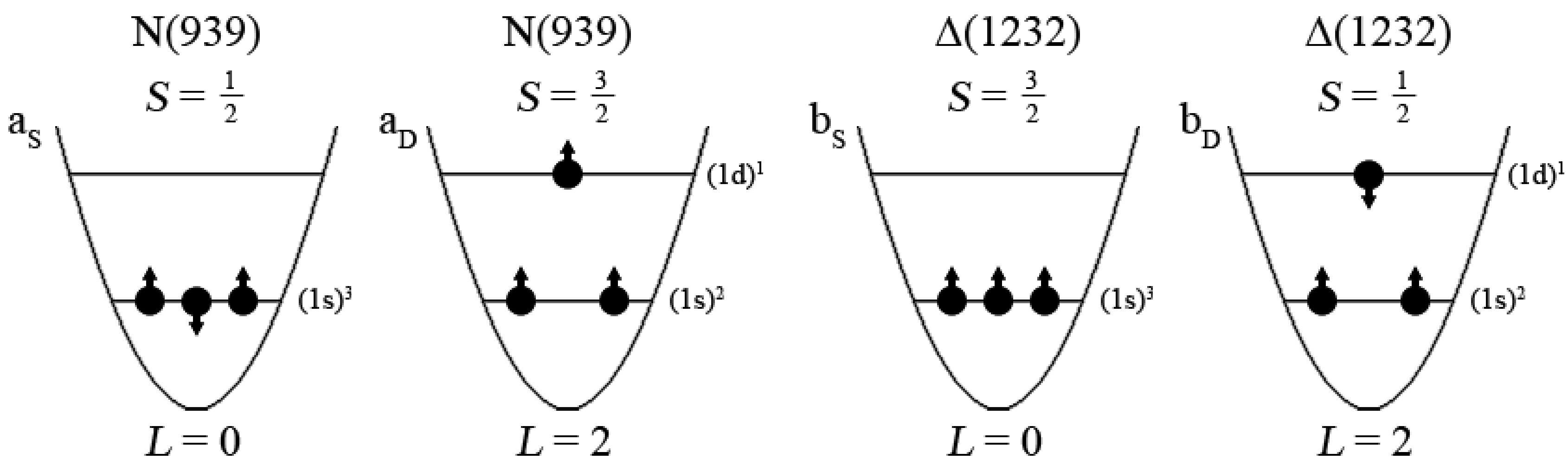
- Motivation: Constituent Quark Model
- Kinematics and Transition Amplitudes
- Response Functions and Amplitude Extraction
- World Data and Models
- Experimental Setup
- Conclusions

Motivation: Constituent Quark Model

- Three "heavy" quarks in nucleon
  - Each quark has mass $^1\!/_3$ of nucleon
  - Each quark has intrinsic spin angular momentum of $^1\!/_2$
  - Combines to give $S = {}^1\!/_2$ or $S = {}^3\!/_2$
    - If $L = 0$
      - $S = {}^1\!/_2$, $J^\pi = {}^1\!/_2{}^+ \rightarrow$ corresponds to $N(939)$
      - $S = {}^3\!/_2$, $J^\pi = {}^3\!/_2{}^+ \rightarrow$ corresponds to $\Delta(1232)$
    - If $L = 2$
      - $\mathrm{S} = {}^1\!/_2$, $J^\pi = {}^3\!/_2{}^+ \rightarrow$ corresponds to $\Delta(1232)$
      - $\mathrm{S} = {}^3\!/_2$, $J^\pi = {}^1\!/_2{}^+ \rightarrow$ corresponds to $N(939)$

Motivation: Constituent Quark Model

- Wave functions created
  - $\left|N(939)\right\rangle = a_S\left|\left(S=\tfrac{1}{2}, L=0\right)J^\pi = \tfrac{1}{2}^+\right\rangle + a_D\left|\left(S=\tfrac{3}{2}, L=2\right)J^\pi = \tfrac{1}{2}^+\right\rangle$
  - $\left|\Delta(1232)\right\rangle = b_S\left|\left(S=\tfrac{3}{2}, L=0\right)J^\pi = \tfrac{3}{2}^+\right\rangle + b_D\left|\left(S=\tfrac{1}{2}, L=2\right)J^\pi = \tfrac{3}{2}^+\right\rangle$

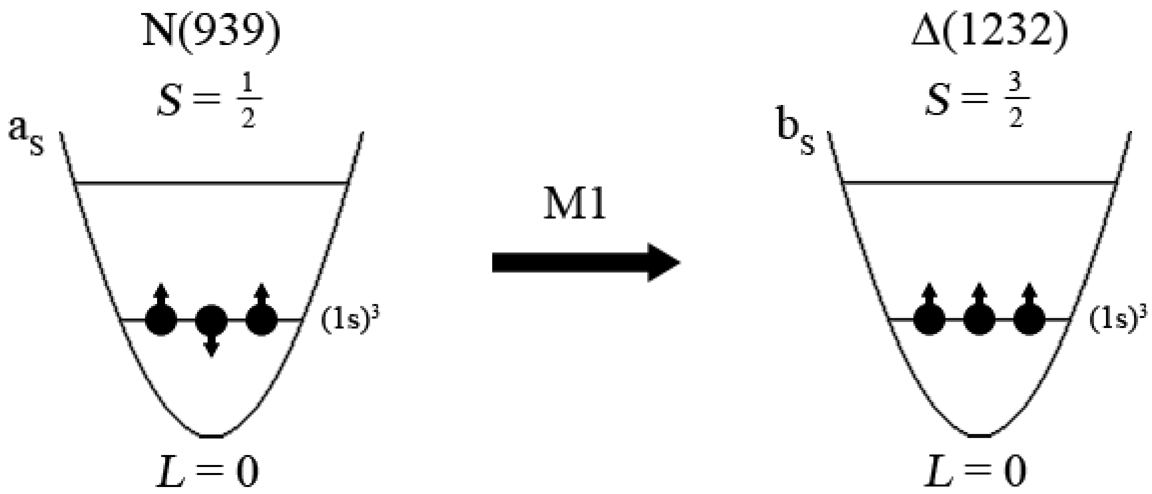


Motivation: Constituent Quark Model

- Measure non-spherical components by measuring quadrupole moment
  - Cannot measure quadrupole moment directly
  - Measure quadrupole moment of $N \rightarrow \Delta$ transition
- Three electromagnetic transitions
  - $M1$ – magnetic dipole
  - $E2$ – electric quadrupole
  - $C2$ – Coulomb/scalar quadrupole

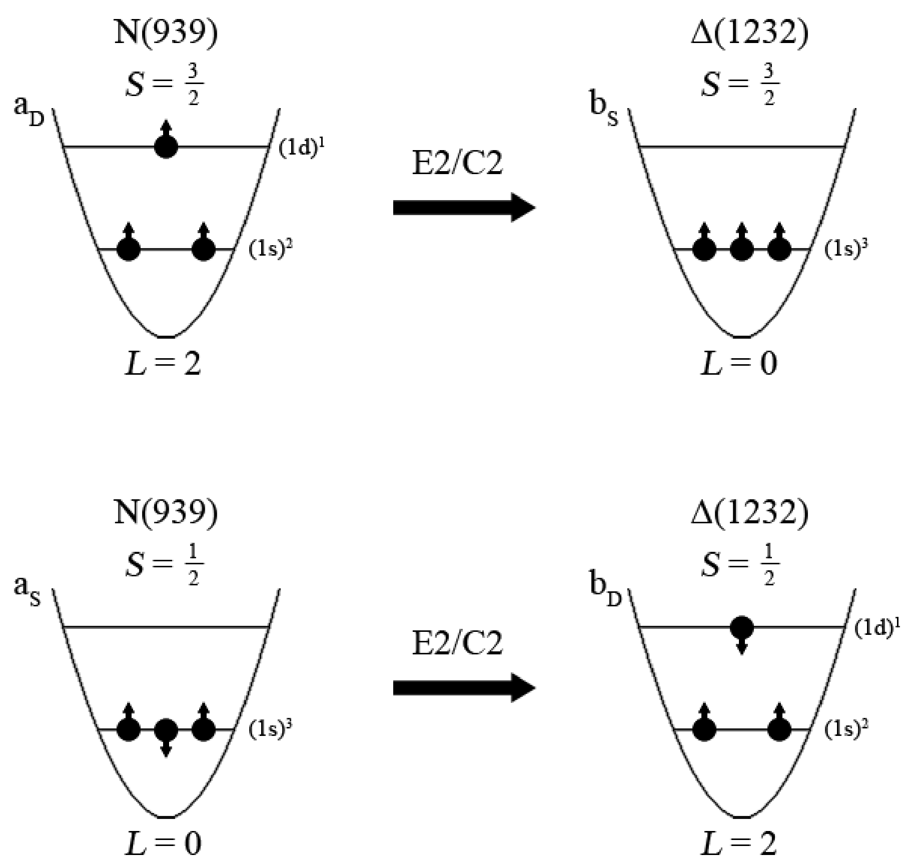
Motivation: Constituent Quark Model

- Magnetic Dipole
  - Spin-flip
  - Dominant



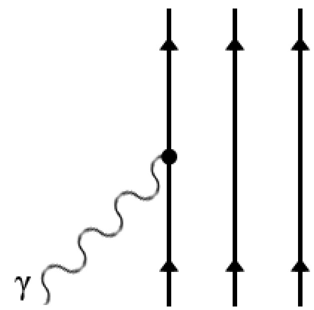
Motivation: Constituent Quark Model

- Electric quadrupole
- Coulomb quadrupole
  - Only with virtual photons

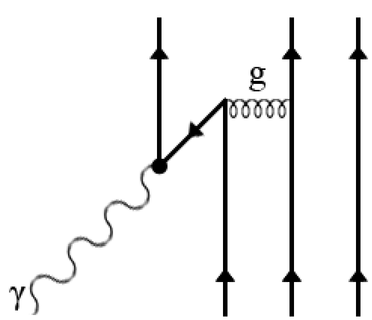


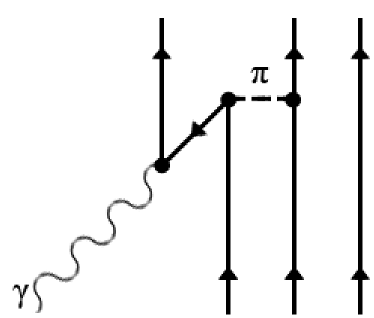
Motivation: Constituent Quark Model

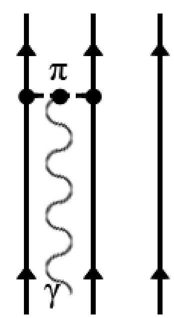
- One-body interactions
  - $$\hat{Q}_{[1]} = \sqrt{\frac{16\pi}{5}\sum_{i=1}^{3} e_i r_i^2 Y_0^2(\vec{r}_i)} = \sum_i e_i \left(3z_i^2 - r_i^2\right)$$



- Two-body interactions
  - $$\hat{Q}_{[2]} = B\sum_{i\neq j=1}^{3} e_i \left(3\sigma_{iz}\sigma_{jz} - \vec{\sigma}_i \cdot \vec{\sigma}_j\right)$$

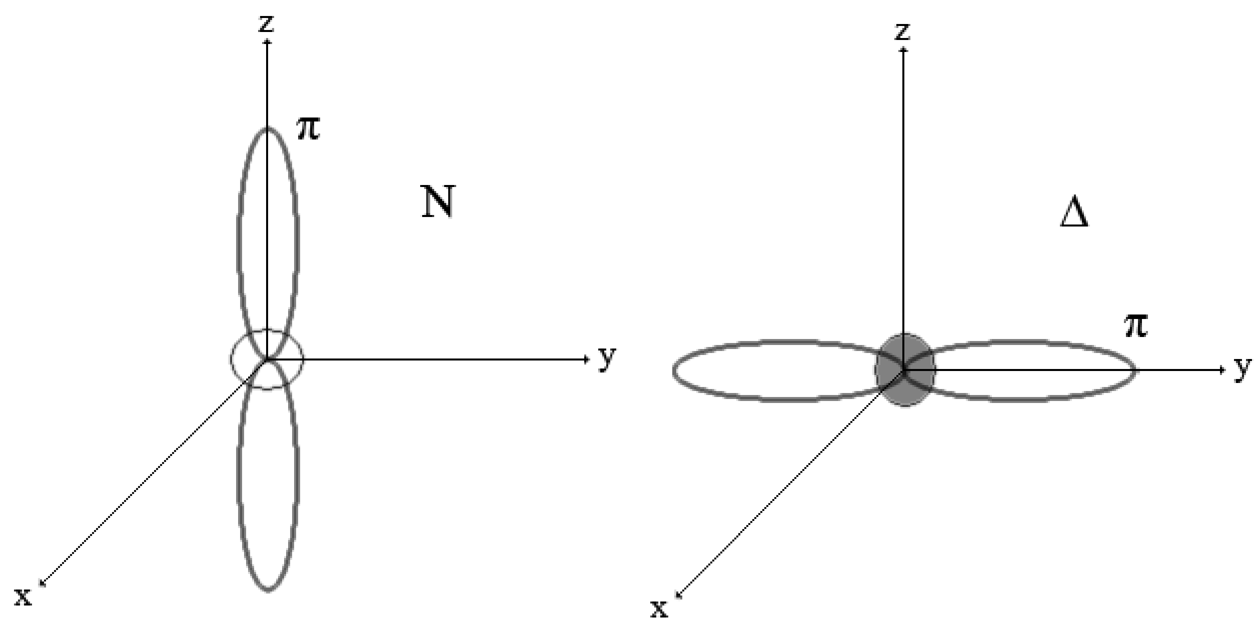






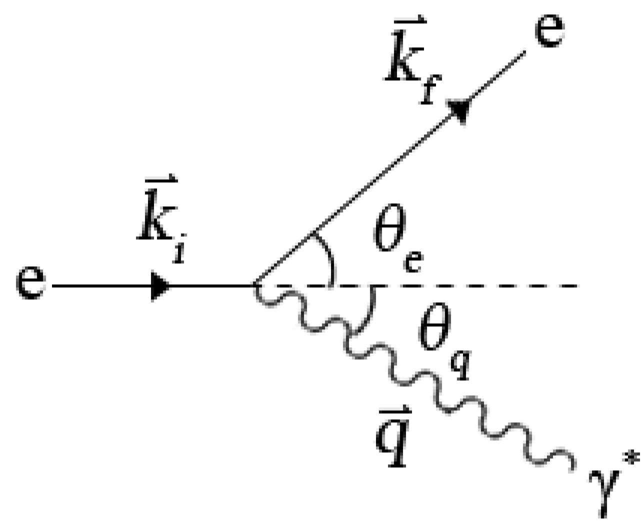
Motivation: Constituent Quark Model

- Pion Cloud



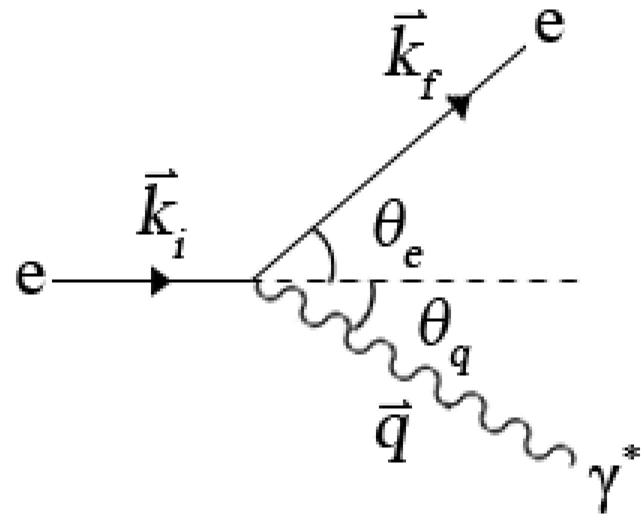
Kinematics – Electronic Vertex

- Incoming electron
  - Energy E
  - Momentum $\vec{k}_i$
- Scattered electron
  - Energy E'
  - Momentum $\vec{k}_f$
  - Angle θ_e
- Virtual photon
  - Energy ω
  - Momentum $\vec{q}$
  - Angle θ_q



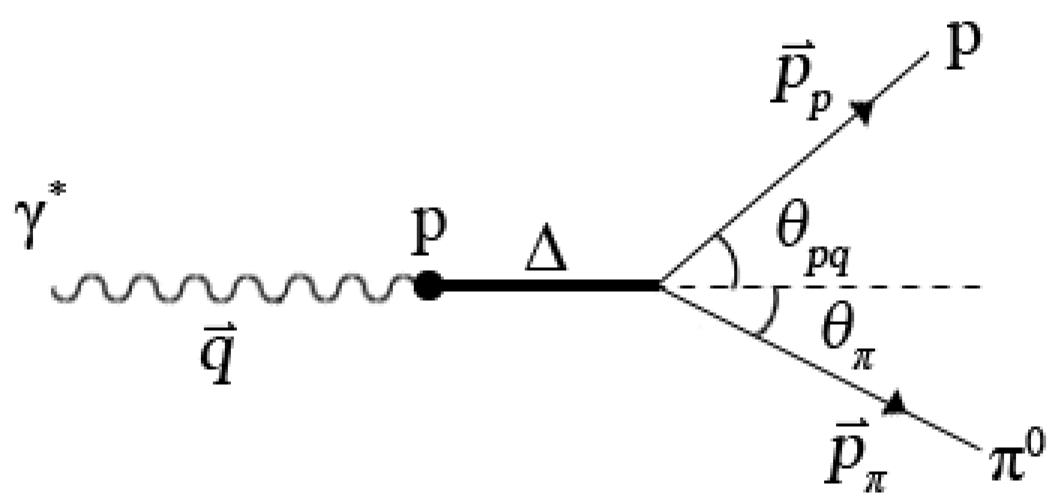
Kinematics – Electronic Vertex

- Momentum transfer, Q^2
  - $Q^2 = -q^2 = -(\omega^2 - \vec{q}^{\,2})$
  - $Q^2 \approx 4EE' \sin^2 \frac{\theta_e}{2}$



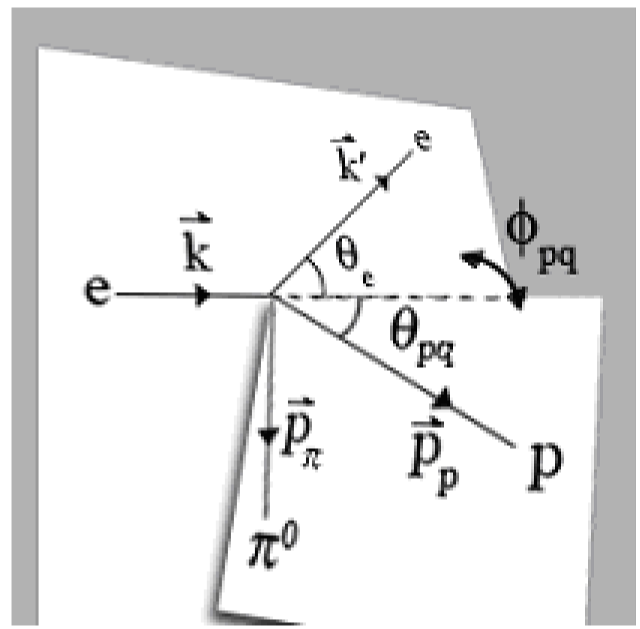
Kinematics – Hadronic Vertex

- Recoil proton
  - Energy E_p
  - Momentum $\vec{p}_p$
  - Angle θ_{pq}
- Recoil pion
  - Energy E_π
  - Momentum $\vec{p}_\pi$
  - Angle θ_π
  - Not detected



Kinematics – Planes

- Scattering plane – $\vec{k}_i$ and $\vec{k}_f$
- Recoil plane – $\vec{p}_p$ and $\vec{p}_\pi$
- Azimuthal angle – ϕ_{pq}



Multipole Amplitudes

- General form of πN Multipoles: $X^{I}_{\ell\pm}$
  - X – type of excitation (M, E, S)
  - I – isospin of excited intermediate state
  - $\ell\pm$ – $J_\Delta = \ell \pm 1/2$
- Magnetic dipole – $M1$ / $M^{3/2}_{1+}$
- Electric quadrupole – $E2$ / $E^{3/2}_{1+}$
- Coulomb quadrupole – $C2$ / $S^{3/2}_{1+}$
  - $\omega S_{1+} = |\vec{q}|L_{1+}$

Multipole Amplitudes

γN-Multipoles	Initial State		Excited State			Final State		πN-Multipoles
C, E, M	L_γ^π	s_N^π	J_R^π	$N^* I_{2I2J} \Delta$		s_N^π	I_π^π	$L_{\ell\pm}, E_{\ell\pm}, M_{\ell\pm}$
$C0$	0^+	$\frac{1}{2}^+$	$\frac{1}{2}^+$	P_{11}	P_{31}	$\frac{1}{2}^+$	1^+	L_{1-}
$C1, E1$	1^-	$\frac{1}{2}^+$	$\frac{1}{2}^-$	S_{11}	S_{31}	$\frac{1}{2}^+$	0^-	L_{0+}, E_{0+}
		$\frac{1}{2}^+$	$\frac{3}{2}^-$	D_{13}	D_{33}	$\frac{1}{2}^+$	2^-	L_{2-}, E_{2-}
$M1$	1^+	$\frac{1}{2}^+$	$\frac{1}{2}^+$	P_{11}	P_{31}	$\frac{1}{2}^+$	1^+	M_{1-}
		$\frac{1}{2}^+$	$\frac{3}{2}^+$	P_{13}	P_{33}	$\frac{1}{2}^+$	1^+	M_{1+}
$C2, E2$	2^+	$\frac{1}{2}^+$	$\frac{3}{2}^+$	P_{13}	P_{33}	$\frac{1}{2}^+$	1^+	L_{1+}, E_{1+}
		$\frac{1}{2}^+$	$\frac{5}{2}^+$	F_{15}	F_{35}	$\frac{1}{2}^+$	3^+	L_{3-}, E_{3-}
$M2$	2^-	$\frac{1}{2}^+$	$\frac{3}{2}^-$	D_{13}	D_{33}	$\frac{1}{2}^+$	2^-	M_{2-}
		$\frac{1}{2}^+$	$\frac{5}{2}^-$	D_{15}	D_{35}	$\frac{1}{2}^+$	2^-	M_{2+}

Multipole Amplitudes

- E_{1+} and S_{1+} at same magnitude as background amplitudes
- Measure ratio to dominant M_{1+}
- $\text{EMR} = R_{EM}^{3/2} = \mathrm{Re}\left(\dfrac{E_{1+}^{3/2}}{M_{1+}^{3/2}}\right) = \dfrac{\mathrm{Re}\left(E_{1+}^{*} M_{1+}\right)}{\left|M_{1+}\right|^2}$
- $\text{CMR} = R_{CM}^{3/2} = \mathrm{Re}\left(\dfrac{S_{1+}^{3/2}}{M_{1+}^{3/2}}\right) = \dfrac{\mathrm{Re}\left(S_{1+}^{*} M_{1+}\right)}{\left|M_{1+}\right|^2}$

Response Functions

- Unpolarized cross section made up of four independent partial cross sections

$$\frac{d^5\sigma}{dk_f d\Omega_e d\Omega^*} = \Gamma_\gamma \frac{k}{q_0}\left(\sigma_L + \sigma_T + \sigma_{LT} + \sigma_{TT}\right)$$

$$\Gamma_\gamma = \frac{\alpha}{2\pi^2}\frac{k_f}{k_i}\frac{k_\gamma}{Q^2}\frac{1}{1-\varepsilon} \qquad \varepsilon_S = \frac{Q^2}{q^2}\varepsilon \qquad k = \sqrt{\frac{W^2 + m_\pi^2 - m_p^2}{4W^2} - m_\pi^2}$$

$$k_\gamma = \frac{W^2 - m_p^2}{2m_p} \qquad \varepsilon = \left(1 + 2\frac{q^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1} \qquad q_0 = \frac{W^2 - m_p^2}{2W}$$

Response Functions

- Unpolarized cross section made up of four independent partial cross sections

$$\frac{d^5\sigma}{dk_f d\Omega_e d\Omega^*} = \Gamma_\gamma \frac{k}{q_0}\left(\sigma_L + \sigma_T + \sigma_{LT} + \sigma_{TT}\right)$$

$$\sigma_L = \varepsilon_S R_L \qquad \sigma_{LT} = \sqrt{2\varepsilon_S\left(1+\varepsilon\right)} R_{LT} \sin\theta^*_{pq} \cos\phi^*_{pq}$$

$$\sigma_T = R_T \qquad \sigma_{TT} = \varepsilon R_{TT} \sin^2\theta^*_{pq} \cos 2\phi^*_{pq}$$

Response Functions

- $R_L\sqrt{\frac{\omega_{cm}^2}{Q^2}} = |L_{0+}|^2 + 4|L_{1+}|^2 + |L_{1-}|^2 - 4\,\mathrm{Re}\{L_{1+}^* L_{1-}\} + 2\cos\theta\,\mathrm{Re}\{L_{0+}^*(4L_{1+} + L_{1-})\} + 12\cos^2\theta\left(|L_{1+}|^2 + \mathrm{Re}\{L_{1+}^* L_{1-}\}\right)$
- $$R_T = |E_{0+}|^2 + \tfrac{1}{2}|2M_{1+} + M_{1-}|^2 + \tfrac{1}{2}|3E_{1+} - M_{1+} + M_{1-}|^2 + 2\cos\theta\,\mathrm{Re}\{E_{0+}^*(3E_{1+} + M_{1+} - M_{1-})\}$$
  $$+\cos^2\theta\left(|3E_{1+} + M_{1+} - M_{1-}|^2 - \tfrac{1}{2}|2M_{1+} + M_{1-}|^2 - \tfrac{1}{2}|3E_{1+} - M_{1+} - M_{1-}|^2\right)$$
- $R_{LT}\sqrt{\frac{\omega_{cm}^2}{Q^2}} = -\sin\theta\,\mathrm{Re}\{L_{0+}^*(3E_{1+} - M_{1+} + M_{1-}) - (2L_{1+}^* - L_{1-}^*)E_{0+} + 6\cos\theta\left(L_{1+}^*(E_{1+} - M_{1+} + M_{1-}) + L_{1-}^* E_{1+}\right)\}$
- $R_{TT} = 3\sin^2\theta\left(\tfrac{3}{2}|E_{1+}|^2 - \tfrac{1}{2}|M_{1+}|^2 - \mathrm{Re}\{E_{1+}^*(M_{1+} - M_{1-}) + M_{1+}^* M_{1-}\}\right)$

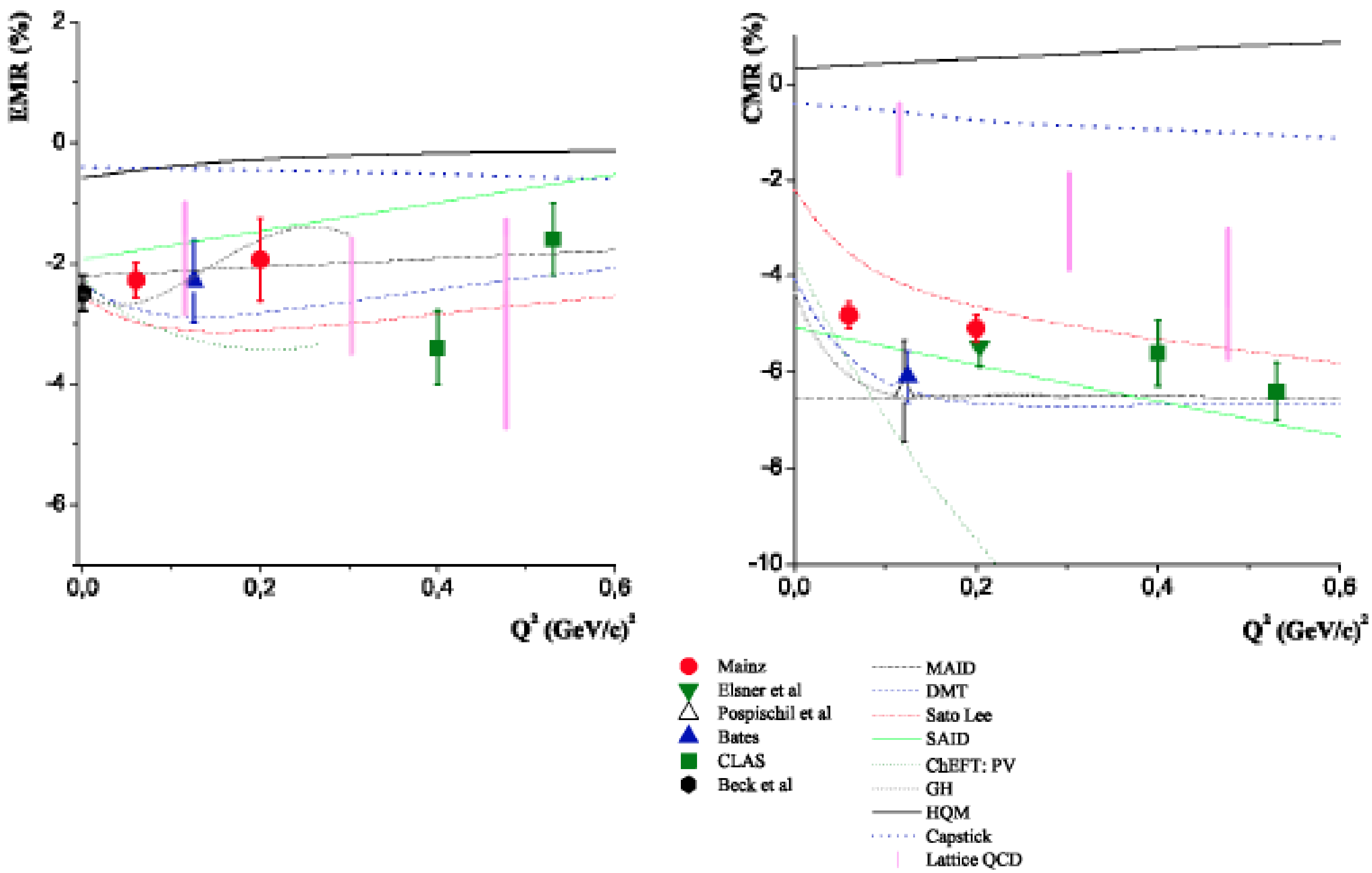
Response Functions

- Truncated Multipole Expansion
  - $R_L \approx 0$
  - $R_T \approx \frac{5}{2}\left|M_{1+}\right|^2 + 2\cos\theta \operatorname{Re}\left\{E_{0+}^* M_{1+}\right\} - \frac{3}{2}\cos^2\theta \left|M_{1+}\right|^2$
  - $R_{LT} \approx \sin\theta \operatorname{Re}\left\{L_{0+}^* M_{1+}\right\} - 6\cos\theta \left(L_{1+}^* M_{1+}\right)$
  - $R_{TT} \approx -3\sin^2\theta \left(\frac{1}{2}\left|M_{1+}\right|^2 + \operatorname{Re}\left\{E_{1+}^* M_{1+} + M_{1+}^* M_{1-}\right\}\right)$
- Model Dependent Extraction
  - Fit theoretical model to existing data
  - Insert model values for background amplitudes

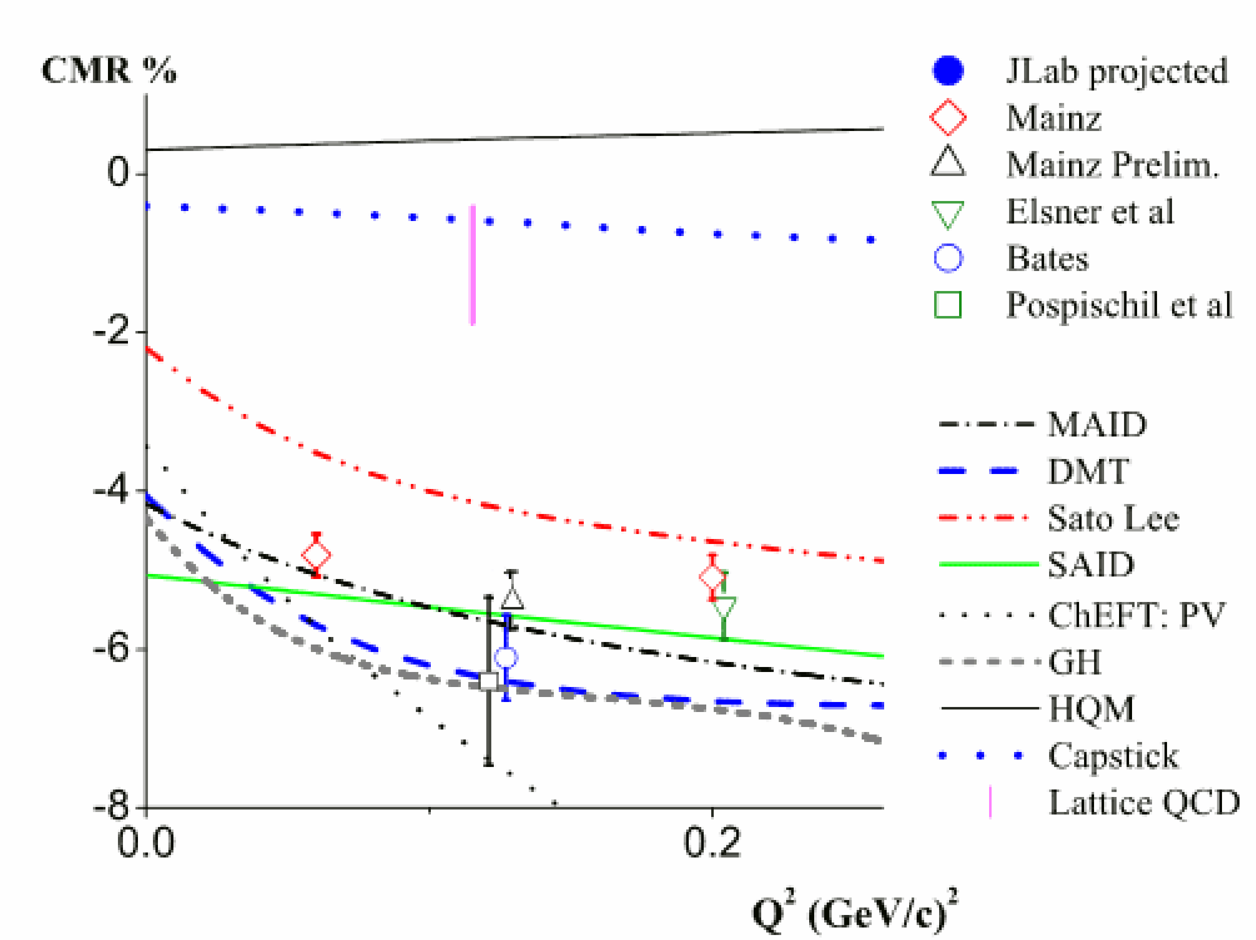
World Data and Models

- Models
  - MAID
  - SAID
  - DMT
  - Sato-Lee
  - Chiral EFT
  - Lattice QCD
- $p(e,e'p)\pi^0$ experiments
  - CEA – 1969
  - DESY – 1970-1972
  - NINA – 1971
  - ELSA – 1997
  - MIT-Bates – 2000
  - MAMI – 2001
  - CLAS – 2002
  - MAMI – 2005-2006

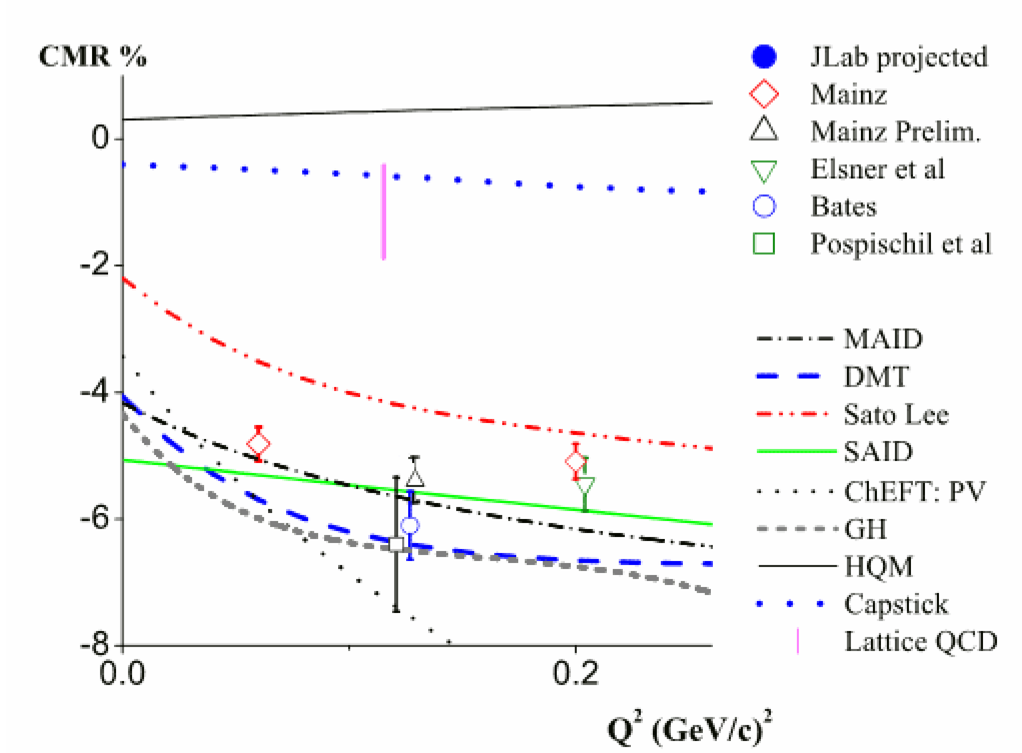
World Data and Models



World Data and Models

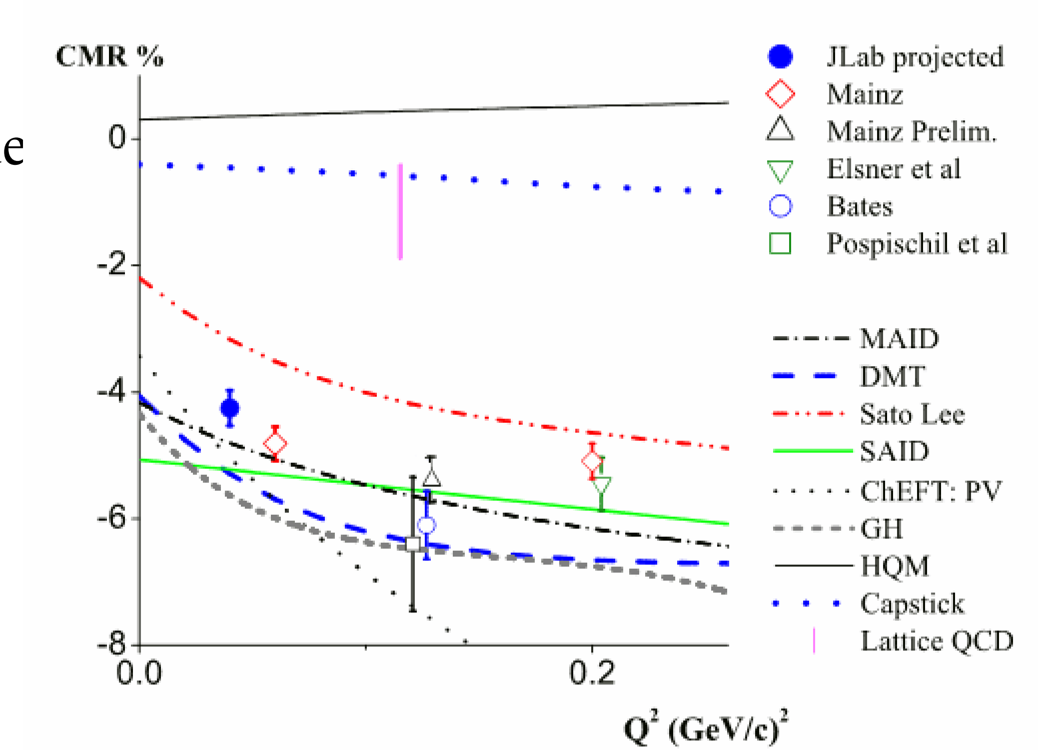


World Data and Models



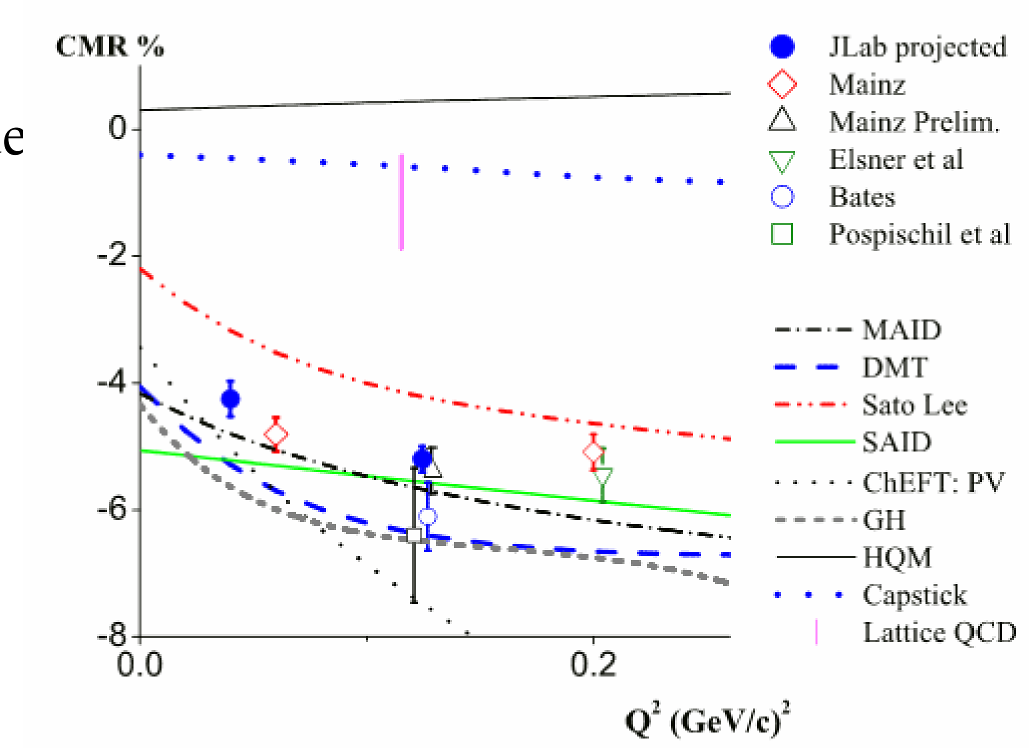
World Data and Models

- $Q^2 = 0.040\ (\text{GeV}/c)^2$
  - New lowest CMR value
  - $\theta_e = 12.5°$



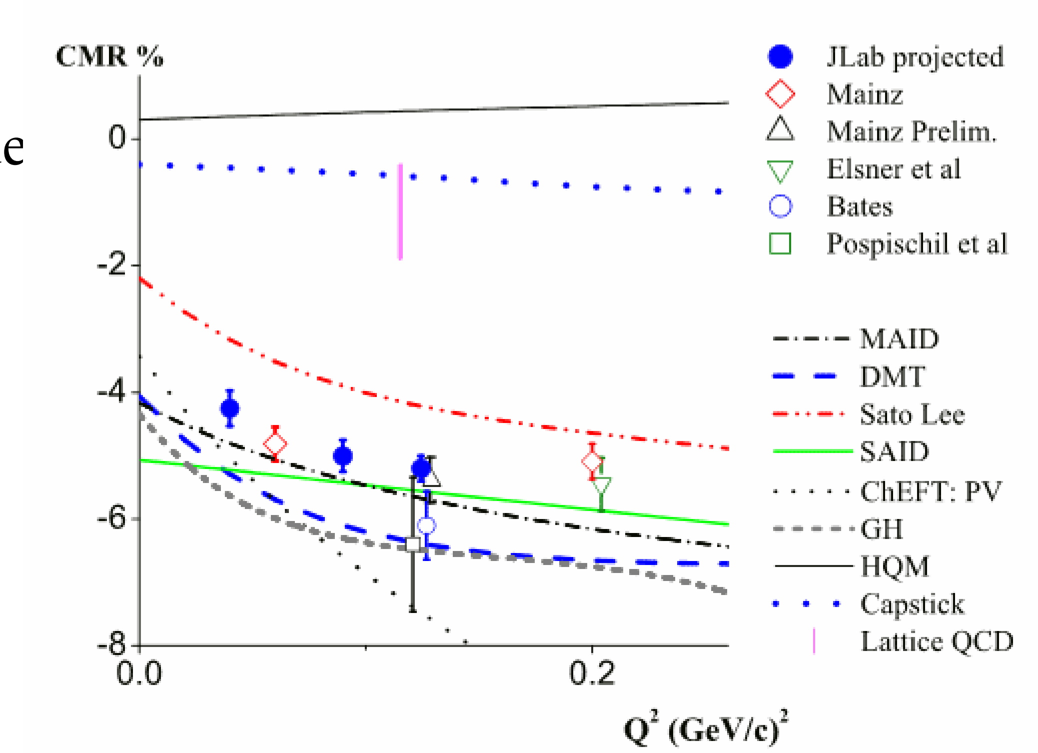
World Data and Models

- $Q^2 = 0.040\ (\mathrm{GeV}/c)^2$
  - New lowest CMR value
  - $\theta_e = 12.5°$
- $Q^2 = 0.125\ (\mathrm{GeV}/c)^2$
  - Validate previous measurements



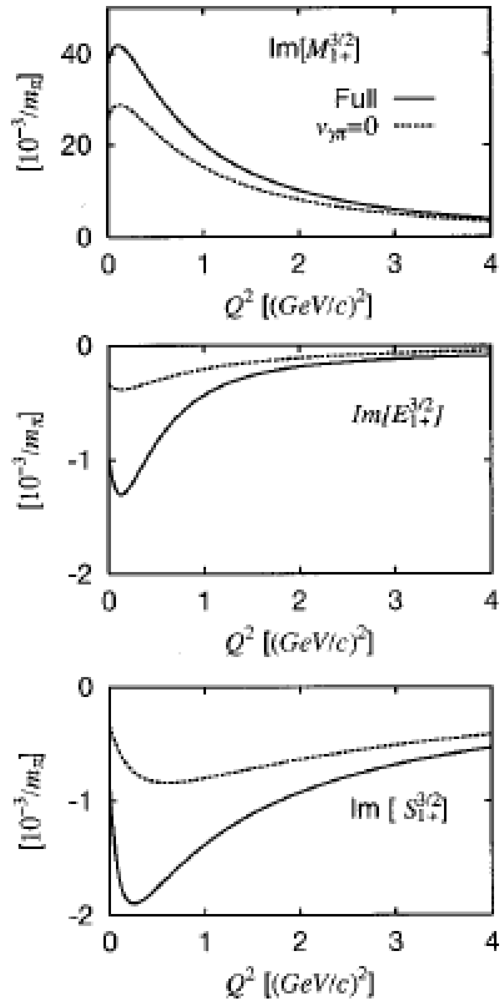
World Data and Models

- $Q^2 = 0.040\ (\mathrm{GeV}/c)^2$
  - New lowest CMR value
  - $\theta_e = 12.5°$
- $Q^2 = 0.125\ (\mathrm{GeV}/c)^2$
  - Validate previous measurements
- $Q^2 = 0.090\ (\mathrm{GeV}/c)^2$
  - Bridge previous measurements



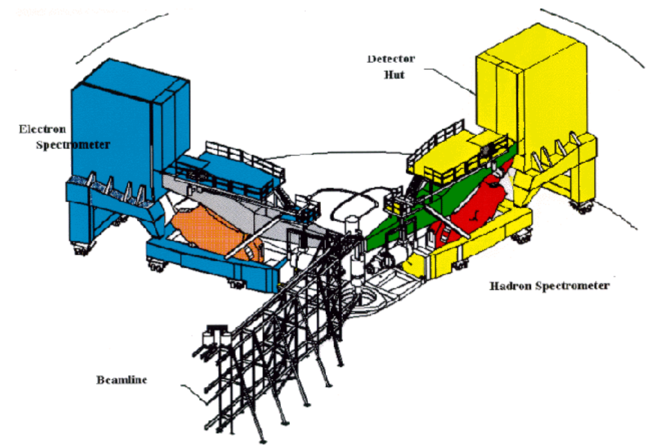
World Data and Models

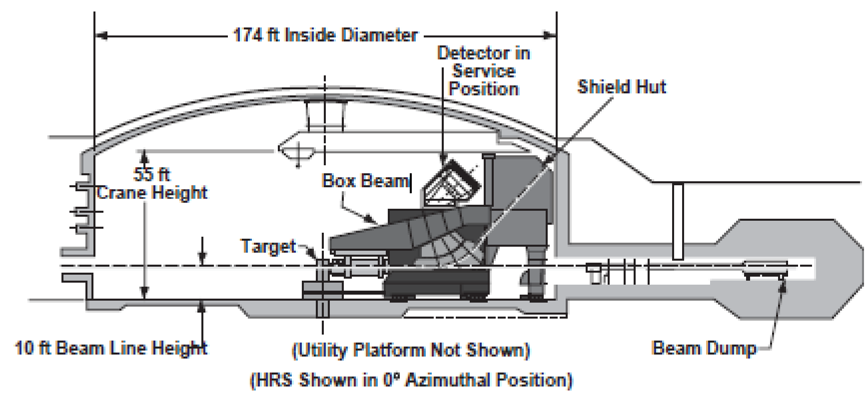
- Sato and Lee
  - Suggest separating nucleon into quark core and pion cloud
  - “bare” quark core links to lattice QCD
  - “full” nucleon links to experimental data



The Experiment

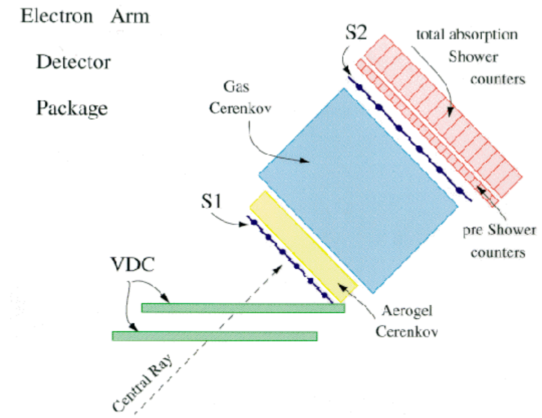
- Jefferson Lab, Hall A
- April 3rd – April 8th, 2011
- 1115 MeV, 75μA e^- beam
- 6 cm LH_2 target
- Two high resolution spectrometers
  - HRSe and HRSh

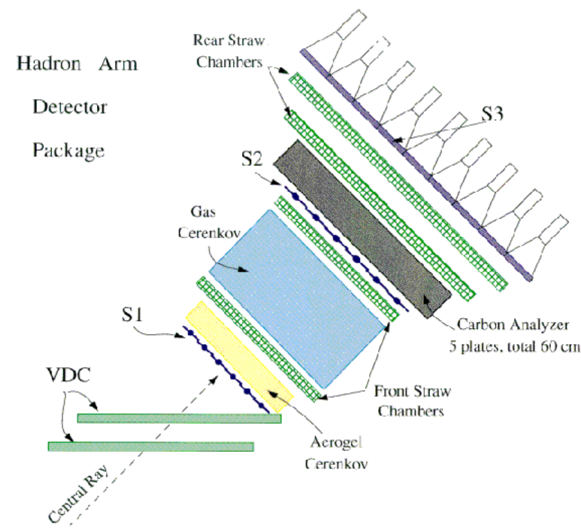




High Resolution Spectrometers

- Vertical drift chambers
  - Particle tracking
- Scintillators
  - Timing information
  - Triggering DAQ
- Cerenkov detectors
  - Aerogel and gas
  - Particle identification
- Lead glass showers
  - Particle identification





Settings

Q^2 (GeV/c)^2	W (MeV)	θ^*_{pq} °	θ_e °	P'_e (MeV/c)	θ_p °	P'_p (MeV/c)	Time (hrs)
0.040	1221	0	12.52	767.99	24.50	547.54	1.5
0.040	1221	30	12.52	767.99	12.52	528.12	2
0.040	1221	30	12.52	767.99	36.48	528.12	3.5
0.040	1260	0	12.96	716.42	21.08	614.44	1.5
0.090	1230	0	19.14	729.96	29.37	627.91	1.5
0.090	1230	40	19.14	729.96	14.99	589.08	3
0.090	1230	40	19.14	729.96	43.74	589.08	4.5
0.125	1232	0	22.94	708.69	30.86	672.56	3.5
0.125	1232	30	22.94	708.69	20.68	649.23	7
0.125	1232	30	22.94	708.69	41.03	649.23	7
0.125	1232	55	22.94	708.69	12.52	596.43	3.5
0.125	1232	55	22.94	708.69	49.19	596.43	3.5
0.125	1170	0	21.74	788.05	37.31	575.57	3
0.125	1200	0	22.29	750.16	34.06	622.63	2
Configuration changes							17
Calibrations							8
				Total:			72

Conclusion

- Important step forward in understanding nucleon's internal structure
- Help bridge and validate experimental world data
- Help theoretical models better understand
  - role of pion cloud in nucleon deformation
  - role of QCD in low momentum transfer region