Date: 21 September 2006

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Re: Request to measure radii with Li,B(e,e')

Summary: We request one week (on the floor) of 362 MeV beam in parallel with G0 to measure the Li and B "radii".

Motivation: Bob Wiringa recently alerted us to a need for better measurements of the radii of Li and B isotopes. While other light nuclei have radius measurements good to about 0.01 fm, including He, Be, and C, Li and B are only known to about 0.05 fm. Improved lithium measurements will give a solid reference point for the charge radius differences measured by atomic spectroscopy, and thus will be important for pinning down absolute radii for all the unstable lithium isotopes too. Rocco Schiavilla is also very interested in the measurements; Rocco and Bob have committed to providing up to date nuclear structure and scattering calculations for the isotopes measured, so we will have a direct comparison of the full range of measured form factors with theory.

Plan: Similar to our recent E05-004 measurements, we will cover a range of Q similar to that used for previous radii extractions on these light nuclei, but with improved uncertainties. The measurements will require about 1 week (floor time) of beam in parallel with G0 during the 362 MeV winter run. We will repeat the procedures of our recently completed E05-004 run. By taking the angular distribution with one spectrometer, while using the second spectrometer at a fixed angle as a luminosity monitor, we will ensure precise relative cross sections. We will use four targets: carbon, lithium, boron, and tantalum targets. The tantalum provides a measure of the beam energy. Carbon provides a check of the measurements, by comparing our results to the existing ~1% NIKHEF data. The two issues we have identified for the measurement are 1) the choice of targets, and 2) the analysis procedure / an indication that we can significantly improve upon the existing radii determinations. We will address these points by presenting the proposed kinematic points, reviewing some of the existing published data and analyzes, indicating how good a measurement we expect to do, describing the targets we plan to use, and discussing improved analyzes.

Possible Kinematic Points: The range of accessible kinematics at 362 MeV beam energy ranges from Q ~ 0.36 fm^{-1} up to about 2.3 fm^{-1}. The table below gives several entries that indicate the limits of the Q range, as well as the central Q values for normal forward angle scattering in Hall A. With the proposed nuclear targets, the spectrometer can be set to a single momentum, p_0 = 353 MeV/c, for the entire experiment, with δ values all comfortably away from the edges of the acceptance. The table omits seven intermediate Q points, at higher Q, for brevity.

Q (fm^{-1})	***θ (deg)***	***δ (%) ^{6}Li(e,e')***	***δ (%) ^{12}C(e,e')***	***δ (%) ^{181}Ta(e,e')***
0.355 (at edge of acceptance)	11.1	2.426	2.487	2.545
0.400	12.5	2.392	2.470	2.544
0.446	14.0	2.354	2.451	2.543
0.507	15.9	2.297	2.423	2.541
0.760	24.0	1.981	2.264	2.531
1.014	32.2	1.538	2.040	2.516
1.267	40.7	0.970	1.751	2.496
1.521	49.5	0.274	1.396	2.472
1.774	58.8	-0.547	0.974	2.444
2.027	68.6	-1.495	0.481	2.410
2.281	79.2	-2.570	-0.082	2.371

Review of old measurements: Lithium

Reference	*Isotope*	*Radius (fm)*	*Q range (fm^{-1})*	*Statistical Precision*	*J^{π}*
L.R. Suelzle, M.R. Yearian, H. Crannell, Phys. Rev. 162, 992–1008 (1967)	^{6}Li	2.54 ± 0.05	0.71 – 2.63	3.4% and up	1^{+}
G.C. Li, I. Sick, R.R. Whitney, M.R. Yearin, Nucl. Phys. A162, 583–592 (1971)	^{6}Li	2.56 ± 0.05	0.56 – 3.66	3.0 % and up	1^{+}
F.A. Bumiller, F.R. Buskirk, J.N. Dyer, W.A. Monson, Phys. Rev. C5, 391–412 (1972)	^{6}Li	2.51 ± 0.10	0.09 – 0.89	2.6% - 3.5%	1^{+}
L.R. Suelzle, M.R. Yearian, H. Crannell, Phys. Rev. 162, 992–1008 (1967)	^{7}Li	2.39 ± 0.03	0.71 – 2.63	2.6% and up	$3/2^{-}$
F.A. Bumiller, F.R. Buskirk, J.N. Dyer, W.A. Monson, Phys. Rev. C5, 391–412 (1972)	^{7}Li	2.35 ± 0.10	0.09 – 0.89	2.6% - 3.3%	$3/2^{-}$

For ^{6}Li, the first excited state is $J^{\pi} = 3^{+}$ at 2.18 MeV, and is not an experimental problem. There is a potential background problem from the ^{7}Li elastic state at low Q – and because the ^{7}Li elastic form factor becomes ~10 times that of ^{6}Li for Q ~ 2.5 fm^{-1}, and from the 7/2-, 4.6 MeV ^{7}Li second excited state, in the region of the ^{6}Li form factor minimum, Q ~ 2.6 fm^{-1}. Since the ^{6}Li ground state is 1^{+}, there are contributions from charge, quadrupole, and magnetic form factors, but the latter two were estimated to be negligible in the measured kinematics; the ^{6}Li quadrupole moment is known to be small.

- Suezle *et al.* found that absolute normalization changes in their data of 5% led to changes of ±0.03 in the extracted radii. Their extractions were model dependent, using a 3-parameter function. Fitting a power series in Q^2 led to larger uncertainties, ±0.1. The quoted absolute normalization systematic uncertainty is less than 5%.
- Li *et al.* similarly used a phenomenological 3 parameter function to determine the radius. They claim their uncertainty is due about equally to statistics and to absolute normalization uncertainties. The absolute normalization uncertainty were apparently 2% on target thickness plus a further 3% from cross normalizing to hydrogen.
- Bumiller *et al.* determined the cross sections by cross normalizing their lithium data to previous carbon form factors at each point. The purely statistical uncertainties in each target were in the range 0.7 – 1.7%; the larger uncertainties in the Table above reflect additional cross normalization uncertainties. The radii quoted are for a model-independent expansion in Q^2; a fit with the functional form of Suezle *et al.* gives 2.53 ± 0.03 fm. No uncertainty is given for the carbon form factor, but it is noted that with the larger radius of more recent carbon measurements, the extracted form factors for lithium would increase by about 0.06 fm. The more recent measurements had a ^{12}C radius of 2.46 fm, vs the 2.395 fm of the normalization data, and the 2.47-2.48 fm of the most precise, most recent measurements.

For ^{7}Li, the first excited state is $J^{\pi} = 1/2^{-}$ at 0.478 MeV, which is potentially an experimental problem. Since the ground state is 3/2-, there are contributions from 4 form factors. The quardupole moment is 57 times that of ^{6}Li, and there is no model independent way to separate the contributions of the monopole and quadrupole form factors to ^{7}Li(e,e').

- Suezle *et al.* fit a harmonic oscillator monopole form factor plus a calculated quadrupole form factor to determine the radius. See also the other notes in the ^{6}Li discussion.
- Bumiller *et al.* similarly follow the same procedures as noted for ^{6}Li above. Corrections for the quadrupole form factor on the radius were claimed to be less than 0.1%, in the low Q range of the experiment. A fit like done by Suezle *et al.* gives 2.29 ± 0.04 fm.

The above discussion indicates that there is much room for experimental improvement in the determination of the lithium radius. It should be possible to improve the lithium statistical precision from about 2.5 – 3.5% to below 1%, and to keep point to point systematic uncertainties at this level. It should further be possible to reduce absolute systematics from the level of 5% to the level of 2-3% - the issue that is difficult to assess at this point is how uniform the target will be.

Natural ^{6,7}Li, is 7.6% ^{6}Li and 92.4% ^{7}Li. The ^{6}Li form factor drops faster than the 7Li form factor with increasing Q, being about equal up to about 0.7 fm^{-1}, but 10 times smaller by 2.5 fm^{-1}. If the peaks were not resolved, the ^{6}Li contamination to ^{7}Li ranges from ~8% down to 0.8%. The relative form factors are known from ~1% - 7%, depending on Q , so a natural target is conceivable for ^{7}Li. However, due to the quadrupole contribution to ^{7}Li(e,e'), it is clearly preferable to measure ^{6}Li(e,e') with an isotopically enriched target. We are currently expecting to try to obtain such material from Oak Ridge.

Review of old measurements: Boron

Reference	*Isotope*	*Radius (fm)*	*Q range (fm^{-1})*	*Statistical Precision*	J^{π}
T. Stovall, J. Goldemberg, D.B. Isabelle, Nucl. Phys. 86 , 225–240 (1966)	^{10}B	2.45 ± 0.12	0.69 – 2.81	2 – 10%	3^+
A. Cichocki, J. Dubach, R.S. Hicks, et al., Phys. Rev C51 , 2406–2426 (1995)	^{10}B	2.58 ± 0.05	0.48 – 2.58	2.2 – 12.7%	3+
T. Stovall, J. Goldemberg, D.B. Isabelle, Nucl. Phys. 86 , 225–240 (1966)	^{11}B	2.42 ± 0.12	0.69 – 2.81	2 – 10%	3/2-
R. Riskalla, Ph.D. thesis, University of Paris, Orsay, France, LAL 1243 (1971)	^{11}B	2.37 (uncertainty not tabulated)	(not tabulated)	(not tabulated)	$3/2^-$

Natural $^{10,11}B$ is 19.8% ^{10}B and 80.2% ^{11}B. The ^{10}B form factor squared varies from about 5 – 20% larger than the ^{11}B form factor in our Q range. The two ground states cannot be separated at low Q, so isotopically enriched targets are needed. For ^{10}B, the first excited state is $J^{\pi} = 1^+$ at 0.718 MeV, and is not an experimental problem. For ^{11}B, the first excited state is $J^{\pi} = 1/2^-$ at 2.125 MeV, and is not an experimental problem. From the theoretical perspective, the ^{10}B structure is at present much more precisely determined by calculations, as ^{11}B is at the limits of modern calculations.

Due to the high spin (3+) of ^{10}B, the measured form factor has contributions from several multipoles (C0, C2, C4, and C6), in addition to having magnetic contributions. The multipoles sum incoherently, and cannot be uniquely determined simply from the cross section data. Theoretical estimates indicate that the monopole and quadrupole contributions dominate at low momentum transfer, and higher contributions can likely be ignored; this assumption can be tested by the quality of a fit to the shape of the data.

We describe two of the three tabulated B measurements.

- The $^{10,11}B$ measurements of Stovall *et al.* have a claimed absolute normalization of 3%, adding their estimates for solid angle, charge, and target thickness in quadrature. The extracted radii are based on fitting the measured form factor with various phenomenological forms of the monopole plus quadrupole form factors.
- We have not obtained a copy of the Riskalla thesis, and do not discuss this measurement further.
- Cichocki *et al.* give a much more modern analysis and a much more thorough presentation of their results than any of the other papers discussed here. The target, made of ^{10}B plus binder, was damaged by beam heating, and the authors suggested a boron carbide target, $^{10}B_4C$, for future experiments. The absolute normalization was determined relative to the ^{12}C(e,e') measurements of Offermann *et al.* The good elastic data and the older elastics from Stovall *et al.* were used to determine form factors, which with a DWBA calculation provided a normalization for the damaged-target runs. Magnetic contributions to the elastic peak were determined from the MIT Bates large scattering angle data of Hicks *et al.* Models were used to separate out the C0 and C2 contributions to the form factor, and the C0 contribution then determined the charge radius of ^{10}B. The larger radius than Stovall *et al.* results from using a three-parameter Fermi charge distribution, as opposed to a spherical harmonic oscillator. Quoting from the article, ``In summary, our final result for the rms size of the ^{10}B charge density is 2.58 ± 0.05 ± 0.05 fm, where the first error represents the statistical uncertainty, and the second error is the estimated systematic uncertainty due to the model dependence of the C0 and C2 form factors. Experimental systematic uncertainties, such as those associated with the overall normalization of the data or incident beam energy, are relatively small."

Oak Ridge does not supply boron isotopes, but they are available from Cambridge Isotopes. We are currently looking into the cost of such material, for construction of a boron-carbide target.

The improvements we can make in the experimental determination of the ^{10}B form factor are two fold. We can reduce the statistical uncertainties from the typical 3% to the 1% level, and we can take data both more finely spaced in Q and to lower Q. These improvements should allow the ± 0.05 fm statistical uncertainty to be reduced to below 0.01 - 0.02 fm.
The model dependence is discussed below.

Improving Model Dependent Analysis?

An issue with older determinations of nuclear radii is the typical use of simple models for the nuclear charge density, such as Gaussians or Fermi functions. While this procedure clearly made sense at the time, it has no real theoretical justification; there is no reason to expect the nuclear charge distribution is really Gaussian or Fermi function shaped, and given a sufficiently precise experiment the differences will be visible. This is clearly implicit in the model dependence discussion reported in Cichocki *et al.*

Given the modern nuclear theory now available, it makes sense to try for an improved analysis technique that does not rely on simple functional forms, or even summation of some series of functions, but instead on the best nuclear theory. Our proposal is that the best way to determine the nuclear radii is through comparison of the full form factor data to a set of modern calculations, varying calculation parameters, such as 3-body force parameters, to find the acceptable range of calculations, and from them the uncertainty range of the nuclear radius. Since the NN force is determined from the NN scattering data, one is not fitting the nuclear densities / form factors simple functional forms. While the 3-body force is a model, it is a model that is being adjusted to fit what we expect to be a small residual difference between the pure NN prediction and the measured form factors, rather than the full form factors. Thus, we hope to reduce the model dependence in the extraction of nuclear radii. Even if the model dependence uncertainties are not reduced, we believe that they will be more realistic estimates of the true uncertainty.

Equipment and Staff

The measurements will be performed as a continuation of the LEDEX experiments run during summer / fall 2006 in Hall A. We have a Ph.D. student already working on the analysis of the data taken during August 2006. The main impact on Hall A is time for a change to the solid target ladder after the new year and lack of access to the hall for one week during the measurement.