Obtaining Cross Sections for E99-114 [1]

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January 7, 2002

1 Introduction

The goal of experiment E99-114 is to obtain cross sections for Real Compton Scattering (RCS). This document outlines how to extract cross sections from the actual experimental data. The general equation between counts Y and cross section $d\sigma/d\Omega$ is given by

$$Y = \kappa N_i \frac{d\sigma}{d\Omega} \Omega \,, \tag{1}$$

where κ is the number of target nuclei per unit area, N_i is the number of incident projectiles, and Ω is the solid angle subtended by the detector. For a liquid Hydrogen target, κ is given by

$$\kappa = \rho t N_A \,, \tag{2}$$

where N_A is Avogadro's number (6.022 x 10^{23}), ρ is the density (0.0708 g/cm^3), and t is the thickness in cm. This implies

$$\kappa = 4.264 \times 10^{23} \left[\frac{t}{10 \text{ cm}}\right] \text{protons/cm}^2 \tag{3}$$

For an incident electron beam, the number of electrons N_e is given by

$$N_e = \frac{Q}{1.602 \times 10^{-13}} \,, \tag{4}$$

where Q is the integrated charge in μC. It is more convenient to measure Q in units of μA-hr, in which case

$$N_e = 2.247 \times 10^{16} \left[\frac{Q}{1\mu\text{A-hr}}\right] . \tag{5}$$

[1] rcs/analysis/jan02run/cross-sections.tex

The solid angle Ω is given by

$$\Omega = 10^{-6}\left[\frac{\Delta\theta_e\Delta\phi_e}{1\mathrm{mr}^2}\right]\mathrm{sr}\,, \tag{6}$$

where the angles are the *geographic* angles of the electron detector (our calorimeter) in mr.

2 Application to *ep* Elastic Scattering without Radiator

In this case, the number of incident particles N_i is just the number of electrons N_e. Putting together all the preceding factors, we arrive at the following handy formula:

$$Y = 9.581\times\left[\frac{t}{10\ \mathrm{cm}}\right]\left[\frac{Q}{1\mu\mathrm{A\text{-}hr}}\right]\left[\frac{\Delta\theta_e\Delta\phi_e}{1\mathrm{mr}^2}\right]\frac{d\sigma}{d\Omega}\,, \tag{7}$$

where the cross section is in units of nb/sr (10^{-33} cm^2/sr). As a numerical example, suppose our target is 10 cm long, the cross section is 0.5 nb/sr (approximately the *ep* cross section at 2.34 GeV and electron scattering angle of 46°), and that our calorimeter angular acceptance is 80 mr x 120 mr (approx. 10 meters from target). If we count at 10 μA for 1 hour, then Q=10 μA-hr and we expect Y=5.56 x 10^5 counts. Of course, we will measure Y and use that to determine the $d\sigma/d\Omega$, then compare it to previously measured values in order to evaluate the systematics of our experiment.

3 Application to RCS with Radiator

Let us suppose that upstream of the target, there is a radiator of thickness t_r[2], where the thickness corresponding to one radiation length is X_0 (for copper, X_0=1.43 cm). Then t_r/X_0 is the fraction of a radiation length (in E99-114, this is 0.06), and the number of photons per unit energy interval is

$$\frac{dN_\gamma}{dE_\gamma} = \frac{t_r}{X_0}f(E_\gamma)N_e\,, \tag{8}$$

[2]The subscript is to distinguish the radiator thickness from the hydrogen target thickness.

where $f(E_\gamma)$ is the bremsstrahlung energy spectrum normalized so that

$$\int_0^{E_0} E_\gamma f(E_\gamma) dE_\gamma = \frac{t_r}{X_0} E_0 \tag{9}$$

with E_0 the incident electron energy. Note that $f(E\gamma)t_r/X_0$ is the number of photons of energy E_γ per unit energy interval per incident electron and that the mean bremsstrahlung photon energy is $E_0 t_r/X_0$. For purposes of estimating yields, we use the approximate expression $f(E_\gamma) \approx 1/E_\gamma$. With this approximation, the number of incident photons N_γ between energies $E_{\rm min}$ and $E_{\rm max}$ is

$$N_\gamma = N_e \frac{t_r}{X_0} \ln\left(\frac{E_{\rm max}}{E_{\rm min}}\right). \tag{10}$$

We then arrive at the following formula:

$$Y_{\rm RCS} = 9.581 \times \left[\frac{t}{10 \text{ cm}}\right] \left[\frac{Q}{1\mu\text{A-hr}}\right] \frac{t_r}{X_0} \ln\left(\frac{E_{\rm max}}{E_{\rm min}}\right) \left[\frac{\Delta\theta_e \Delta\phi_e}{1\text{mr}^2}\right] \frac{d\sigma}{d\Omega}. \tag{11}$$

Of course, in the final analysis of the data, we will use a more accurate expression for $f(E_\gamma)$. The energies limits $E_{\rm min}$ and $E_{\rm max}$ will be imposed by appropriate cuts on the proton recoil kinematics, as measured by the HRS.

4 Application to *ep* Elastic Scattering with Radiator

After passing through the radiator with an initial energy of E_0, there are electrons in the beam with energy E between $E_{\rm min}$ and $E_{\rm max}$ (the same energy range for the photons we are using). These electron radiated a (low energy) photon in the energy range E_0 - $E_{\rm max}$ to E_0 - $E_{\rm min}$. Therefore, the formula relating $Y_{\rm ep}$ to cross section looks identical to the RCS formula with the substitutions

$$\begin{aligned} E_{\rm max} &\rightarrow E_0 - E_{\rm min} \\ E_{\rm min} &\rightarrow E_0 - E_{\rm max} \end{aligned} \tag{12}$$

5 Complications Due to the HRS

All of the preceding discussion assumed that every scattered electron or photon hitting the calorimeter is counted as an event. If in addition we demand that the recoil proton be detected in the HRS, then things potentially get more complicated. In general, only a subset of events get counted in the coincidence, except for the case where the calorimeter is far enough away from the target that it is the limiting solid angle. In fact, that is the case (purposely) for our initial kinematic point, partly to simplify the initial calculation of cross sections and partly to assure that the *ep* coincidence event fully illuminate the front of the calorimeter (if the calorimeter were too close, then the coincidences would not populate the outer blocks due to limitations imposed by the HRS acceptance). Generally speaking, for most of the kinematics we will run, the vertical acceptance of the calorimeter is matched to that of the HRS, so we are safe using $\Delta\phi$ for the calorimeter. Also, only about 70% of the horizontal acceptance of the calorimeter gives rise to an event in the HRS. A simple thing to do is just assume that $\Delta\theta$ is 70% of the angle subtended by the calorimeter. By the way, there is an additional complication due to the y acceptance of the HRS, which imposes limitations on the effective target length:

$$t \rightarrow \min(t, \frac{y}{\sin\theta_p}) . \tag{13}$$

6 Normalization of RCS to ep Scattering

Because of the complicated acceptance of the HRS, it might be advantageous to avoid this problem altogether by effectively measuring the ratio of RCS to ep cross section, taking advantage of the fact that, despite the complicated acceptance, it is the same for both processes and therefore cancels in the ratio:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{RCS}} = \left[\frac{Y_{\mathrm{RCS}}/N_\gamma}{Y_{\mathrm{ep}}/N_e}\right]\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} \tag{14}$$

The term in brackets on the right-hand-side of the equation is the measured quantity. It is the ratio of RCS events per incident photon to *ep* events per incident electron, where both photon and electron are in identical energy ranges (and indeed, are measured simultaneously). In this ratio, the following factors cancel out: calorimeter-HRS acceptance; all target quantities; the absolute number of incident electrons in the primary beam; the thickness of the radiator. The cost of doing the normalization in this manner is that we need to know separately the number of incident photons N_γ and the number of incident electrons N_e. We need to know N_γ anyway, but we now have to determine independently N_e. Of course, both come from the same bremsstrahlung spectrum $f(E_\gamma)$, but they come from opposite ends of the spectrum: the photons are near the endpoint E_0 whereas the electrons come from photons that are very low in energy. We will use a calculation of the thick-target bremsstrahlung to *calculate* N_γ. The experts[3] claim this can be done to 3% accuracy. We will use the same code to calculate N_e, but will supplement the calculation with measurements. In the E99-114 run plan, these measurement come under the heading **$\mathbf{N(E_e)}$ Mesurements** and the essence of the technique is described in the next section.

There is one complication due to the fact that the deflection magnet bends the electrons (in the horizontal plane toward the beam line). Since the calorimeter acceptance is roughly matched to that of the HRS, bending the electrons effectively removes some of them from the coincidence acceptance (but, of course, not from the HRS acceptance, since the associated recoil protons are not affected). In order to make our ratio technique viable, we will use the proton kinematics, as measured by the HRS, to reconstruct the *ep*/RCS scattering angle. We will then make a cut to assure that the *same* scattering angle acceptance is used for both the *ep* and RCS events. Of course, this means that we effectively lose some of our RCS acceptance to compensate for the lost *ep* acceptance. In order to minimize this, the magnet current will be adjusted so that electrons are deflected by no more than about 25% of the full calorimeter (i.e., $\leq$ 22 cm). In effect, this reduces our overall acceptance by 25%. To compensate, we will simply run a bit longer. The current run plan calls for approximately 1500 RCS events per kinematic point, before applying any cuts. Under a worst case scenario, we might lose half of our events by cutting, but this still will safely allow the 5% statistical precision we aim for in the cross sections.

To get a rough idea about what kind of cross sections to expect, we can

[3]Dave Meekins has written a nice document on this which can be obtained from him or found in Appendix B of his thesis (downloadable from the JLab web site).

use the following simple estimate based on the old Cornell data:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm RCS} \approx \frac{10E'^2}{s^6}\mu b/sr\,, \tag{15}$$

where E' is the scattered photon energy in GeV and $s = M^2 + 2ME$ is the usual Mandelstam variable in units of GeV^2. The *ep* scattering cross section is approximated by the Rosenbluth formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm ep} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \left[A + B\tan^2\theta/2\right]\,, \tag{16}$$

where

$$A = \frac{G_E^2 + \eta G_M^2}{1+\eta} \qquad B = 2\eta G_M^2 \qquad \eta = \frac{Q^2}{2M}\,, \tag{17}$$

$$G_E = \frac{1}{(1+\frac{Q^2}{.71\,GeV})^2} \qquad G_M = 2.79G_E \qquad Q^2 = 2EE'(1-\cos\theta)\,, \tag{18}$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \left[\frac{\hbar c\alpha}{2E}\right]^2 \frac{\cos^2\theta/2}{\sin^4\theta/2}\left\{\frac{1}{1+\frac{2E}{M}\sin^2\theta/2}\right\} \tag{19}$$

7 Measuring N(E$_e$)

The essential idea for measuring the post-radiator electron energy spectrum N(E$_e$) is to perform two sequential coincidence measurements of the *ep* elastic cross section at a fixed calorimeter angle θ_e. For any incident electron energy E and electron scattering angle θ_e, the corresponding momentum and angle, p_p(E, θ_e) and θ_p(E, θ_e) are uniquely determined. For the first measurement, the radiator is removed, so that all electrons have the beam energy E$_0$, and the HRS is set for the corresponding momentum and angle. For the second measurement, the radiator is inserted and the HRS is set for the momentum and angle corresponding to energy E$_1$, the energy at which we desire to measure N(E$_e$). In each case, we measure the number of *ep* coincidence events Y per incident electron in the primary beam N$_e$:

$$\begin{aligned}\frac{Y_{ep}(E_0)}{N_e} &= K\frac{d\sigma}{d\Omega}(E_0,\theta_e)\\ \frac{Y_{ep}(E_1)}{N_e} &= K\frac{d\sigma}{d\Omega}(E_1,\theta_e)\left[\frac{N_e(E_1)}{N_e}\right]\,,\end{aligned} \tag{20}$$

where K lumps together all the other factors (target, solid angle, etc.) The essential point is that since the slice of incident energy centered on E_1 is set to illuminate the *same* region of HRS acceptance as the no-radiator measurement, then the factor K is *identical* in the two measurements. Taking the ratio and rearranging, we find

$$\frac{N_e(E_1)}{N_e} = \left[\frac{Y_{ep}(E_1)/N_e}{Y_{ep}(E_0)/N_e}\right]\left\{\frac{d\sigma/d\Omega(E_0,\theta_e)}{d\sigma/d\Omega(E_1,\theta_e)}\right\}, \tag{21}$$

so that the desired ratio is determined directly by the measured quantity in brackets on the right-hand-side, with a small correction in braces due to the (known) energy dependence of the *ep* cross section. In particular, the acceptance of the calorimeter-HRS system completely cancels out of the ratio.[4]

[4]One cautionary note: Since the HRS angle changes slightly (few degrees), the length of target illuminating the y acceptance of the spectrometer is different in the two measurements. That is, the target is effectively thicker when the HRS angle is smaller. It is essential to keep the y acceptance the same in the two measurements, so that we will have to correct for the slightly different target thickness.