Event Reconstruction for RCS Experiment [1]

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For the RCS experiment, we use the proton kinematic quantities to reconstruct the event at the target and calorimeter. The geometry of the experiment is shown in the figure. We define three distinct coordinate systems, all of which have their origin at the spectrometer pivot. The target (or Hall A) coordinate system has its z-axis pointing along the beam direction, x-axis pointing to beam left, and y-axis pointing up. The HRS coordinate system (denoted by *) has z^* pointing from the target to the spectrometer. Looking along the z^* axis, then y^* points to the left and x^* points down. The angle of z^* with respect to z is Θ_{HRS}, and it is assumed the spectrometer is on the left side of the beam (i.e., the proton always has positive x). The angles ϕ^* and θ^* (the usual ESPACE angles) increase in the same directions as y^* and x^*, respectively. The Calorimeter coordinate system (denoted by $\bullet$) has $z^\bullet$ pointing from target to calorimeter. Looking along the $z^\bullet$ axis, then $x^\bullet$ points to the left and $y^\bullet$ points up. The angle of $z^\bullet$ with respect to z is Θ_{CALO}, and it is assumed the calorimeter is on the right side of the beam (i.e., the scattered photon/electron always has negative x). The distance from the origin to the front face of the calorimeter is D_0.

The goal is to start with the proton parameters p_p, ϕ_p, θ_p, and y_p and predict the energy and position of the associated particle (photon or electron) on the calorimeter. The two proton angles determine the angle of the proton θ^*_{pz} with respect to the z^* axis:

$$\cos\theta^*_{pz} = \frac{1}{\sqrt{1 + tan^2\phi_p + tan^2\theta_p}}\,, \tag{1}$$

which in turn determines the components of the proton momentum p^* in the HRS system:

$$\begin{aligned} p^*_x &= p_p \cos\theta^*_{pz} \tan\theta_p \\ p^*_y &= p_p \cos\theta^*_{pz} \tan\phi_p \\ p^*_z &= p_p \cos\theta^*_{pz}\,. \end{aligned} \tag{2}$$

[1] rcs/analysis/jan02run/kinematic-reconstruction.tex

We next rotate about the y-axis to determine the components of the proton momentum in the Hall A system:

$$\begin{aligned} p_x &= p_y^* \cos\Theta_{HRS} + p_z^* \sin\Theta_{HRS} \\ p_y &= -p_x^* \\ p_z &= -p_y^* \sin\Theta_{HRS} + p_z^* \cos\Theta_{HRS} \,. \end{aligned} \tag{3}$$

This determine the proton scattering angle θ_{pz} in the Hall A frame (i.e, the angle relative to the beam or z axis):

$$\cos\theta_{pz} = p_z/p. \tag{4}$$

We assume a reaction

$$\gamma + p \to p + X\,, \tag{5}$$

where X has mass M_X and is a photon γ, an electron e, or a π^0. We assume no kinematic distinction between an incident photon and incident electron. Then the proton momentum p, energy $E_p = \sqrt{p^2 + M_p^2}$, and scattering angle θ_{pz} uniquely determine the incident photon energy E_{in}:

$$E_{in} = \frac{M_p(E_p - M_p) + 0.5M_X^2}{M_p - E_p + p_p \cos\theta_{pz}}\,. \tag{6}$$

We then use 4-momentum conservation to determine the energy E, three-momentum k, and scattering angle θ_k of X:

$$\begin{aligned} E &= E_{in} + M_p - E_p \\ k &= \sqrt{E^2 - M_X^2} \\ \cos\theta_k &= \frac{E_{in}E - M_p(E_p - M_p) - 0.5M_X^2}{E_{in}k}\,. \end{aligned} \tag{7}$$

Moreover, conservation of out-of-plane momentum determines the azimuthal angle ϕ_k:

$$\sin\phi_k = -\frac{p_y}{k\sin\theta_k}\,. \tag{8}$$

From this we determine the component of the X momentum in the Hall A system:

$$\begin{aligned} k_x &= -k\sin\theta_k\cos\phi_k \\ k_y &= +k\sin\theta_k\sin\phi_k \\ k_z &= +k\cos\theta_k\,. \end{aligned} \tag{9}$$

We next rotate about the y-axis to determine the components of the X momentum in the Calorimeter frame:

$$\begin{aligned} k_x^\bullet &= k_x \cos\Theta_{CALO} + k_z \sin\Theta_{CALO} \\ k_y^\bullet &= k_y \\ k_z^\bullet &= -k_x \sin\Theta_{CALO} + k_z \cos\Theta_{CALO} \,. \end{aligned} \tag{10}$$

We define the in-plane and out-of-plane calorimeter angles, $\theta^\bullet$ and $\phi^\bullet$, respectively:

$$\begin{aligned} \tan\theta^\bullet &= \frac{k_x^\bullet}{k_z^\bullet} \\ \tan\phi^\bullet &= \frac{k_y^\bullet}{k_z^\bullet} \,. \end{aligned} \tag{11}$$

In terms of these angles, the $x^\bullet$ and $y^\bullet$ positions of X on the front face of the calorimeter are given by

$$\begin{aligned} x^\bullet &= x_0 + D\theta^\bullet \\ y^\bullet &= D\phi^\bullet \,, \end{aligned} \tag{12}$$

where the offset x_0 and distance D_0 are given by

$$\begin{aligned} x_0 &= z_v \sin\Theta_{CALO} \\ D &= D_0 - z_v \cos\Theta_{CALO} \end{aligned} \tag{13}$$

and the vertex position z_v is given by

$$z_v = -y_p / \sin\Theta_{HRS} \,. \tag{14}$$

The analysis proceeds by making histograms of the difference between the calculated calorimeter parameters (E, $x^\bullet$, and $y^\bullet$) and the measured ones (E_m, x_m, and y_m) to form the quantities δE, δx, and δy, assuming that X is a photon or electron (i.e., $M_X = 0$). For RCS or *ep* events, these quantities should be peaked at zero value and have a width determined by measurement resolutions, except for δx for *ep* events, which is peaked at a nonzero value due to the deflection in the magnet. For π^0 events, the events are confined to a circle in the $\delta y - \delta x$ plane centered at $\delta x \approx 0$ and $\delta y = 0$ and with radius

$r_0 = DM_\pi/E$. Moreover, δE is approximately flat between 0 and $E/2$. On the other hand, it is possible to reconstruct the event assuming the X is a π^0 by using $M_X = M_\pi$ in Eqs. 6-7. We then form the quantity

$$r^2 \; = \; \delta x^2 + \delta y^2 \, . \tag{15}$$

Then the quantity z_π, which is the cosine of the decay angle of the π^0 relative to the π^0 direction in the π^0 rest frame, is given by

$$z_\pi \; = \; \frac{r_0^2 - r^2}{r_0^2 + r^2} \, , \tag{16}$$

and the distribution of z_π should be flat between 0 and 1, at least to the extent that acceptance is not a limitation. The energy of the two decay photons

$$E_\pm \; = \; \frac{E}{2}(1 \pm z_\pi) \, . \tag{17}$$

The quantity $\Delta E = E_+ - E_m$ is centered at zero with a width determined by resolutions. The signature of a π^0 event is the correct distribution of ΔE.

It is also useful to work backwards for *ep* measurements without a radiator in order to use the known beam energy, angles determined by the calorimeter, and y to determine the proton parameters p_p, ϕ_p, and θ_p. We start by effectively inverting Eq. 12-14 in order to determine the calorimeter angles $\theta^\bullet$ and $\phi^\bullet$ from measurements of the proton y and the calorimeter coordinates $x^\bullet$ and $y^\bullet$:

$$\begin{aligned} \theta^\bullet \; &= \; (x_0 - x^\bullet)/D \\ \phi^\bullet \; &= \; y^\bullet/D \, , \end{aligned} \tag{18}$$

Just as in Eq. 1, these two angles determine the angle of the electron $\theta^\bullet_{ez}$ with respect to the $z^\bullet$ axis:

$$\cos\theta^\bullet_{ez} = \frac{1}{\sqrt{1 + tan^2\phi^\bullet + tan^2\theta^\bullet}} \, , \tag{19}$$

which in turn determines the electron direction cosines in the CALO system:

$$\begin{aligned} \hat{k}^\bullet_x \; &= \; \cos\theta^*_{ez} \, \tan\theta^\bullet \\ \hat{k}^\bullet_y \; &= \; \cos\theta^*_{ez} \, \tan\phi^\bullet \\ \hat{k}^\bullet_z \; &= \; \cos\theta^*_{ez} \, . \end{aligned} \tag{20}$$

We next rotate into the Hall A system, effectively inverting Eq. 10, to determine the electron direction cosines in that frame:

$$\begin{aligned}
\hat{k}_x &= \hat{k}_x^\bullet \cos\Theta_{CALO} - \hat{k}_z^\bullet \sin\Theta_{CALO} \\
\hat{k}_y &= \hat{k}_y^\bullet \\
\hat{k}_z &= \hat{k}_x^\bullet \sin\Theta_{CALO} + \hat{k}_z^\bullet \cos\Theta_{CALO}\,.
\end{aligned} \tag{21}$$

The angle $\hat{k}_z \equiv \cos\theta_k$ and the beam energy E_{in} determines the scattered electron energy via the Compton formula (assuming a massless electron):

$$E = \frac{E_{in}}{1 + \frac{E_{in}}{M_p}(1 - \cos\theta_k)}\,. \tag{22}$$

The preceding two equations determine all the components of the electron momentum in the Hall A frame. The components of proton momentum in the Hall A frame can simply be determined from 4-momentum conservation:

$$\begin{aligned}
E_p &= E_{in} + M_p - E \\
p_p &= \sqrt{E_p^2 - M_p^2} \\
p_x &= -E k_x \\
p_y &= -E k_y \\
p_z &= E_{in} - E k_z\,.
\end{aligned} \tag{23}$$

We next invert Eq. 3 to rotate back into the HRS frame:

$$\begin{aligned}
p_y^* &= p_x \cos\Theta_{HRS} - p_z \sin\Theta_{HRS} \\
p_x^* &= -p_y \\
p_z^* &= p_x \sin\Theta_{HRS} + p_z \cos\Theta_{HRS}\,.
\end{aligned} \tag{24}$$

Finally we invert Eq. 2 to obtain the proton angles in the HRS frame:

$$\begin{aligned}
\theta_p &= \arctan\frac{p_x^*}{p_z^*} \\
\phi_p &= \arctan\frac{p_y^*}{p_z^*}\,.
\end{aligned} \tag{25}$$

This completes the task.

A useful diagnostic for the HRS is to use the measured proton angle θ_{pz} in the Hall A frame and the known beam energy E_{in} to predict the proton momentum p, assuming ep elastic scattering is the reaction. The correct expression is

$$p = \frac{2M_p E_{in} \cos\theta_{pz}}{(E_{in} + M_p)\left(1 - \left[\frac{E_{in}\cos\theta pz}{E_{in}+M_p}\right]^2\right)} . \tag{26}$$

This can then be compared to the actual momentum (from ESPACE) to see if the event is consistent with the ep hypothesis.

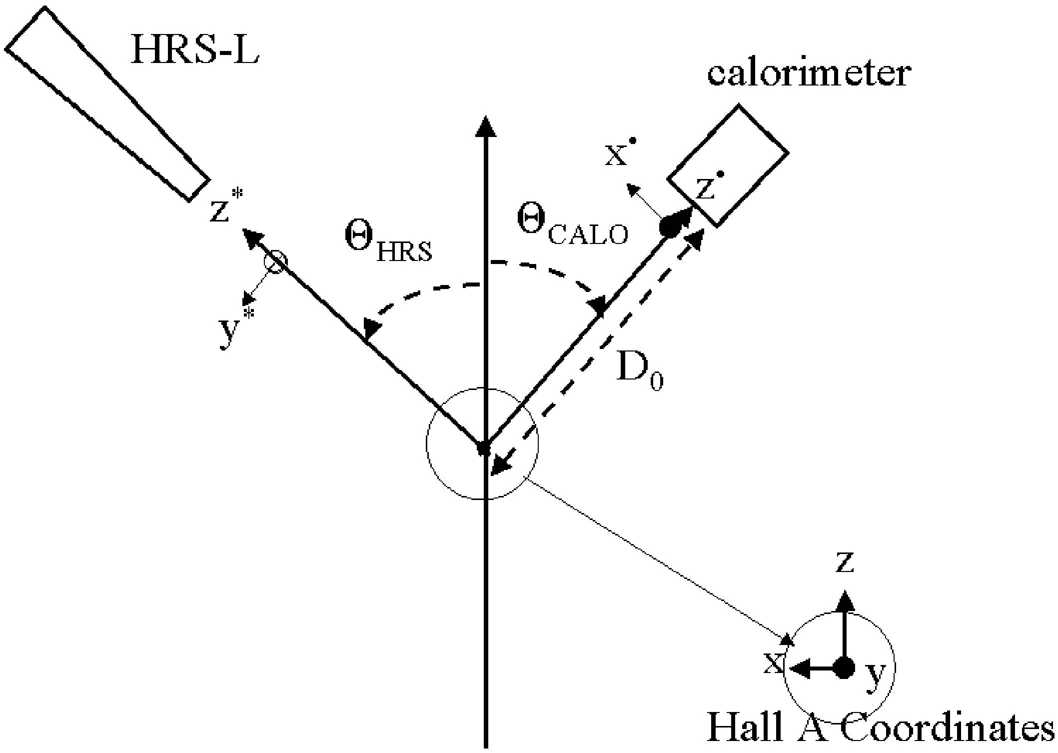

Figure 1: Geometry for the RCS experiment. The three different coordinate systems are explained in the text.