# Collection of Formulae

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http://hallaweb.jlab.org/physics/experiments/he3/A1n/formula.ps.gz

## 1 Basic Formula

### 1.1 Unpolarized and polarized structure functions

Cross section:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\mathrm{d}E'} = \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega}\right)_{Mott}\left[W_2(Q^2,\nu)+2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right] \tag{1}$$

Scaling at $Q^2 \to$, with fixed $x$: $MW_1(\nu,Q^2)\to F_1(x)$, $\nu W_2(\nu,Q^2)\to F_2(x)$, then

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{2\pi x^2}{Q^4}s[1+(1-y)^2]F_2(x) \tag{2}$$

$$F_1(x,Q^2) = \frac{F_2(x,Q^2)(1+\gamma^2)}{(2x(1+R(x,Q^2))} \tag{3}$$

The cross section difference between a target polarized along $\alpha$ and $\alpha+\pi$ is

$$\begin{aligned}\frac{\mathrm{d}^3\Big(\sigma(\alpha)-\sigma(\alpha+\pi)\Big)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} &= \frac{e^4}{4\pi^2Q^2}\Big\{\cos\alpha\{(1-\frac{y}{2}-\frac{y^2}{4}\gamma^2-\frac{y}{2}\gamma^2g_2(x,Q^2)\} \\ &\quad -\sin\alpha\cos\phi\sqrt{\gamma^2(1-y-\frac{y^2}{4}\gamma^2}\{\frac{y}{2}g_1(x,Q^2)+g_2(x,Q^2)\}\Big\}\end{aligned} \tag{4}$$

Within the quark-parton model:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2[q_i^{\uparrow}(x)+q_i^{\downarrow}(x)] \tag{5}$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2[q_i^{\uparrow}(x)-q_i^{\downarrow}(x)] \tag{6}$$

Ignoring quark mass effects, $g_2^n(x,Q^2)$ can be decomposed into the sum of two terms:

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2)+\bar{g}_2(x,Q^2) \tag{7}$$

where $g_2^{WW}(x,Q^2)$ is purely twist-2 and can be expressed in terms of $g_1(x,Q^2)$ as follows:

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2)+\int_x^1\frac{g_1(y,Q^2)}{y}\mathrm{d}y \tag{8}$$

### 1.2 Virtual photon asymmetries

$$A_1(x,Q^2) = \frac{g_1(x,Q^2)-\gamma^2g_2(x,Q^2)}{F_1(x,Q^2)} \tag{9}$$

$$A_2(x,Q^2) = \frac{\gamma[g_1(x,Q^2)+g_2(x,Q^2)]}{F_1(x,Q^2)} \tag{10}$$

## 1.3 Electron DIS asymmetries

$$A_{\|} = \frac{\sigma\uparrow\downarrow - \sigma\uparrow\uparrow}{\sigma\uparrow\downarrow + \sigma\uparrow\uparrow} = \frac{1-\epsilon}{(1-\epsilon R)W_1}\Big[M(E+E'\cos\theta)G_1 - Q^2G_2\Big] \tag{11}$$

$$A_{\perp} = \frac{\sigma\downarrow\leftarrow - \sigma\downarrow\rightarrow}{\sigma\downarrow\leftarrow - \sigma\downarrow\rightarrow} = \frac{(1-\epsilon)E'}{(1-\epsilon R)W_1}\Big[MG_1 + 2EG_2\Big]\cos\theta \tag{12}$$

Scaling: $\frac{\nu}{M}G_1(\nu,Q^2) \to g_1(x)$, $\frac{\nu^2}{M^2}G_2(\nu,Q^2) \to g_2(x)$, then

## 1.4 Extracting structure functions from physics asymmetries

Extracting $g_1^n$ and $g_2^n$ from raw asymmetries $A_{\|}$ and $A_{\perp}$:

$$g_1(x,Q^2) = \frac{F1(x,Q^2)}{D'}\Big[A_{\|} + \tan(\theta/2)\cdot A_{\perp}\Big] \tag{13}$$

$$g_2(x,Q^2) = \frac{F1(x,Q^2)}{D'}\frac{y}{2\sin\theta}\Big[\frac{E+E'\cos\theta}{E'}A_{\perp} - \sin\theta\cdot A_{\|}\Big] \tag{14}$$

$$\tag{15}$$

## 1.5 Extracting physics asymmetry from raw data

$$A_{phy} = \frac{\Delta}{P_bP_tf}$$

Then the target asymmetry $A_1^{^3\mathrm{He}}$ can be extracted from the measured asymmetries $A_{\perp}$ and $A_{\|}$ as:

$$A_1 = \frac{1}{D(1+\eta\xi)}A_{\|} - \frac{\eta}{d(1+\eta\xi)}A_{\perp} \tag{16}$$

$$A_2 = \gamma\Big\{\frac{1-\frac{y}{2}}{D'}A_{\|} + \frac{1}{D'}\Big[\tan(\theta/2) + \frac{y}{2\sin\theta}\frac{1+(1-y)\cos\theta}{1-y}\Big]A_{\perp}\Big\} \tag{17}$$

where $A_{\|}$ is the asymmetry when the beam polarization is parallel and antiparallel to the target polarization: $A_{\|} = \frac{\sigma^{\uparrow\downarrow}-\sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow}+\sigma^{\uparrow\uparrow}}$. And $A_{\perp}$ is the asymmetry when the beam polarization is transverse to the target polarization: $A_{\perp} = \frac{\sigma^{\downarrow\leftarrow}-\sigma^{\uparrow\leftarrow}}{\sigma^{\downarrow\leftarrow}+\sigma^{\uparrow\leftarrow}}$.

## 1.6 Statistics

The statistical error of the asymmetries are:

$$\Delta A_1^2 = (\frac{1}{P_tP_bf})^2[(\frac{1}{D(1+\eta\xi)}\Delta A_{\|})^2 + (\frac{\eta}{d(1+\eta\xi)}A_{\perp})^2]$$

The statistical error of $A_{\|}$ and $A_{\perp}$ can be obtained from:

$$\Delta A_{\|} = \sqrt{\frac{1-A_{\|}^2}{N_{\|}}} \sim \frac{1}{\sqrt{N_{\|}}} \qquad \Delta A_{\perp} = \sqrt{\frac{1-A_{\perp}^2}{N_{\perp}}} \sim \frac{1}{\sqrt{N_{\perp}}}$$

where $N_{||}$ ($N_{\perp}$) is the number of scattered electrons when the beam polarization is parallel or antiparallel (perpendicular) to the target polarization.

The error of $A_1$ for each kinematics can be written as:

$$\Delta A_1^2 = \alpha^2 \Delta A_{||}^2 + \beta^2 \Delta A_{\perp}^2$$

To minimize $\Delta A_1$ we need $t_{||} = \frac{\alpha}{\beta} t_{\perp}$.

### 1.7 Kinematic variables

$$\begin{aligned}
Q^2 &= 2EE'(1-\cos\theta) && (18)\\
y &= \frac{\nu}{E} && (19)\\
R &= \sigma_L/\sigma_T = \frac{1+\gamma^2}{2x}\frac{F_2}{F_1} - 1 && (20)\\
D' &= \frac{(1-\epsilon)(2-y)}{y(1+\epsilon R(x,Q^2))} && \\
\gamma^2 &= \frac{Q^2}{\nu^2} = \frac{(2Mx)^2}{Q^2} && (21)\\
\epsilon &= 1/[1+2(1+1/\gamma^2)\tan^2(\theta/2)) && (22)\\
D &= \frac{1-\epsilon E'/E}{1+\epsilon R} && (23)\\
\eta &= (\epsilon\sqrt{Q^2})/(E-E'\epsilon) && (24)\\
\xi &= \eta(1+\epsilon)/(2\epsilon) && (25)\\
d &= D\sqrt{2\epsilon/(1+\epsilon)} && (26)
\end{aligned}$$

## 2 From $^3$He to neutron

### 2.1 Convolution Approach

In the deep inelastic region an effective neutron spin structure function can be extracted to first order from that of $^3$He if only S, S' and D states of the $^3$He wave function are included:

$$\begin{aligned}
\tilde{g_1^n} &= \frac{1}{\rho_n}(g_1^{^3\mathrm{He}} - 2\rho_p g_1^p) && (27)\\
\tilde{A_1^n} &= \frac{W_1^{^3\mathrm{He}}}{W_1^n}\frac{1}{\rho_n}(A_1^{^3\mathrm{He}} - 2\frac{W_1^p}{W_1^{^3\mathrm{He}}}\rho_p A_1^p) &&
\end{aligned}$$

where $\tilde{g_1^n}$, $g_1^p$ and $g_1^{^3\mathrm{He}}$ are the spin structure functions of an effective free neutron, a free proton and $^3$He, respectively. $\rho_n = (87 \pm 2)\%$ and $\rho_p = (-2.7 \pm 0.3)\%$ are the effective nucleon polarizations of the neutron and proton in $^3$He due to the S, S' and D states of the wave function.

## 2.2 Complete Analysis - $\Delta(1232)$

(ref.: F. Bissey *et. al.*, hep-ph/0109069)

Taken into account the presence of $\Delta$ isobar in $^3He$ wavefuntion:

$$A_1^n = \frac{F_2^{^3He}}{P_n F_2^n(1+\frac{0.056}{P_n})}[A_1^{^3He} - 2\frac{F_2^p}{F_2^{^3He}} P_p A_1^p(1-\frac{0.014}{2P_p})] \tag{28}$$

- $R(x, Q^2)$ from E143 fit: K. Abe et. al., hep-ex/9808028
- $F_2^p$, $F_2^d$ from NMC fit: M. Arneodo et. al., hep-ph/9509406, assuming $F_2^{^3He} = F_2^p + 2F_2^d$, $F_2^n = 2F_2^d - F_2^p$
- $P_p = -0.021$,$P_n = 0.879$ from Paris N-N potential with 5 channels: F. Bissey et. al, Phys.Rev.**C64**, 024004(2001)
- $A_1^p$ from E155 fit, P. L. Anthony et. al., Phys.Lett.**B493**:19-28,(2000) (hep-ph/0007248)