Technical Note on EPR

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1 Introduction

This Technical Note is meant to make the analysis of EPR data (hopefully) a little easier. Further it might help tuning the F=3 ^{85}Rb transitions. In section 2. I describe how an $|F=3, \Delta m_F = 1\rangle$ transition can help to determine the magnetic holding field B_0 quite precisely and, if B_0 is known to about ± 0.5 G, how such a transition can be used to determine whether the ^{85}Rb atoms are in the $|F=3, m_F=+3\rangle$ or $|F=3, m_F=-3\rangle$ state. The following section 3. describes how the polarization can be extracted from the EPR frequency and section 4. gives a first pass analysis of the EPR data collected on the cell Nepheli.

2 Determination of the Magnetic Field using EPR

Since Rb (I=5/2, S=1/2) is an alkali metal, the energy dependence of the the Rb atoms in a magnetic field is exactly described by the Breit-Rabi formula [1]:

$$E_{F=I\pm\frac{1}{2},m_F} = -\frac{\Delta E_{hf}}{2(2I+1)} + g_N\mu_N B m_F \pm \frac{\Delta E_{hf}}{2}\Big(1+\frac{4m_F}{2I+1}x+x^2\Big)^{\frac{1}{2}}. \tag{1}$$

Here x is defined as

$$x = \frac{g_s\mu_B - g_N\mu_N}{\Delta E_{hf}}\cdot\Big(B_0+\delta B\Big), \tag{2}$$

where B_0 is the external holding field and δB is the field generated be the polarized ^{3}He. The δB term can be associated with the Fermi contact interaction. According to [2] δB can be expressed as $\delta B = \frac{8\pi}{3}\frac{\mu_0}{4\pi}g_{^3He}\mu_N\kappa_0 n_{^3He}P_{^3He}$. Here $\mu_0 = 4\pi\cdot 10^{-7}N/A^2 = 2.01336\cdot 10^{-25}\frac{Vs^2}{me}$, $n_{^3He}$ is the density of the helium-3 atoms in m^{-3}, and $P_{^3He}$ is the polarization. κ_0 is given in [3] as $\kappa_0 = 4.52 + 0.00934\cdot$ T (°C). For a helium-3 polarisation of 30% we get an additional field of about 0.04 G.

The following table lists a few quantities relevant for the ^{85}Rb-^{3}He gas mixture:

Table 1 Some useful quantities.

Atom	^{3}He	^{85}Rb
Electronic Spin	0	1/2
Nuclear Spin	1/2	5/2
$\mathbf{g_s}$	0	2.0023
$\mathbf{g_N}$	-2.127544	+1.35268
$\mathbf{g_N}$ $\mu_\mathbf{N}$ (eV/G)	-6.70698^{-12}	4.26426^{-12}
$\mathbf{E}_{hf}$ (GHz)	–	3.0357

So now we can use the Breit-Rabi formula to calculate the $\Delta m_F = 1$ transition frequencies as a function of the B-field. In order to get the value of the holding field B_0 one has to take the average value of spins of the ^{3}He atoms aligned parallel and antiparallel to the holding field. We measured a transition frequency of 11.97 MHz (to the accuracy of the Wavetek reading). The following table lists the transition frequencies as a function of the holding field for the possibleB-field values.

Table 2 Frequencies in MHz.

B_0 [G]	**+3↔+2**	**+2↔+1**	**+1↔0**	**0↔-1**	**-1↔-2**	**-2↔-3**
25.12	11.5185	11.6048	11.6931	11.7835	11.8760	**11.9707**
26.12	**11.9680**	12.0611	12.1565	12.2542	12.3543	12.4569

Since according to Sebastien Incerte the holding B-field is about 25 G, we can see that the field is 25.12 G and we have polarized ^{85}Rb in the $|F = 3, m_F = -3\rangle$ state. This can easily be checked by the EPR method: if one finds more resonances above 11.97 MHz, then we are in the $m_F = +3$ state, if the resonances are below 11.97 MHz, then we are in the m_F = -3 state. Also, measuring the B-field in the hall should confirm this.

3 The EPR Transition Frequencies

Now we can calculate the transition frequencies after formula (5.22) in M. Romalis' thesis [4]:

$$\Delta\nu_{_{EPR}} = \frac{8\pi}{3}\frac{\mu_0}{4\pi}\frac{d\nu_{_{EPR}}(F, m_F)}{dB}\kappa_0\mu_{_{^3He}}n_{_{^3He}}P_{_{^3He}}. \tag{3}$$

We can express $d\nu_{_{EPR}}(F, m_F)/dB$ as $d\nu_{_{EPR}}(F, m_F)/dB = \mu_B g_s/h(2I+1)$ for the case of $\mathrm{B}_0 = 0$ G. In the case of finite, but small, fields we can use the following equations:

$$\frac{d\nu_{_{EPR}}}{dB}(-2 \leftrightarrow -3) = 0.4670779 + 7.37904 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{4}$$

$$\frac{d\nu_{_{EPR}}}{dB}(-1 \leftrightarrow -2) = 0.4670779 + 4.25140 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{5}$$

$$\frac{d\nu_{_{EPR}}}{dB}(0 \leftrightarrow -1) = 0.4670779 + 1.26135 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{6}$$

$$\frac{d\nu_{_{EPR}}}{dB}(+1 \leftrightarrow 0) = 0.4670779 - 1.59878 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{7}$$

$$\frac{d\nu_{_{EPR}}}{dB}(+2 \leftrightarrow +1) = 0.4670779 - 4.33604 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{8}$$

$$\frac{d\nu_{_{EPR}}}{dB}(+3 \leftrightarrow +2) = 0.4670779 - 6.95702 \cdot 10^{-4}B \qquad \frac{MHz}{G} \tag{9}$$

Note that the coefficients in front of the second term were determined by a fit and therefore they are not exact – but they should be close. Note also that the above equations agree with the insert of Fig. 5.5 of Romalis' thesis.

Now we can calculate the EPR frequencies. The following table lists a few transition EPR frequencies for a cell of 11.5 amagat (1 amagat = $2.68839 \cdot 10^{25} atoms/m^3$) and a pumping cell temperature of 170°C.

Table 3   Typiclal EPR frequencies for different values of the pumping cell polariztion. $\nu_{EPR} \times 2$ *(so that is the measured frequency for the whole dip) in* kHz *(11.5 amagat,* $\mathbf{T_{pump} = 170°C}$*).*

P_{pump}	-3 ↔ -2	-2 ↔ -1	-1 ↔ 0	0 ↔ +1	+1 ↔ +2	+2 ↔ +3
0.25	-41.46	-40.79	-40.14	-39.53	-38.94	-38.38
0.26	-43.11	-42.42	-41.75	-41.11	-40.50	-39.92
0.27	-44.77	-44.05	-43.36	-42.69	-42.06	-41.45
0.28	-46.43	-45.68	-44.96	-44.27	-43.62	-42.99
0.29	-48.09	-47.31	-46.57	-45.86	-45.17	-44.52
0.30	-49.75	-48.94	-48.17	-47.44	-46.73	-46.06
0.31	-51.41	-50.57	-49.78	-49.02	-48.29	-47.59
0.32	-53.06	-52.21	-51.38	-50.60	-49.85	-49.13
0.33	-54.72	-53.84	-52.99	-52.18	-51.41	-50.66
0.34	-56.38	-55.47	-54.60	-53.76	-52.96	-52.20
0.35	-58.04	-57.10	-56.20	-55.34	-54.52	-53.73
0.36	-59.70	-58.73	-57.81	-56.92	-56.08	-55.27
0.37	-61.35	-60.36	-59.41	-58.51	-57.64	-56.80
0.38	-63.01	-61.99	-61.02	-60.09	-59.19	-58.34
0.39	-64.67	-63.63	-62.62	-61.67	-60.75	-59.88
0.40	-66.33	-65.26	-64.23	-63.25	-62.31	-61.41

Table 4 Typiclal EPR frequencies for different values of the pumping cell polariztion. $\nu_{EPR} \times 2$ *(so that is the measured frequency for the whole dip) in* kHz *(11.5 amagat,* $\mathbf{T_{pump}} = \mathbf{190°C}$*).*

P_{pump}	-3 ↔ -2	-2 ↔ -1	-1 ↔ 0	0 ↔ +1	+1 ↔ +2	+2 ↔ +3
0.25	-42.72	-42.03	-41.37	-40.74	-40.13	-39.56
0.26	-44.43	-43.71	-43.03	-42.37	-41.74	-41.14
0.27	-46.14	-45.40	-44.68	-44.00	-43.35	-42.72
0.28	-47.85	-47.08	-46.34	-45.63	-44.95	-44.30
0.29	-49.56	-48.76	-47.99	-47.26	-46.56	-45.88
0.30	-51.27	-50.44	-49.65	-48.89	-48.16	-47.47
0.31	-52.98	-52.12	-51.30	-50.52	-49.77	-49.05
0.32	-54.69	-53.80	-52.96	-52.15	-51.37	-50.63
0.33	-56.40	-55.48	-54.61	-53.78	-52.98	-52.21
0.34	-58.10	-57.16	-56.27	-55.41	-54.58	-53.80
0.35	-59.81	-58.85	-57.92	-57.04	-56.19	-55.38
0.36	-61.52	-60.53	-59.58	-58.67	-57.79	-56.96
0.37	-63.23	-62.21	-61.23	-60.29	-59.40	-58.54
0.38	-64.94	-63.89	-62.89	-61.92	-61.00	-60.12
0.39	-66.65	-65.57	-64.54	-63.55	-62.61	-61.71
0.40	-68.36	-67.25	-66.19	-65.18	-64.22	-63.29

A comparison of Table 3 and Table 4 shows that the precise knowledge of the pumping cell temperature is very important.

4 Some Data taken on Nepheli

The following EPR data were taken with the cell Nepheli.

Table 5 Note: The errors in the last column are the linear sum of the errors for ν_1 and ν_2.

Date	Time	ν_1 [Hz]	ν_2 [Hz]	$\Delta\nu$ [kHz]
11/5	10:54 pm	$1.19011 \cdot 10^7 \pm 692$	$1.19074 \cdot 10^7 \pm 916$	6.30 ± 1.61
11/5	11:20 pm	$1.18871 \cdot 10^7 \pm 640$	$1.181957 \cdot 10^7 \pm 720$	8.60 ± 1.36
11/6	1:29 am	$1.18417 \cdot 10^7 \pm 635$	$1.18599 \cdot 10^7 \pm 744$	18.20 ± 1.38
11/6	12:19 pm	$1.18933 \cdot 10^7 \pm 808$	$1.19041 \cdot 10^7 \pm 725$	10.80 ± 1.53
11/6	2:02 pm	$1.19250 \cdot 10^7 \pm 634$	$1.19408 \cdot 10^7 \pm 765$	15.80 ± 1.40
11/6	3:10 pm	$1.19327 \cdot 10^7 \pm 662$	$1.19518 \cdot 10^7 \pm 817$	19.10 ± 1.48
11/6	5:17 pm	$1.19546 \cdot 10^7 \pm 457$	$1.19775 \cdot 10^7 \pm 586$	22.90 ± 1.04
11/6	6:20 pm	$1.19157 \cdot 10^7 \pm 451$	$1.19418 \cdot 10^7 \pm 530$	26.10 ± 0.98
11/6	7:13 pm	$1.19150 \cdot 10^7 \pm 478$	$1.19428 \cdot 10^7 \pm 589$	27.80 ± 1.07
11/6	7:31 pm	$1.19149 \cdot 10^7 \pm 480$	$1.19427 \cdot 10^7 \pm 575$	27.80 ± 1.06
11/6	7:43 pm	$1.19506 \cdot 10^7 \pm 539$	$1.19778 \cdot 10^7 \pm 662$	27.20 ± 1.20
11/6	9:01 pm	$1.19149 \cdot 10^7 \pm 1219$	$1.19446 \cdot 10^7 \pm 1491$	29.70 ± 2.71
11/6	10:50 pm	$1.19471 \cdot 10^7 \pm 660$	$1.19794 \cdot 10^7 \pm 554$	32.30 ± 1.21
11/6	11:10 pm	$1.19497 \cdot 10^7 \pm 1052$	$1.19805 \cdot 10^7 \pm 567$	30.80 ± 1.62
11/7	3:48 pm	$1.18943 \cdot 10^7 \pm 617$	$1.19327 \cdot 10^7 \pm 613$	38.40 ± 1.23
11/7	11:38 pm	$1.19231 \cdot 10^7 \pm 693$	$1.19653 \cdot 10^7 \pm 889$	42.20 ± 1.58

In order to extract the polarization in the *pumping cell* at this point I have to assume that the pumping cell temperature is somewhere between 170°C and 190°C and I further will calculate the polarization values for the m_F: -2↔-3 (B_0=25.12 G) and m_F: +2↔+3 (B_0=26.12 G).

Table 6 Possible polarization values for measured $\Delta\nu_{EPR}$. Signs for polarizations are ignored.

	T = 170°C -2↔-3	T = 170°C +2↔+3	T = 190°C -2↔-3	T = 190°C +2↔+3
$\Delta\nu$ [kHz]	P	P	P	P
6.30	0.038	0.041	0.037	0.040
8.60	0.052	0.056	0.050	0.054
18.20	0.110	0.119	0.106	0.115
10.80	0.065	0.071	0.063	0.068
15.80	0.095	0.103	0.092	0.100
19.10	0.115	0.125	0.112	0.121
22.90	0.138	0.149	0.134	0.145
26.10	0.157	0.170	0.153	0.165
27.80	0.168	0.181	0.163	0.176
27.80	0.168	0.181	0.163	0.176
27.20	0.164	0.177	0.159	0.172
29.70	0.179	0.194	0.174	0.188
32.30	0.195	0.211	0.189	0.204
30.80	0.186	0.201	0.180	0.195
38.40	0.232	0.251	0.225	0.243
42.20	0.254	0.275	0.247	0.267

Clearly, it is easy to measure the B-field and find out whether it is 25.12 G or 26.12 G.

If I remember correctly, the polarization values calculated here are *lower* than the ones extracted from EPR. I will include the error analysis in the next version.

References

[1] N.F. Ramsey; *Moelcular Beams*, Oxford Science Publications (1990).

[2] S.R. Schaefer *et al.*; Phys. Rev. A39, 5613 (1989).

[3] M.V. Romalis *et al.*; *Polarized Gas Targets and Polarized Beams*, AIP Conf. Proc. 421, 46 (1997).

[4] M.V. Romalis, Ph.D Thesis, Princeton.