Electromagnetic Structure of Few-Body Nuclei from keV to GeV Energies

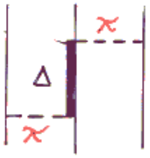
- NNN forces: i) pd scattering
  ii) spectra of light nuclei ($A \leq 12$)
- Nuclear EM currents: i) radiative captures
  ii) $A = 2-3$ magnetic structure
- Approaches to relativistic dynamics: deuteron f.f.
- Quasielastic inclusive response: FSI and many-body currents
- FSI at GeV energies: i) Glauber approximation to $^3\text{He}(e,e'p)d$
  ii) $^4\text{He}(\vec{e},e'\vec{p})^3\text{H}$ and in medium p f.f.
- Coulomb sum rule and correlations
- Conclusions and acknowledgments

The Nuclear Hamiltonian

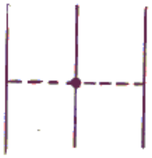
- NN interactions alone fail to predict:
  - i) spectra of light nuclei
  - ii) pd scattering
  - iii) nuclear matter $E_0(\rho)$

- NNN interactions:

$$V^{2\pi} = A^{PW}_{2\pi}$$



$$+ A^{SW}_{2\pi}$$



p-wave

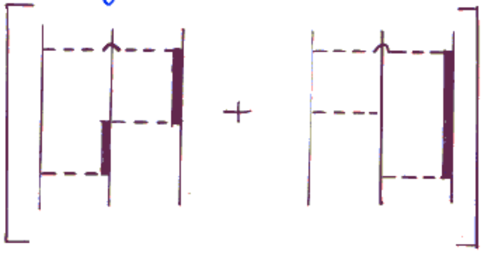
s-wave (very weak)

- EFT w/o explicit Δ's overestimates strength of $A^{PW}_{2\pi}$
(Pandharipande, Phillips, and van Kolk, PRC ('05))

- $V^{2\pi}$ alone does not fix problems i) – iii)

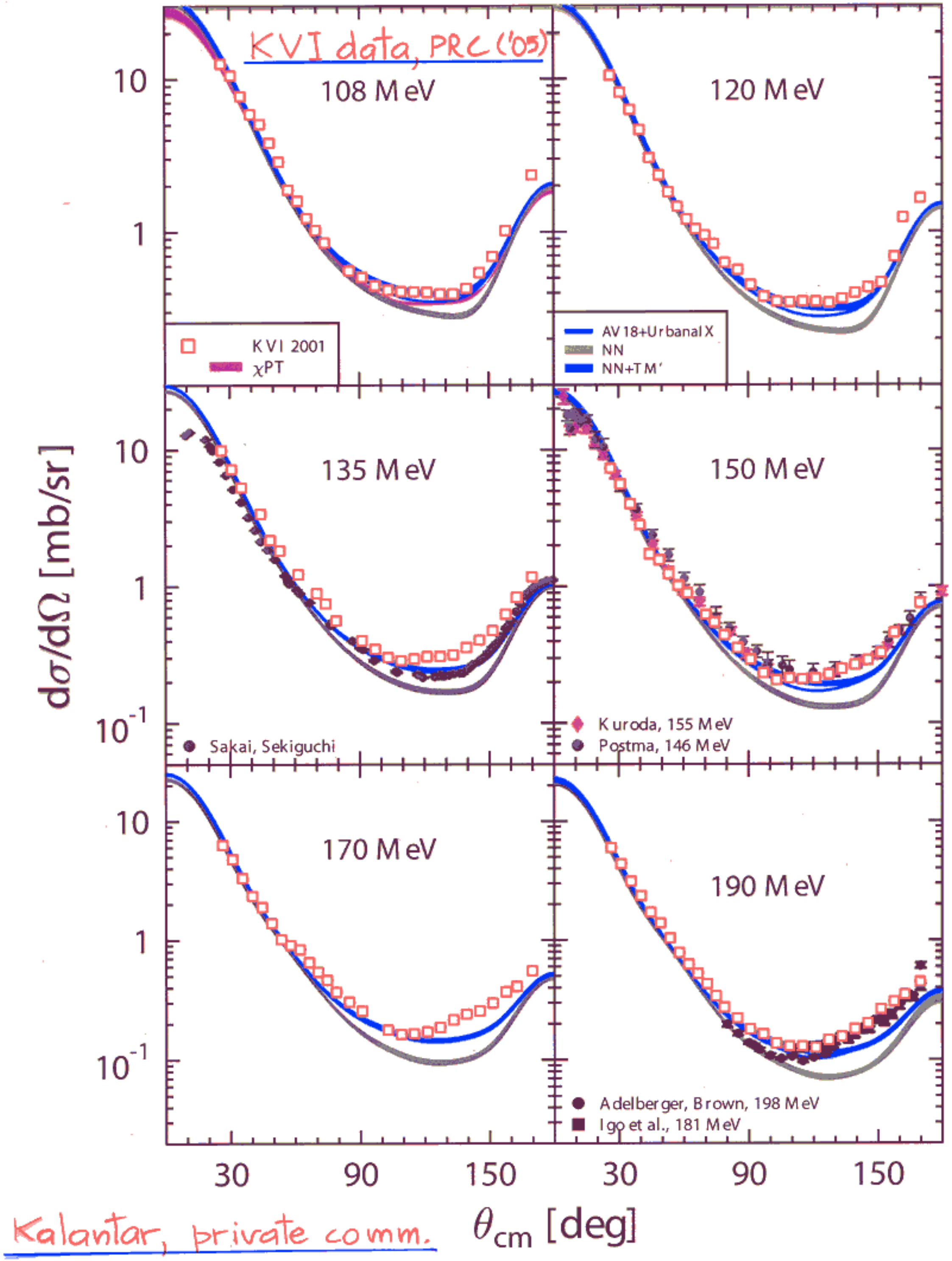
- Beyond 2π-exchange:

$$V = V^{2\pi} + A_{3\pi}$$



$$+ A_R \sum_{cyc} T^2(r_{ij})\, T^2(r_{jk})$$

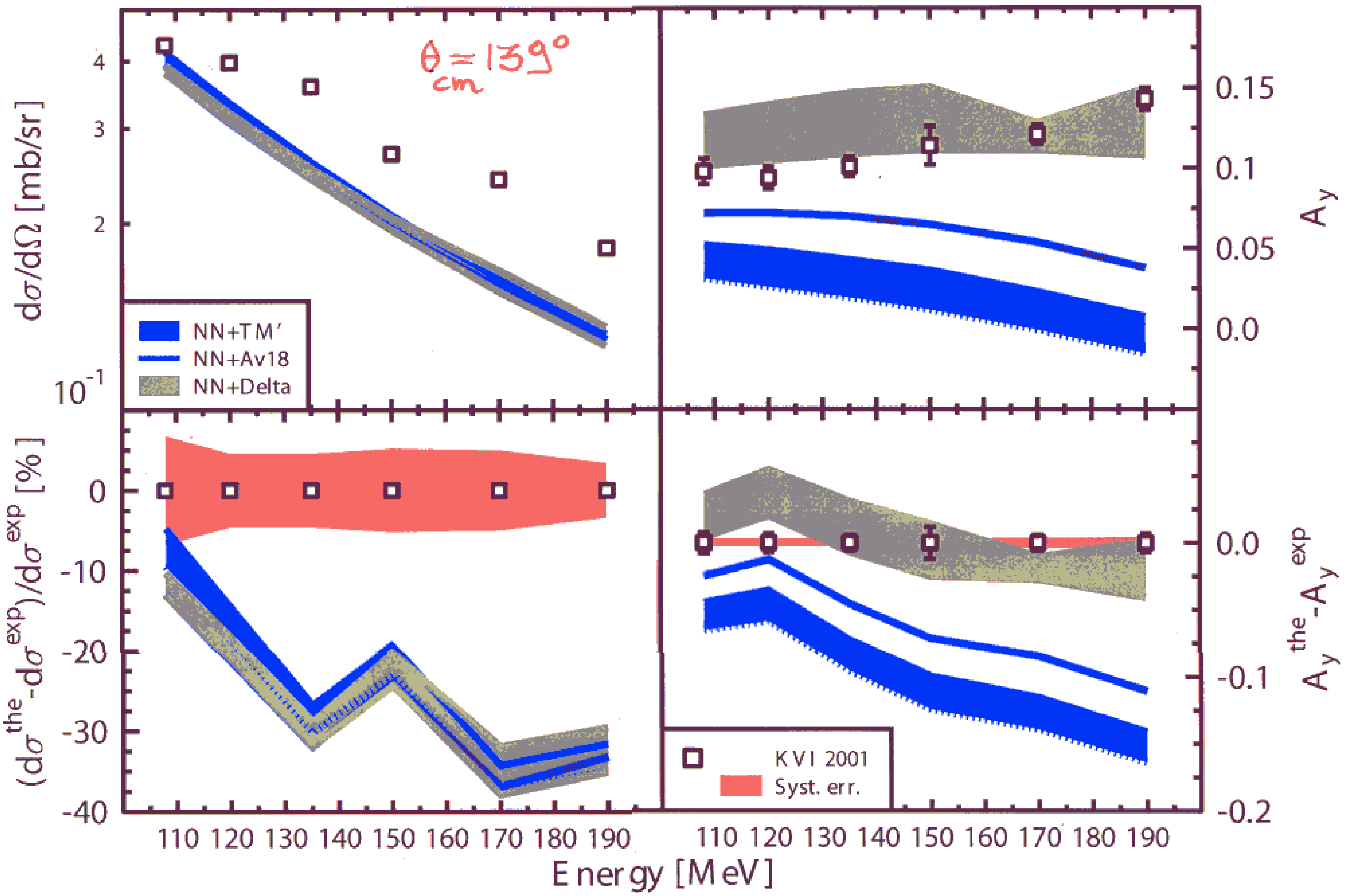
parameters (~3) determined by fitting energies of 17 low-lying states of $A \leq 8$



KVI data, PRC ('05)

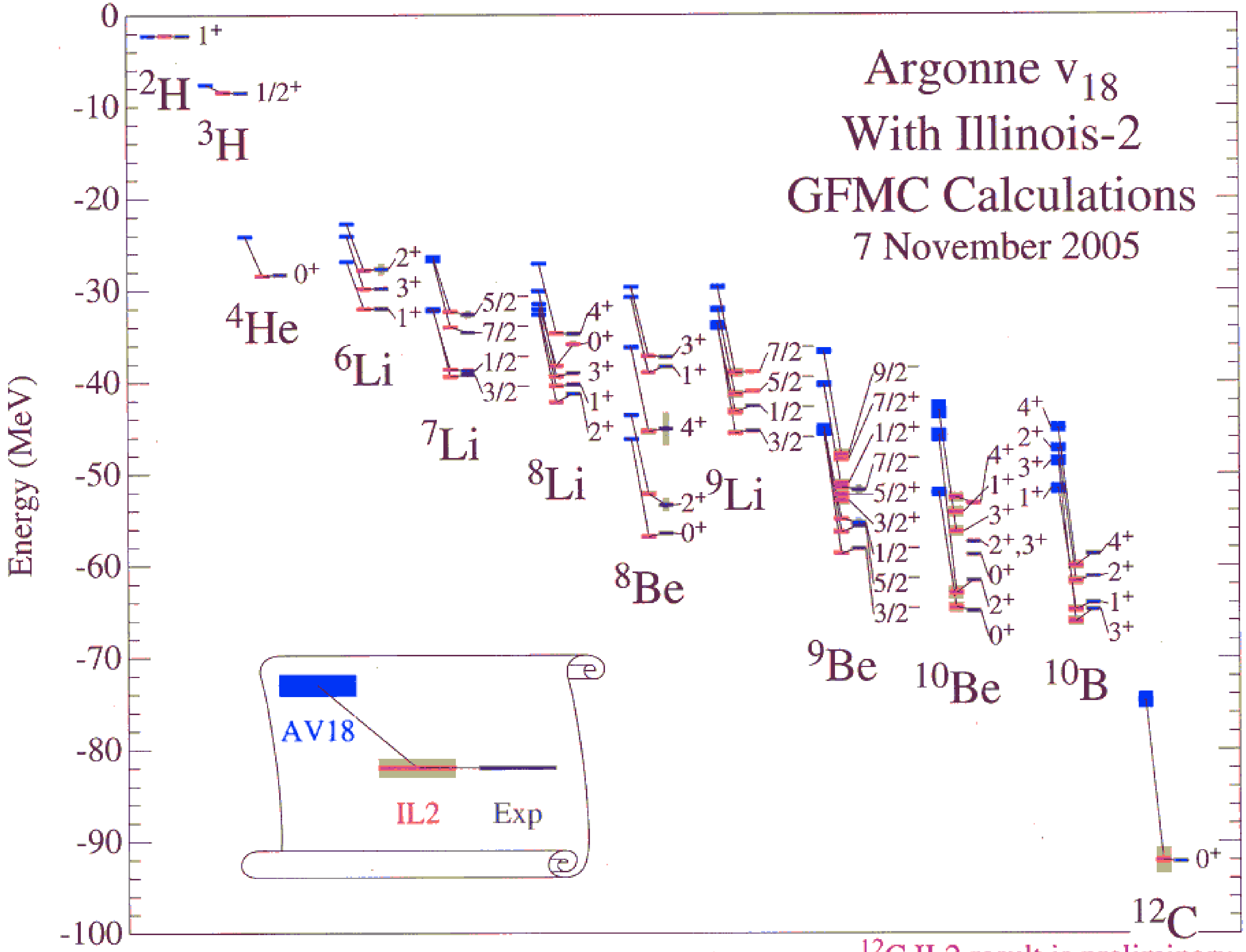
Kalantar, private comm.

KVI data, PRC ('05)



Kalantar, private comm.

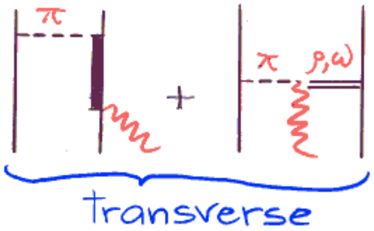
Pieper and Wiringa, private comm.



12C IL2 result is preliminary.

Nuclear Electromagnetic Currents*

$$\vec{j} = \vec{j}^{(1)} + \vec{j}^{(2)}(v) + \vec{j}^{(3)}(V^{2\pi})$$



- gauge invariant:

$$\vec{q}\cdot[\vec{j}^{(1)} + \vec{j}^{(2)}(v) + \vec{j}^{(3)}(V^{2\pi})] = [T + v + V^{2\pi}, \rho]$$

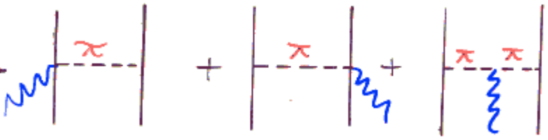
charge operator

- leading terms from static part of v:

$$\vec{j}^{(2)}(v_0) = 3i(\vec{\tau}_i \times \vec{\tau}_j)_z G_E^V \Big[v_{PS}(k_j)\, \vec{\sigma}_i\, (\vec{\sigma}_j \cdot \vec{k}_j) + \frac{\vec{k}_i - \vec{k}_j}{k_i^2 - k_j^2}\, v_{PS}(k_i)(\vec{\sigma}_i \cdot \vec{k}_i)(\vec{\sigma}_j \cdot \vec{k}_j) \Big] + i \rightleftarrows j$$

$v_{PS} = (v^{\sigma\tau} - 2v^{t\tau})/3$

$\vec{j}^{(2)}(v_0) \longrightarrow$



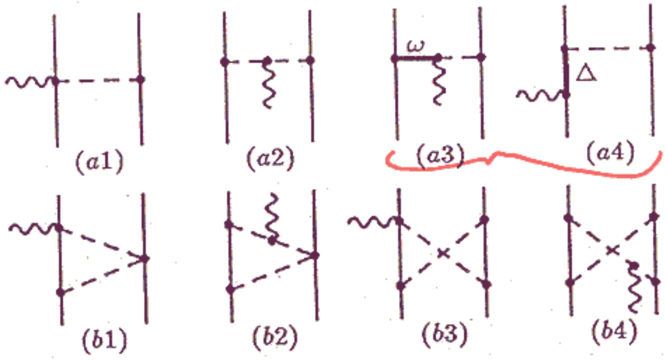
at long range

* Marcucci et al., PRC 72 ('05)

The $np \to d\gamma$ capture at thermal energy

	σ(mb)	
SNPA	332.8 ± 1.4	← interaction model dependence
EFT*	334 ± 2	T.-S. Park, D.-P. Min, and M. Rho PRL 74, 4153 (1995)
EXP	332.6 ± 0.7	

EFT* =



countertems obtained via ω- and Δ-resonan saturation

+ AV18 and radial cutoff r_c

PV A_γ in $\vec{n}p \to d\gamma$: $A_\gamma \vec{\sigma}_n \cdot \hat{q}$

$$A_\gamma = -\sqrt{2}\ \mathrm{Re}(M_1^* E_1)/|M_1|^2$$

E_1 : $|{}^3S_1, PC\rangle \longrightarrow |d, PV\rangle$ induced by v_π^{PV} predominantly

$|{}^3P_1, PV\rangle \longrightarrow |d, PC\rangle$

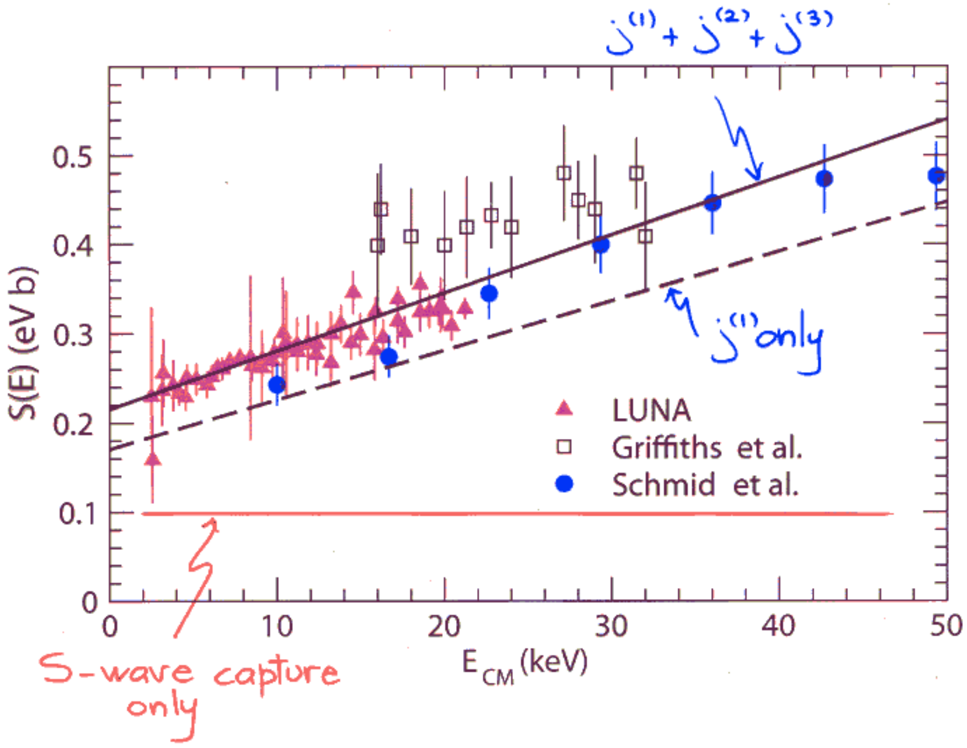
	$A_\gamma \times 10^8$
SNPA	$-4.9 \pm 0.1 \longrightarrow -.107 \times h_\pi^1$
EFT*	$-4.6 \pm \quad \longrightarrow -.10 \times$

↖ C.-O. Hyun, T.-S. Park, and D.-P. Min, PLB 516, 321 (2001)

pd radiative capture at $E \leq 50$ keV

(Marcucci et al., PRC 72 ('05))

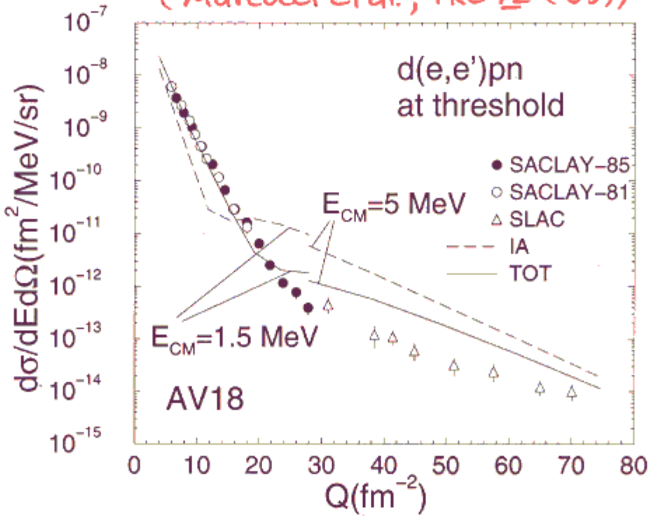
- suppressed process, S- and P-wave capture channels are both important



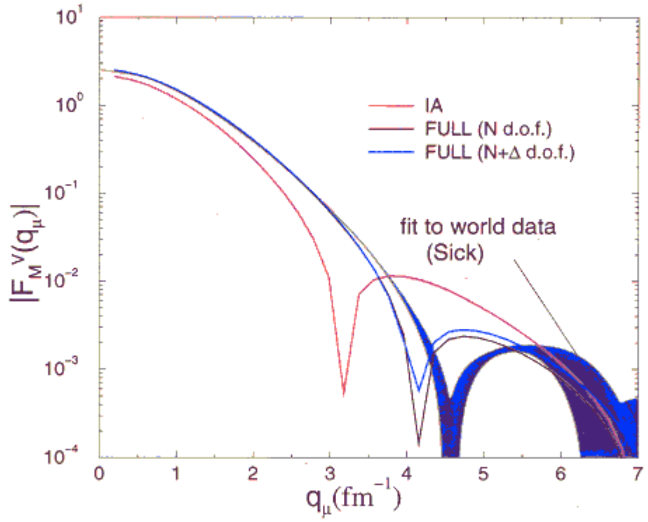
	S(0) (eV·b)	
Theory	0.219	but overestimates $\sigma^{\gamma}(nd)$ by 9%
Luna-exp	0.216 ± 0.010	

Isovector Magnetic Structure

(Marcucci et al., PRC 72 ('05))

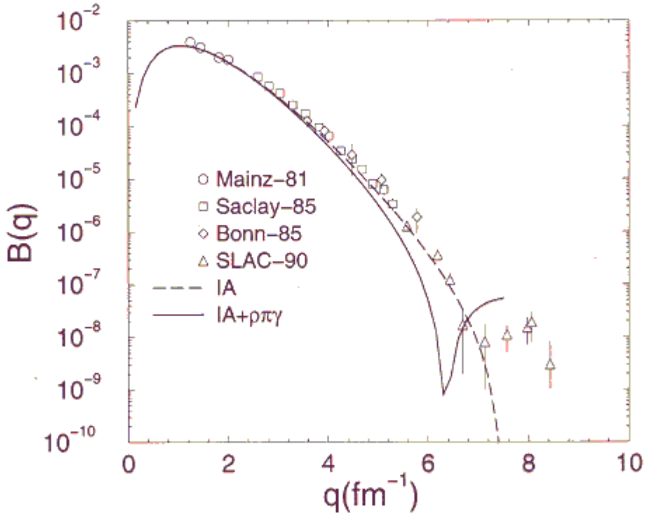


$$F_M^V = \left[F_M(^3He) - F_M(^3H)\right]/2$$

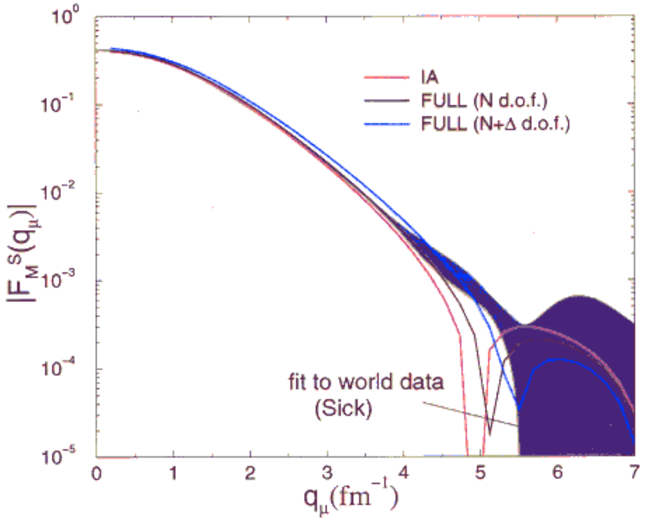


Isoscalar Magnetic Structure

(Marcucci et al., PRC 72 ('05))



$$F_M^S = \left[F_M(^3He) + F_M(^3H) \right] / 2$$



Relativistic Dynamics and Deuteron Form Factors

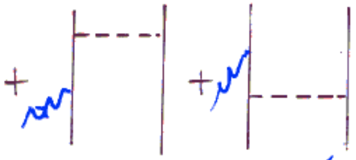
- Hamiltonian dynamics : $\mathcal{N}$ d.o.f. only

i) rest frame : $H = 2\sqrt{p^2+m^2} + \underset{\nwarrow}{v}$ includes relativistic OPEP

ii) boost :

$$\psi_{\vec{\beta}}(\vec{p}) = \frac{1}{\sqrt{\gamma}}\Big[1 - \frac{i}{4m}\underbrace{\vec{\beta}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}}_{\text{Thomas precession}}\Big]\psi_0(\underbrace{\vec{p}_{\parallel}/\gamma}_{\text{Lorentz contraction}}, \vec{p}_{\perp})$$

iii) $j^\mu = \frac{1}{2}\bar{u}'\left[F_1^S\gamma^\mu + \frac{i}{2m}F_2^S\sigma^{\mu\nu}q_\nu\right]u +$



2-body

- Spectator formalism (Gross eq.) : explicit $\mathcal{N}, \bar{\mathcal{N}}$ d.o.f.

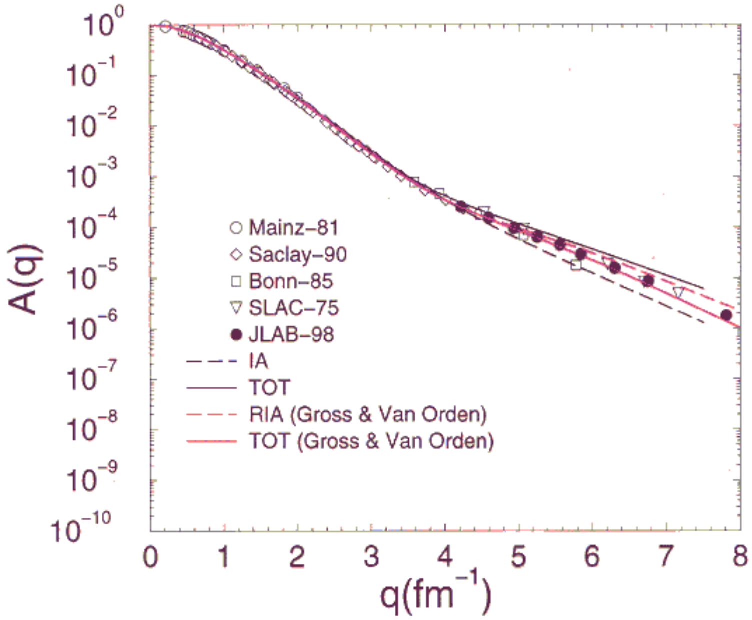
i) bound state :

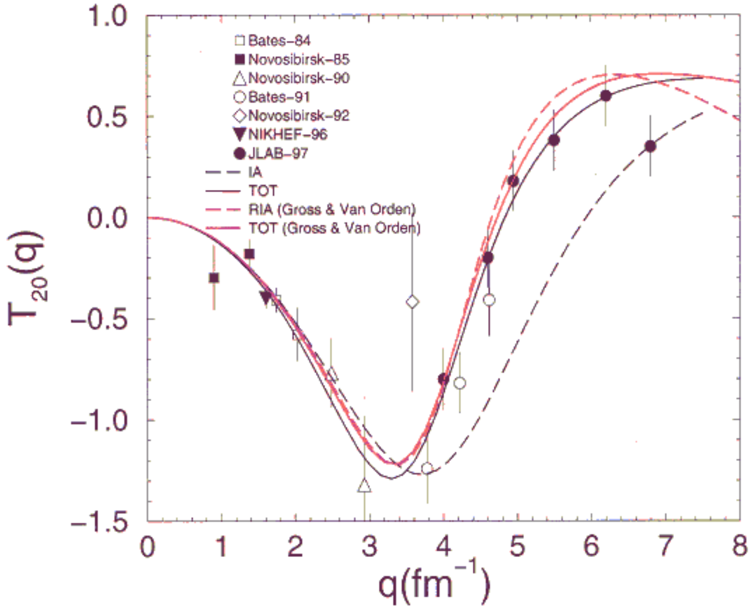
$$V = \sum_i \text{OBE}_i$$

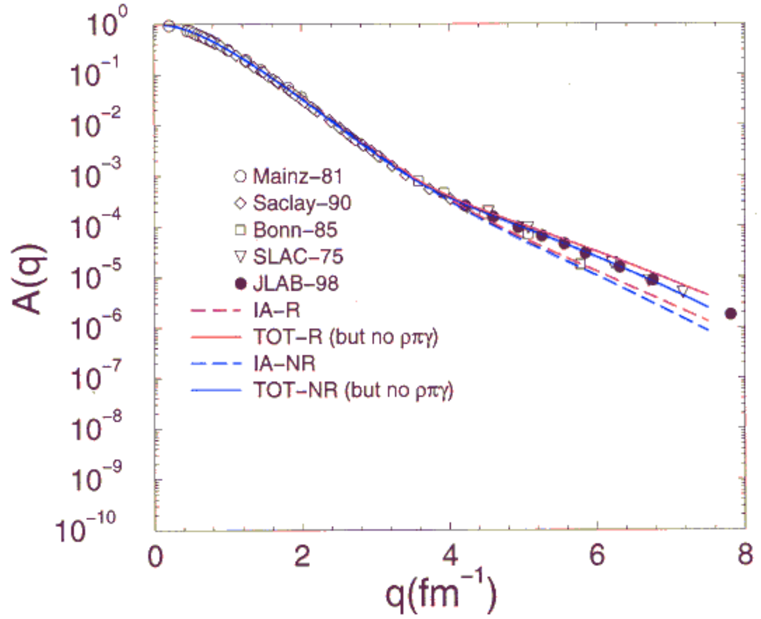
$$\psi_0 = \psi_S + \psi_D + \underbrace{\psi_{P,S=1} - \psi_{P,S=0}}_{\text{P-waves because of } \bar{\mathcal{N}} \text{ d.o.f.}}$$

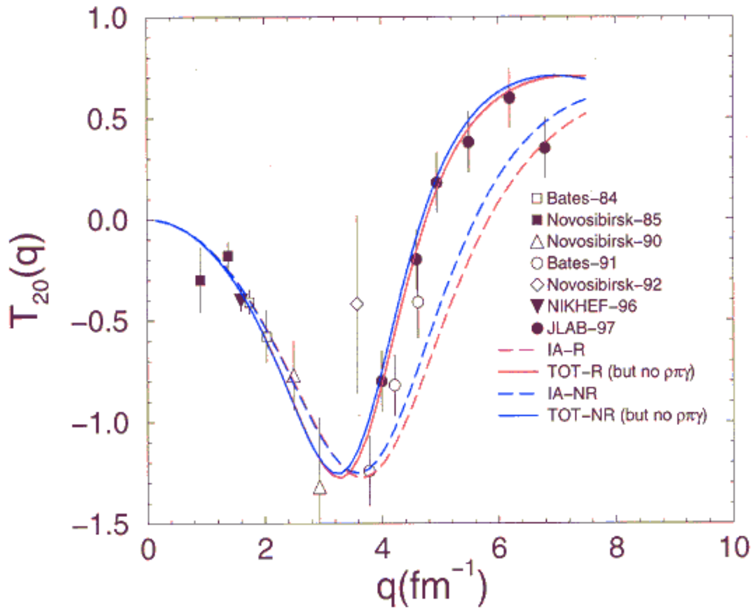
ii) V and j^μ include off-shell couplings

iii) boosts kinematical, and exactly included


A(q)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
1
2
3
4
5
6
7
8
$q(fm^{-1})$
○ Mainz−81
◇ Saclay−90
□ Bonn−85
▽ SLAC−75
● JLAB−98
IA
TOT
RIA (Gross & Van Orden)
TOT (Gross & Van Orden)

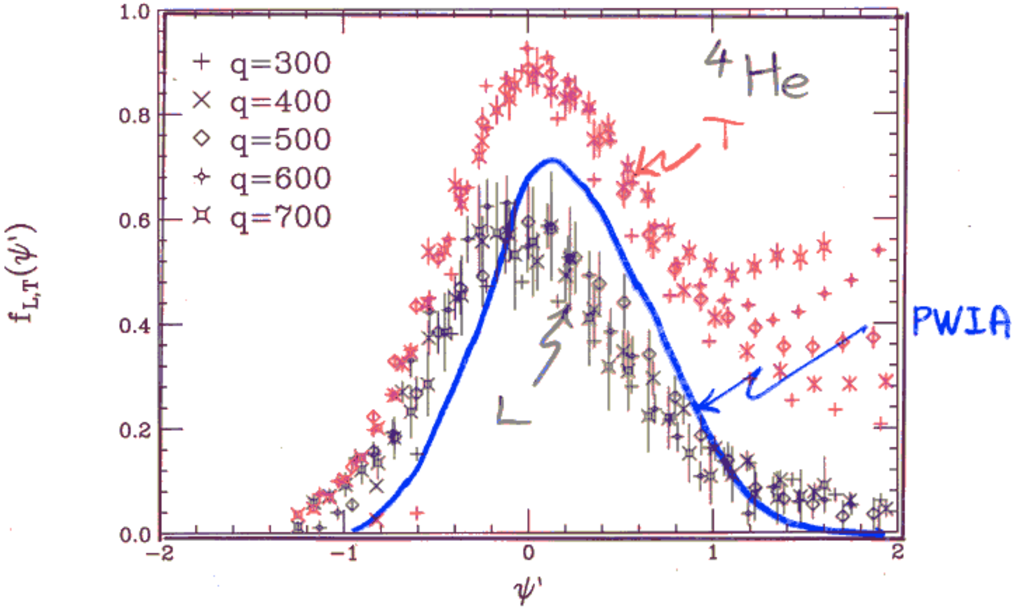

$T_{20}(q)$
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
6
7
8
$q(fm^{-1})$
□ Bates−84
■ Novosibirsk−85
△ Novosibirsk−90
○ Bates−91
◇ Novosibirsk−92
▼ NIKHEF−96
● JLAB−97
IA
TOT
RIA (Gross & Van Orden)
TOT (Gross & Van Orden)


A(q)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0
2
4
6
8
q(fm^-1)
○ Mainz−81
◇ Saclay−90
□ Bonn−85
▽ SLAC−75
● JLAB−98
IA−R
TOT−R (but no ρπγ)
IA−NR
TOT−NR (but no ρπγ)


T20(q)
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10
q(fm^-1)
□ Bates−84
■ Novosibirsk−85
△ Novosibirsk−90
○ Bates−91
◇ Novosibirsk−92
▼ NIKHEF−96
● JLAB−97
IA−R
TOT−R (but no ρπγ)
IA−NR
TOT−NR (but no ρπγ)

Inclusive (e,e') Scattering

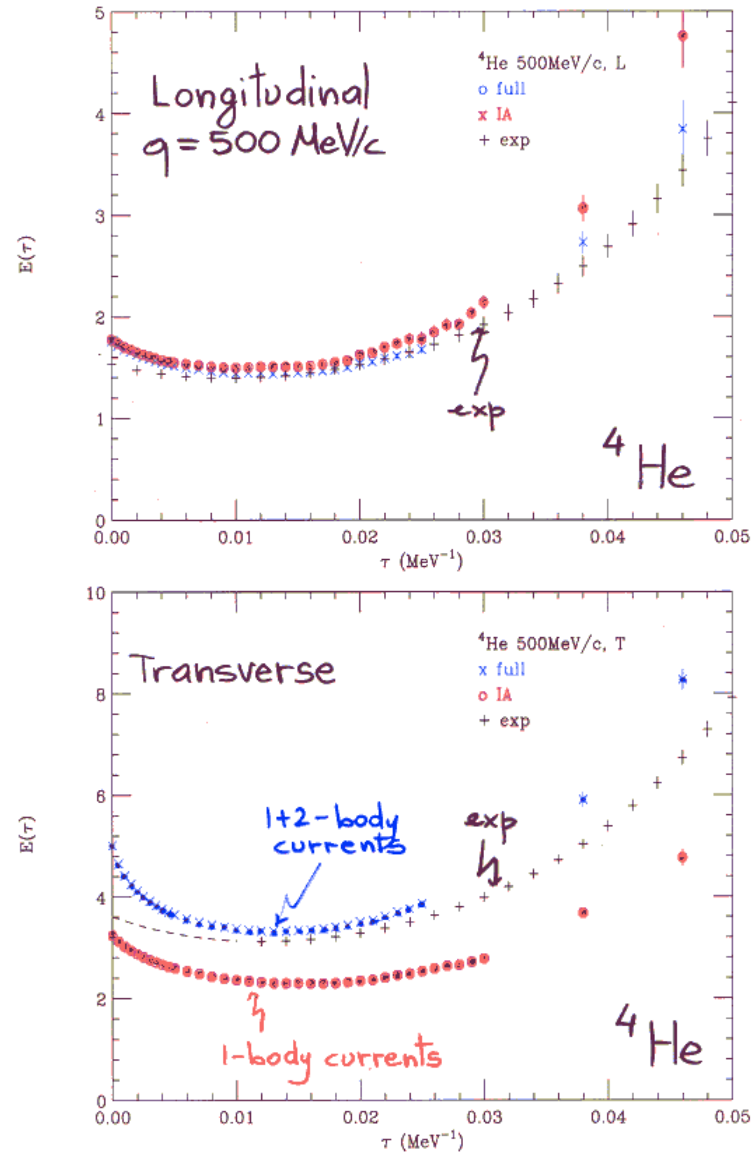
scaling analysis
(Jourdan and Sick)



$$f_{L,T} \equiv k_F \frac{R_{L,T}}{G_{L,T}}$$ in PWIA: $f_L = f_T$

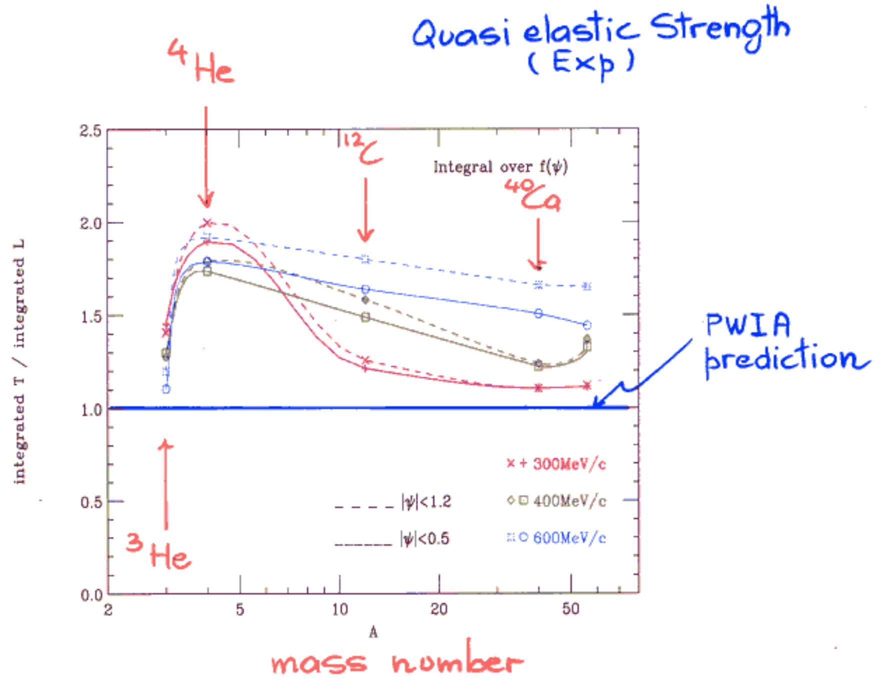
$$E_{L,T}(q,\tau) = e^{\tau \frac{q^2}{2m}} \int_{\omega_{th}}^{\infty} d\omega \, e^{-\tau\omega} R_{L,T}(q,\omega)$$

exact GFMC calculation*

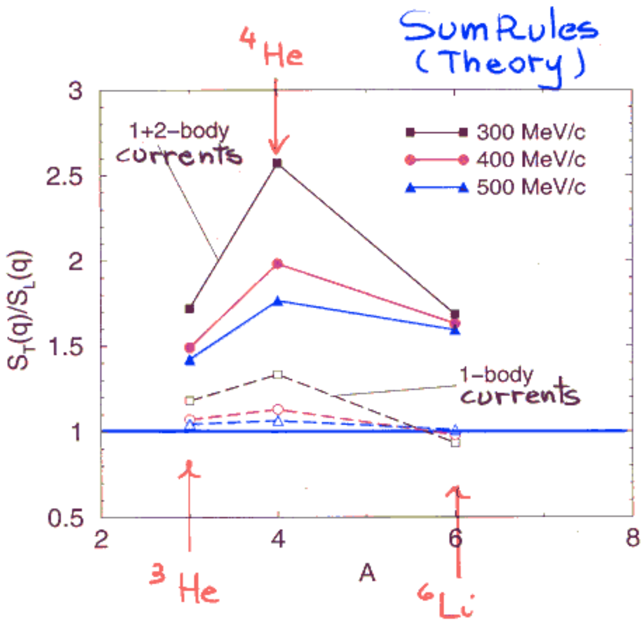


* Carlson and Schiavilla, PRL 68 ('92); PRC 49 ('94)

$$\frac{\int_{qe} d\omega\, f_T(\omega)}{\int_{qe} d\omega\, f_L(\omega)}$$



$$\frac{\int_{\omega_{th}}^{+\infty} d\omega\, f_T(\omega)}{\int_{\omega_{th}}^{+\infty} d\omega\, f_L(\omega)}$$



Carlson, Jourdan, Schiavilla, and Sick , PRC 65 ('02)

FSI in the Glauber Approximation

FSI inducing operator

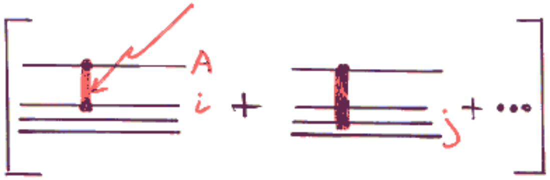
$$\psi(p + \overset{(A-1)}{f}; GLB) = \frac{1}{\sqrt{A}} \sum_{\wp} \varepsilon_{\wp}\, G(A; 1 \cdots A-1)\, \phi(PW)$$

$$\phi(PW) \to e^{i\vec{p}\cdot\vec{r}_A} \chi_{\sigma}(A)\, \phi_{\sigma_f}(1 \cdots A-1; f)$$

- $G(A; 1 \cdots A-1) = 1\!\!1$

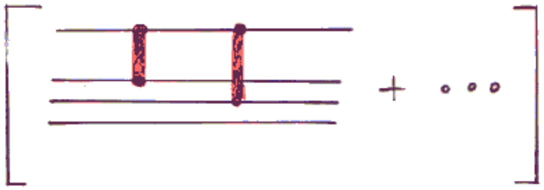
$\overset{NN}{\Gamma}(b_{iA}; s_{iA})$

$-$



single rescattering

$+$



double rescattering

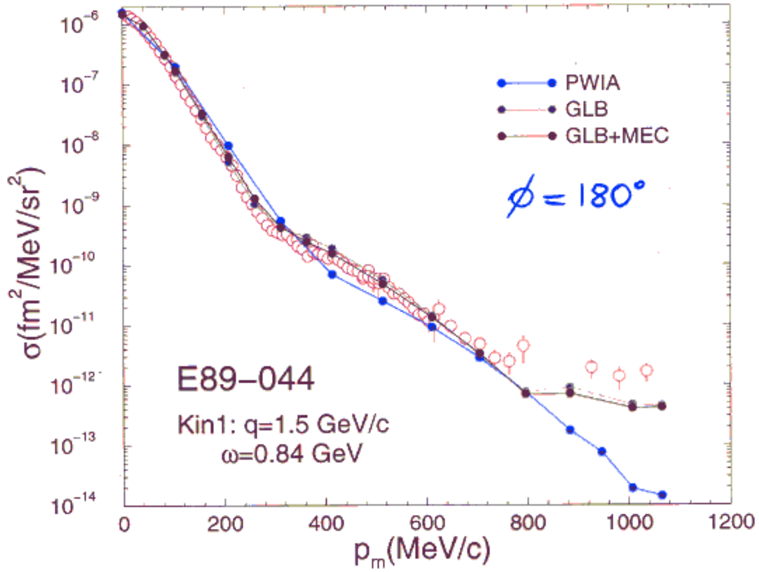
$- \cdots$ up to (A-1) rescattering terms

- $\overset{NN}{\Gamma}_{ij}(\vec{b}; s) = \frac{1}{2\pi i p} \int d^2k\, e^{-i\vec{k}\cdot\vec{b}}\, \overset{NN}{F}_{ij}(\vec{k}; s)$ ← NN amplitude

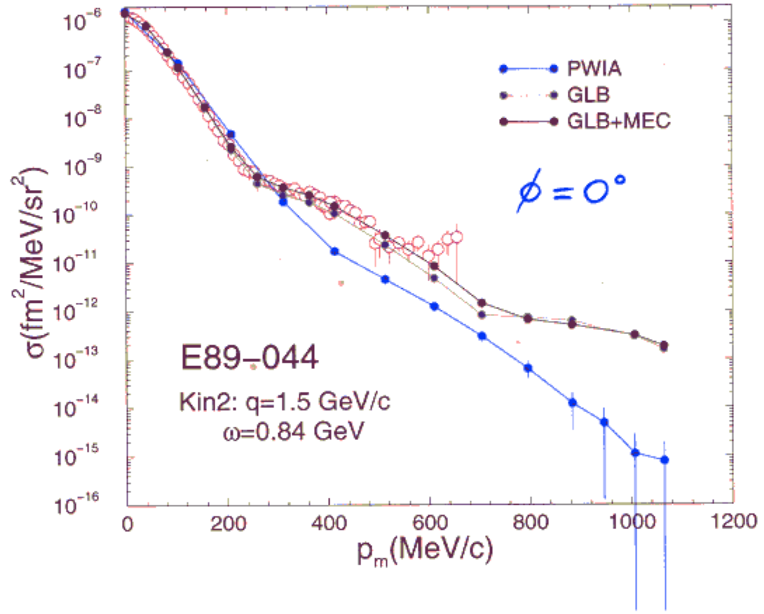
i) in CM : $F^{NN} = F_c^{NN} + \underbrace{4 \text{ spin flip terms}}_{\text{mostly ignored: Ciofi et al., Laget, ...}}$

ii) boost : CM frame ⟶ Rescattering frame

$^3He(e,e'p)d$

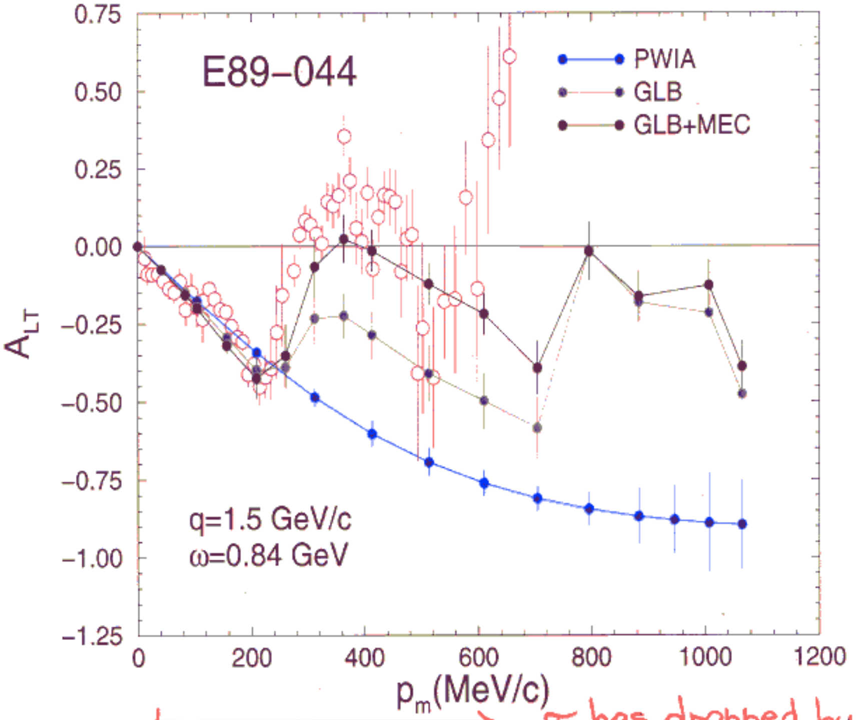


PWIA
GLB
GLB+MEC
$\phi = 180°$
E89–044
Kin1: q=1.5 GeV/c
ω=0.84 GeV
$\sigma(fm^2/MeV/sr^2)$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
0
200
400
600
800
1000
1200
p_m(MeV/c)



PWIA
GLB
GLB+MEC
$\phi = 0°$
E89–044
Kin2: q=1.5 GeV/c
ω=0.84 GeV
$\sigma(fm^2/MeV/sr^2)$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
0
200
400
600
800
1000
1200
p_m(MeV/c)

$^3He(e,e'p)d$

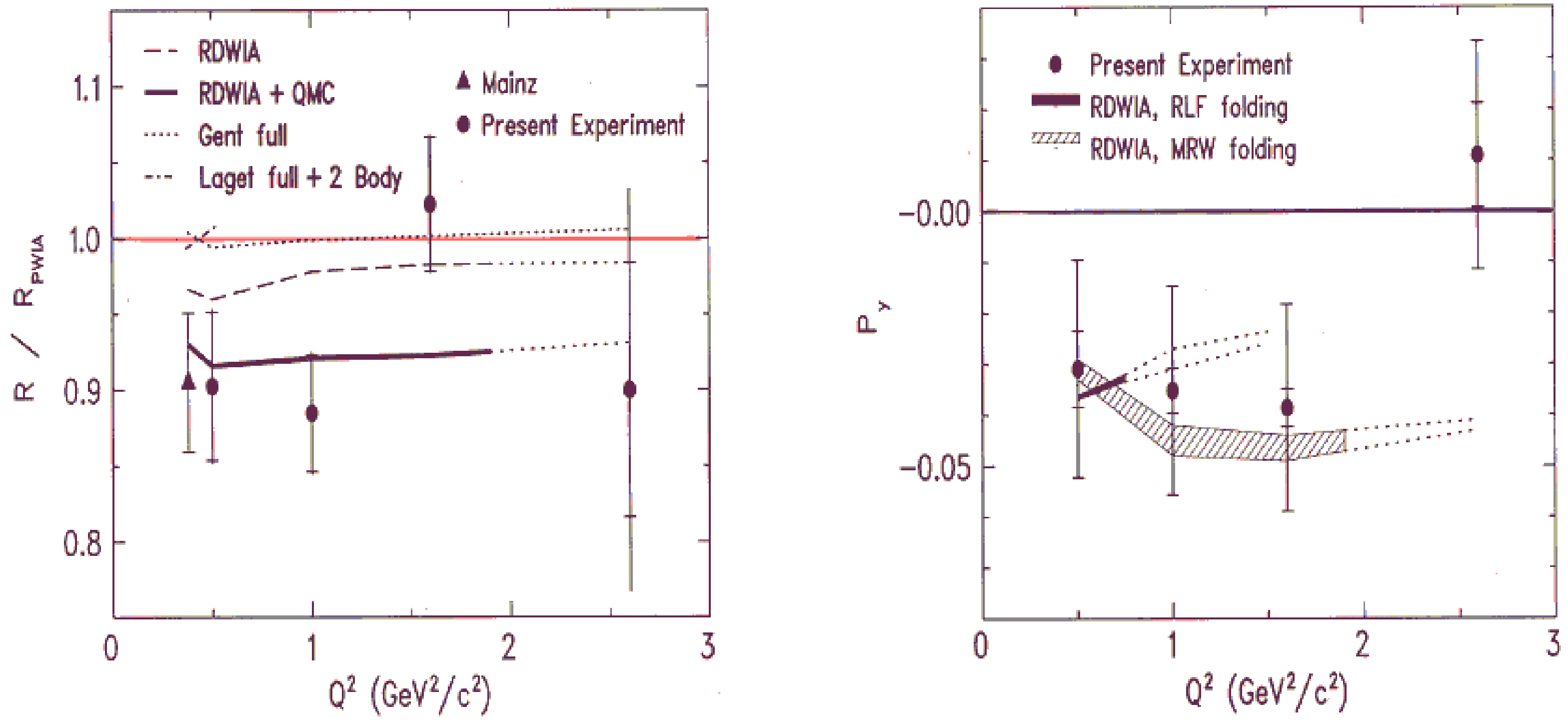


$\longmapsto$ σ has dropped by $\gtrsim 10^{-4}$

$$A_{LT} = \frac{\sigma(\phi=0^\circ) - \sigma(\phi=180^\circ)}{\sigma(\phi=0^\circ) + \sigma(\phi=180^\circ)} \propto R_{LT}$$

$^4He(\vec{e},e'\vec{p})^3H$

Strauch et al., PRL 91 ('03)



$R = P_x' / P_z' \propto G_{Ep}/G_{Mp}$ on a free proton

FSI in A=4: Optical Potential

- 1+3 complex, energy-dependent potential:

relative energy

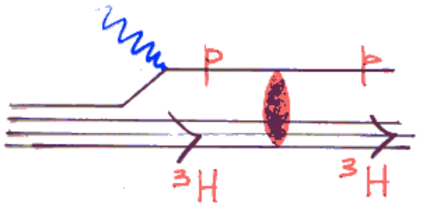
$$v^{OPT}_{1+3}(r;T_k) = v^{c}(r;T_k) + v^{b}(r;T_k)\,\vec{\ell}\cdot\vec{s} + \left[v^{c\tau}(r;T_k) + v^{b\tau}(r;T_k)\,\vec{\ell}\cdot\vec{s}\right]\vec{\tau}\cdot\vec{\tau}_3$$

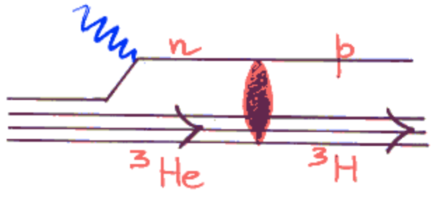
i) $v^{x}(r;T_k)$ constrained by fits to:

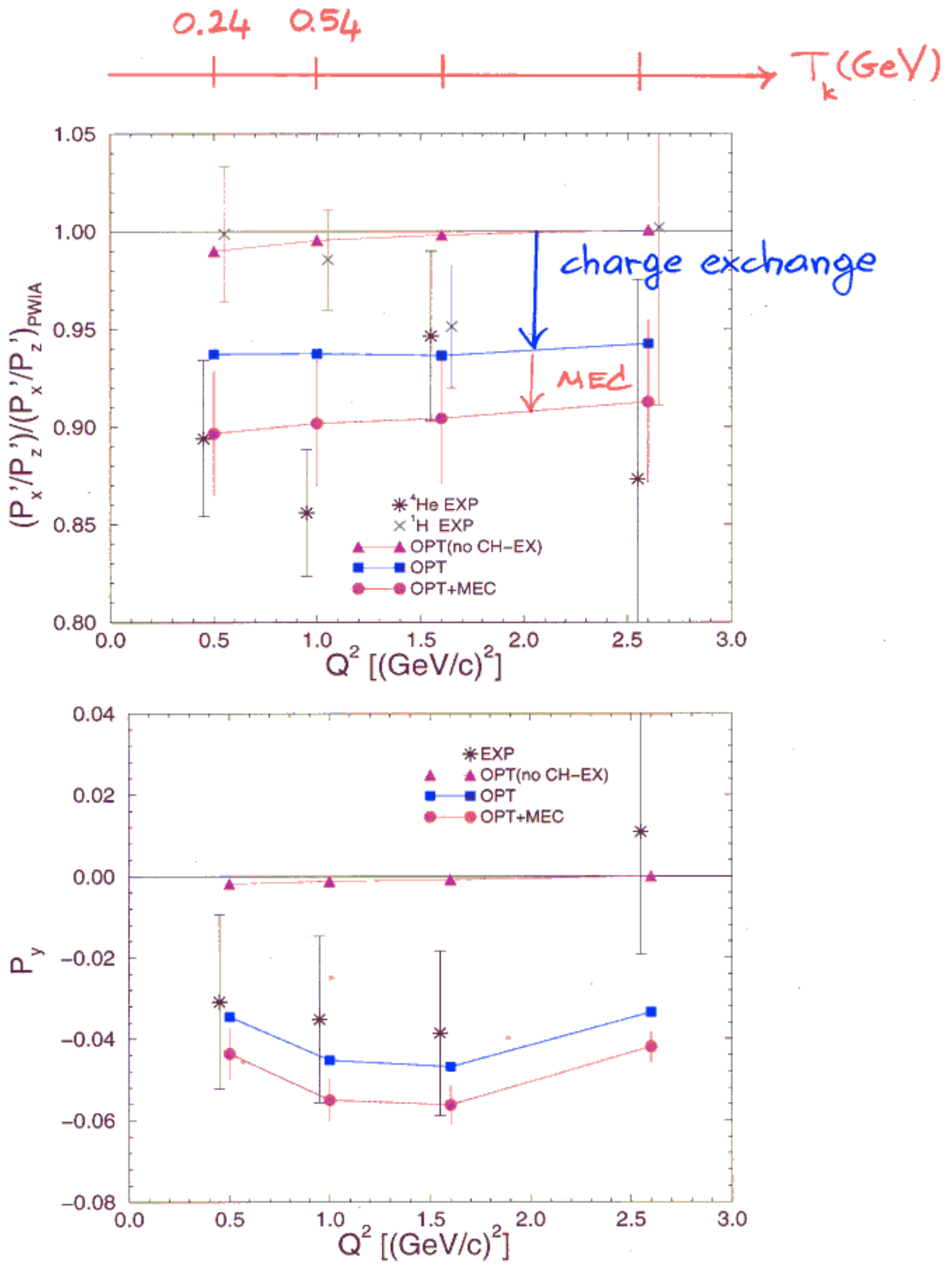
$p\,^3H \longrightarrow p\,^3H$ $T_k = (110-450)$ MeV (Van Oers et al., PRC 25)

$\longrightarrow n\,^3He$ $T_k = (43, 117)$ MeV (Schiavilla et al., PRL 94)

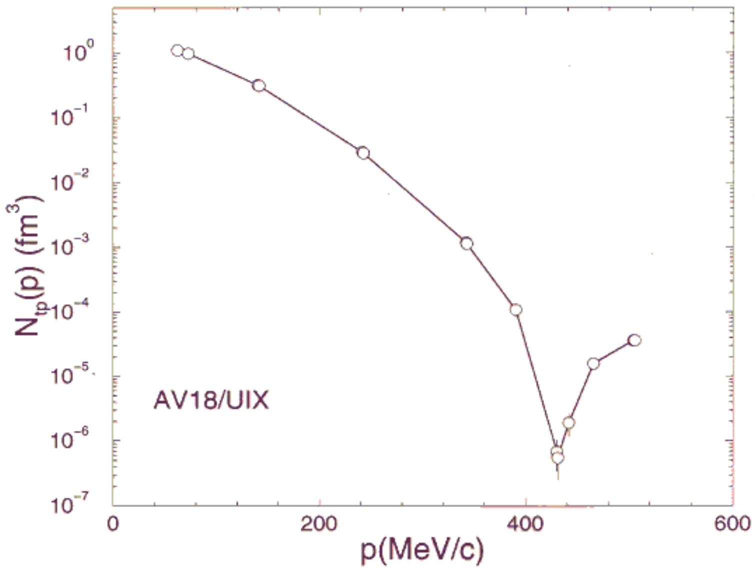
ii) charge-exchange mechanism plays important role in $^4He(\vec{e}, e'\vec{p})^3H$

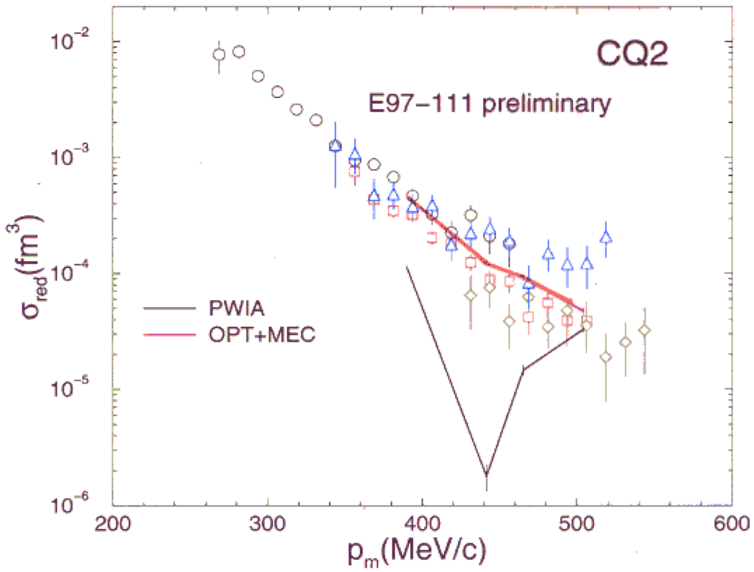




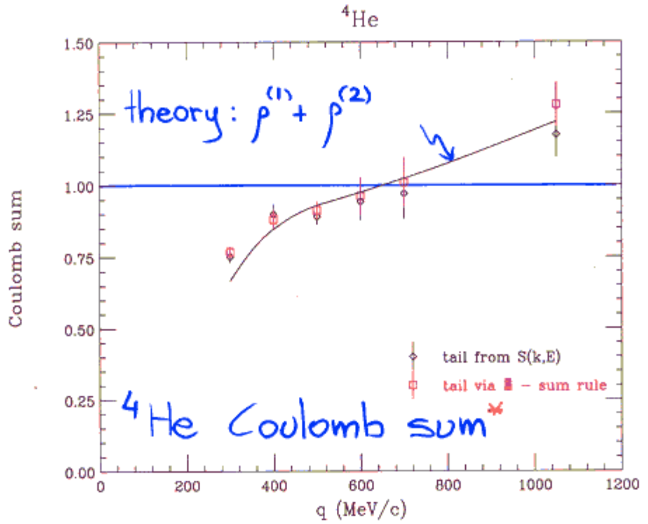

0.24
0.54
T_k (GeV)
charge exchange
MEC
1.05
1.00
0.95
0.90
0.85
0.80
(P_x'/P_z')/(P_x'/P_z')_PWIA
⁴He EXP
¹H EXP
OPT(no CH-EX)
OPT
OPT+MEC
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Q² [(GeV/c)²]
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
P_y
EXP
OPT(no CH-EX)
OPT
OPT+MEC
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Q² [(GeV/c)²]

$^4He(e,e'p)^3H$



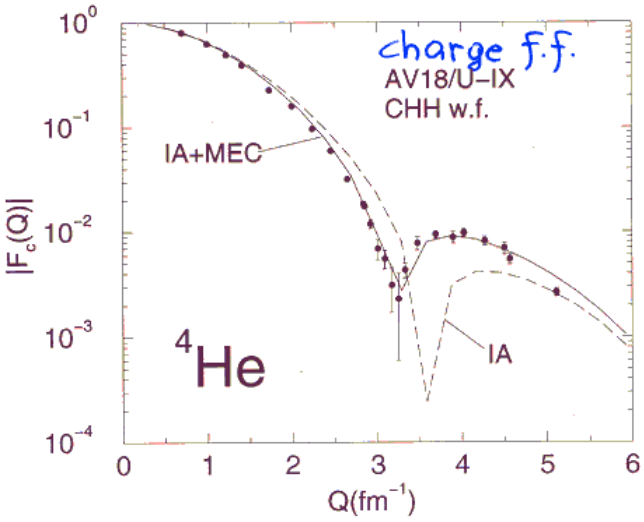


$$\sigma_{red} = \sigma / (p \cdot E_p \cdot \text{recoil} \cdot \sigma_{cc1})$$



$$S_L(q) = \frac{1}{Z} \int d\omega \, \frac{R_L(q,\omega)}{\widetilde{G}_E(Q^2)^2} \longrightarrow 1$$

q large and
1-body $\rho^{(1)}$



* Carlson, Jourdan, Schiavilla, and Sick, PLB 553 ('03)

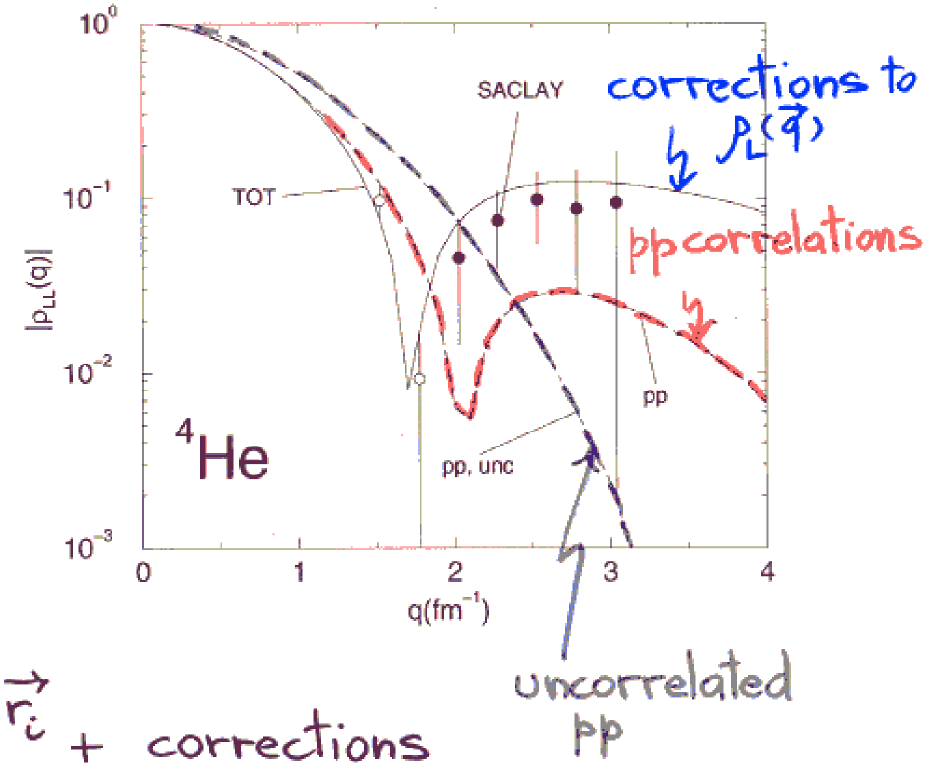
Short-range correlations

$$\rho_{LL}(q) = \frac{1}{Z}\langle 0|\rho_L^\dagger(\vec{q})\,\rho_L(\vec{q})|0\rangle - 1$$ ~ Fourier transform of pp distribution fnt. + corrections

$$= S_L(q) - 1 + Z|F_c(q)|^2$$

charge f.f.

Coulomb sum



$$\rho_L(\vec{q}) = \sum_i \frac{1+\tau_{zi}}{2}\, e^{i\vec{q}\cdot\vec{r}_i} + \text{corrections}$$

Conclusions and Acknowledgments

- Nuclei seem to be well described in terms of N d.o.f. up to GeV energies
- Evidence for medium modification of p f.f. is controversial

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