Interactions, Currents, and Light Nuclei: a Review

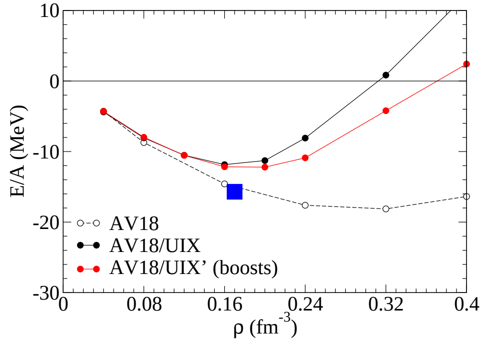
Rocco Schiavilla (JLab/ODU)

- A realistic model of strong and electromagnetic interactions in nuclei: an update (NNN forces, nuclear EM f.f.'s, ...)
- Tensor forces and ground state structure: probing tensor correlations via two-nucleon knock-out processes
- Isospin mixing in the nucleon and ^{4}He, and the PV asymmetry in $^4\mathrm{He}(\vec{e}, e')^4\mathrm{He}$
- Summary(ies)

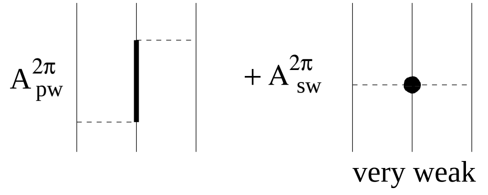
I. Nuclear Interactions and Currents: an Update

Nuclear Interactions

- NN interactions alone fail to predict:
  1. spectra of light nuclei
  2. Nd scattering
  3. nuclear matter $E_0(\rho)$



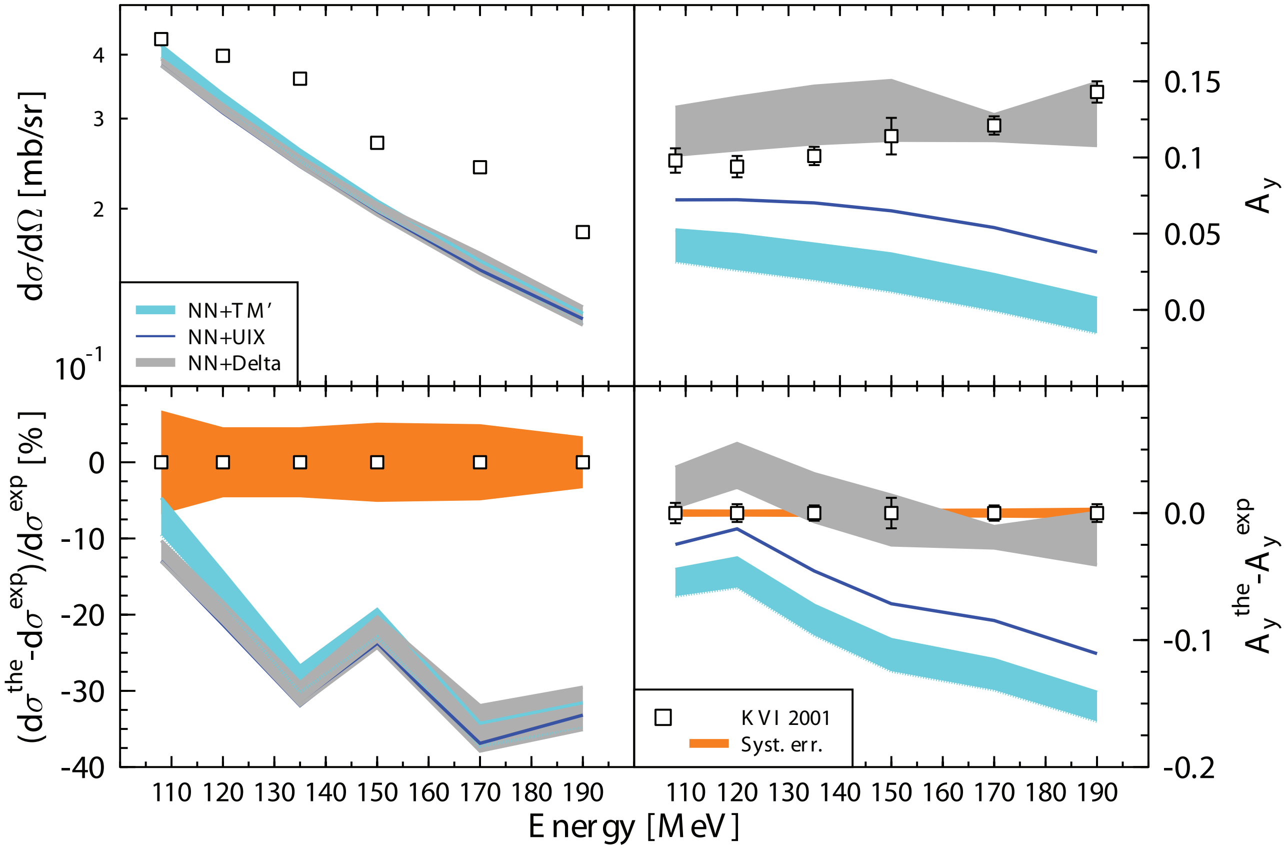
- 2π-NNN interactions [EFT w/o explicit Δ's overestimates strength of $V_{\rm pw}^{2\pi}$, Pandharipande *et al.*, PRC**71**, 064002 (2005)]:



- $V^{2\pi}$ alone does not fix problems above

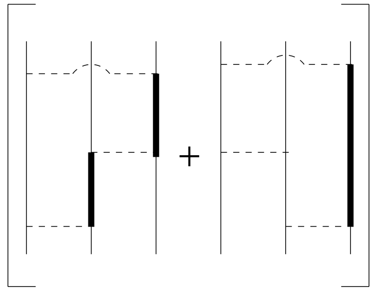
Proton-Deuteron Elastic Scattering

Ermisch *et al.* (KVI collaboration), PRC**71**, 064004 (2005); Kalantar-Nayestanaki, private communication



Beyond 2π-exchange (IL2 model, with important $T = 3/2$ terms)

$V^{2\pi} + A^{3\pi}$



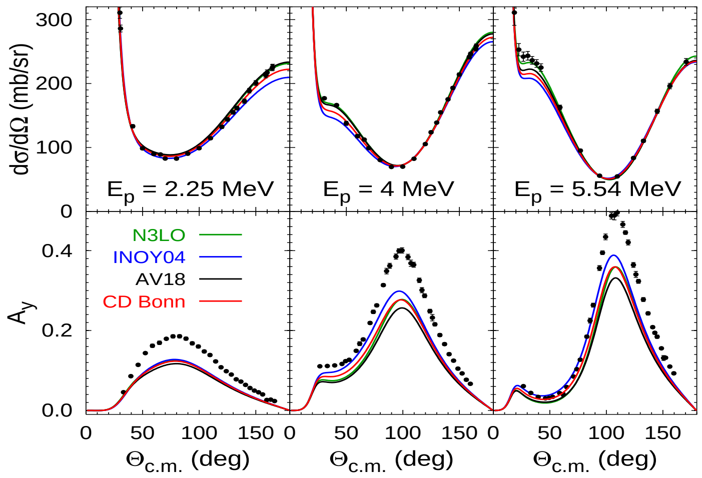
$+ A^R \sum_{cyc} T^2(r_{ij})\, T^2(r_{jk})$

parameters (~ 3) fixed by a best fit to the energies of low-lying states of nuclei with $A \leq 8$

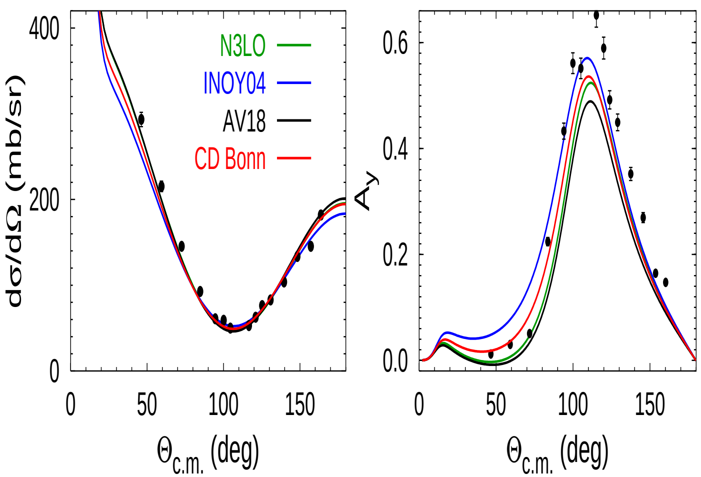
- AV18/IL2 Hamiltonian reproduces well spectra of A=9–12 nuclei (attraction provided by IL2 in $T = 3/2$ triplets crucial for p-shell nuclei)
- but needs to be tested in three- and four-nucleon scattering (work by the Pisa group is in progress)
- A_y puzzle in 4-body scattering: strong isospin dependence, discrepancy in ^{3}H-p or ^{3}He-n much reduced relative to ^{3}He-p

Deltuva and Fonseca, PRL **98**, 162502 (2007) and nucl-th/0703066

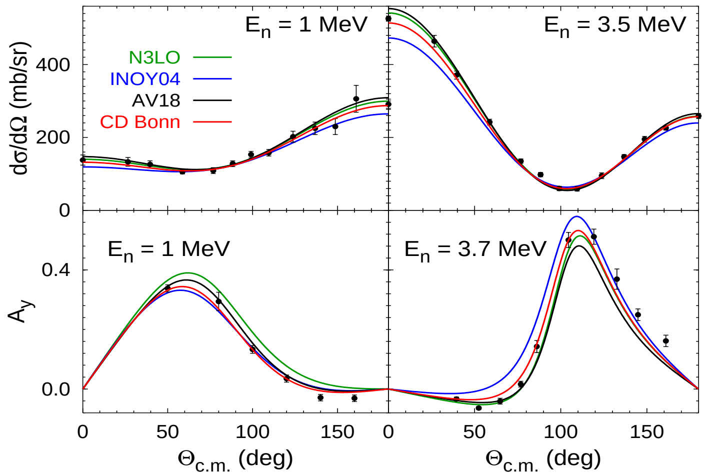
p-^{3}He



p-^{3}H



n-^{3}He

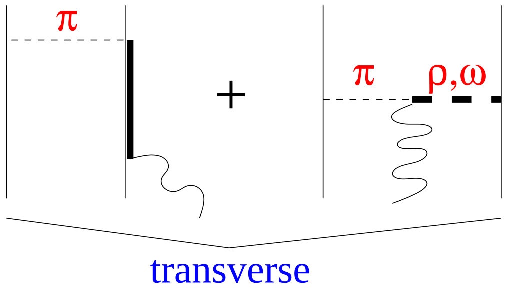


Nuclear Electromagnetic Currents

Marcucci *et al.*, PRC**72**, 014001 (2005)

$$\mathbf{j} = \mathbf{j}^{(1)}$$

$$+\, \mathbf{j}^{(2)}(\mathrm{v}) +$$



$$+\, \mathbf{j}^{(3)}(\mathrm{V}^{2\pi})$$

- Gauge invariant:

$$\mathbf{q} \cdot \left[\mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V^{2\pi})\right] = \left[T + v + V^{2\pi}\,,\ \rho\right]$$

ρ is the nuclear charge operator

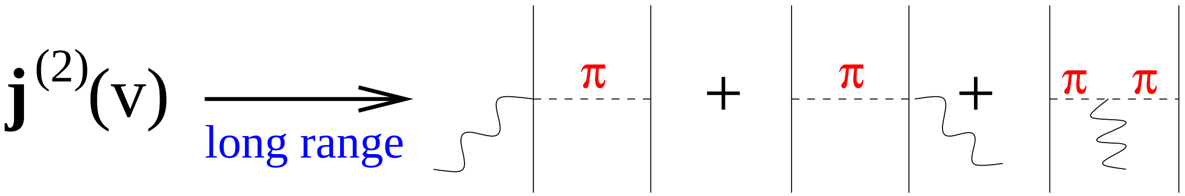
- Terms from static part v_0 of v (and $V^{2\pi}$) assumed to arise from pion-like (PS) and rho-like (V) exchanges:

$$\mathbf{j}_{ij}(v_0; PS) = \mathrm{i}\,(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z \Bigg[v_{PS}(k_j) \boldsymbol{\sigma}_i\, (\boldsymbol{\sigma}_j \cdot \mathbf{k}_j) + \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} v_{PS}(k_i)\, (\boldsymbol{\sigma}_i \cdot \mathbf{k}_i)\, (\boldsymbol{\sigma}_j \cdot \mathbf{k}_j) \Bigg] + i \leftrightharpoons j$$

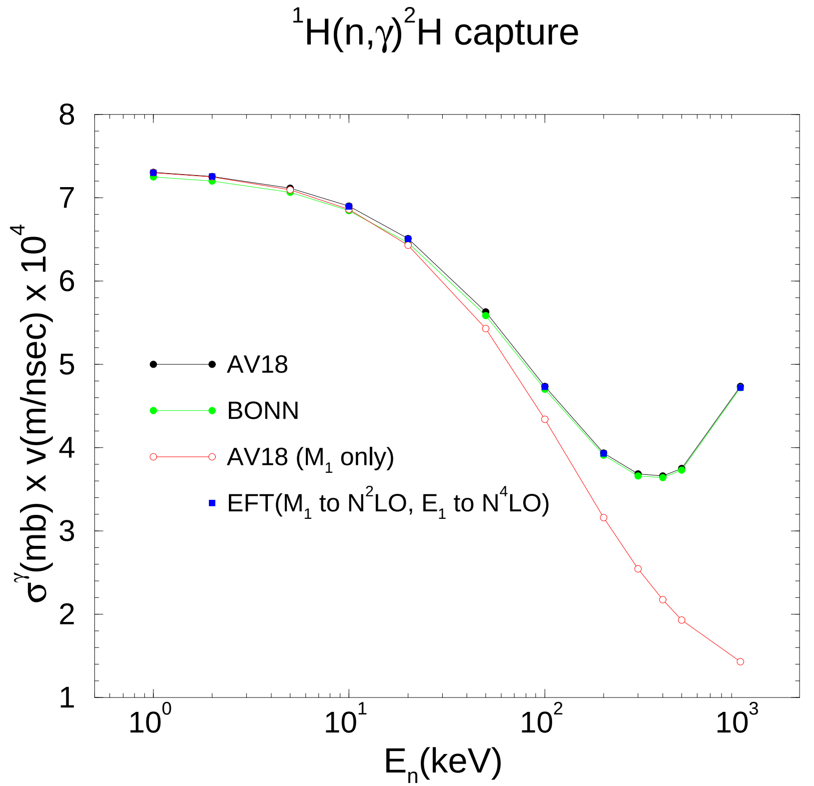
with $v_{PS} = v^{\sigma\tau} - 2\, v^{t\tau}$

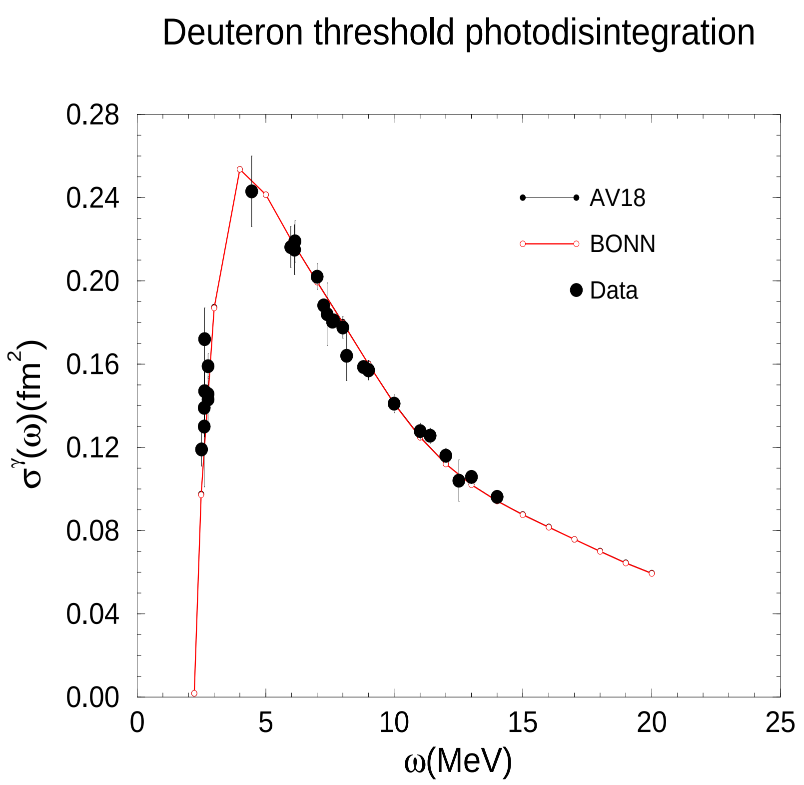
- Terms from velocity-dependent part v_1 of v by minimal substitution: $\mathbf{p}_i \rightarrow \mathbf{p}_i - e\,\mathbf{A}(\mathbf{r}_i)$

- $\mathbf{j}^{(2)}(v)$ satisfies:



Low-Energy Photoreactions in the np System

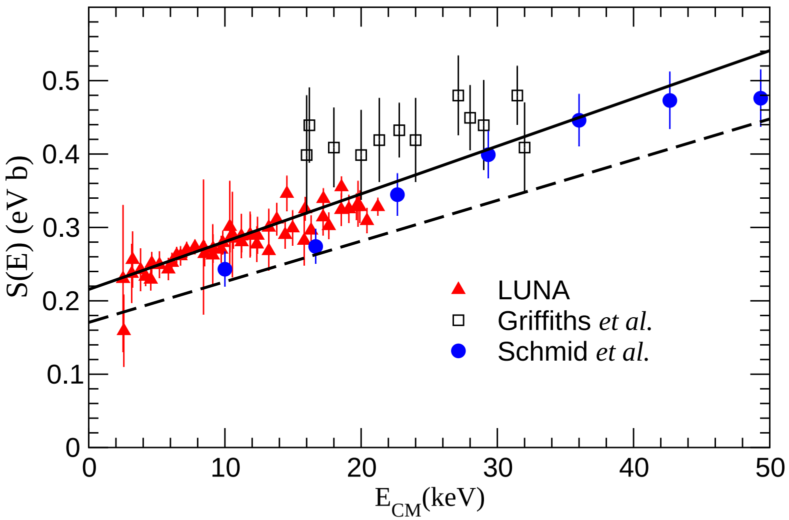




^{2}H$(p,\gamma)^3$He Radiative Capture at $E \leq 50$ keV

Marcucci *et al.*, PRC**72**, 014001 (2005)

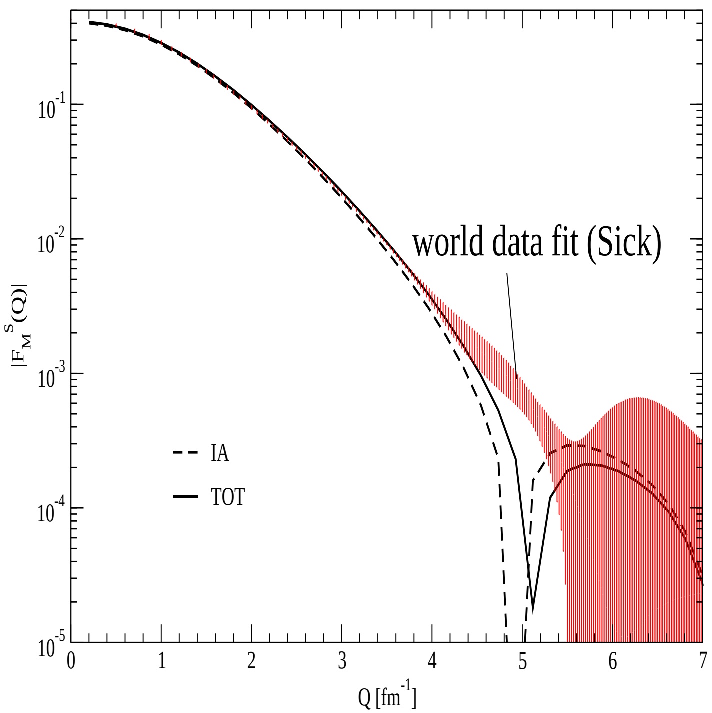
- Suppressed process, S- and P-wave capture both important

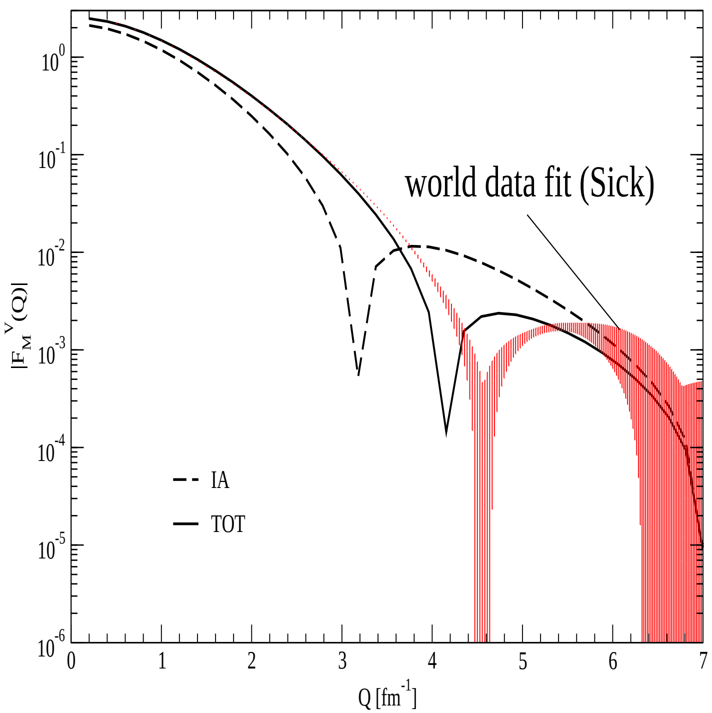


	S($E = 0$) (eV b)
Theory	0.219
LUNA	0.216±0.010

however, ^{2}H$(n,\gamma)^3$H experimental cross section at thermal energies is overestimated by theory by ≈ 9 %

Isoscalar and Isovector Magnetic Structure in A=3 Nuclei

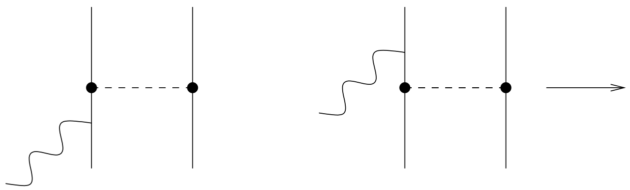




- diffraction region in F_M^V "problematic" for (present) theory: similar trend seen in deuteron threshold e-disintegration (Arriaga and Schiavilla, arXiv:0704.2514)

Nuclear Charge Operators

Leading two-body charge operator derived from analysis of the virtual pion photoproduction amplitudes:

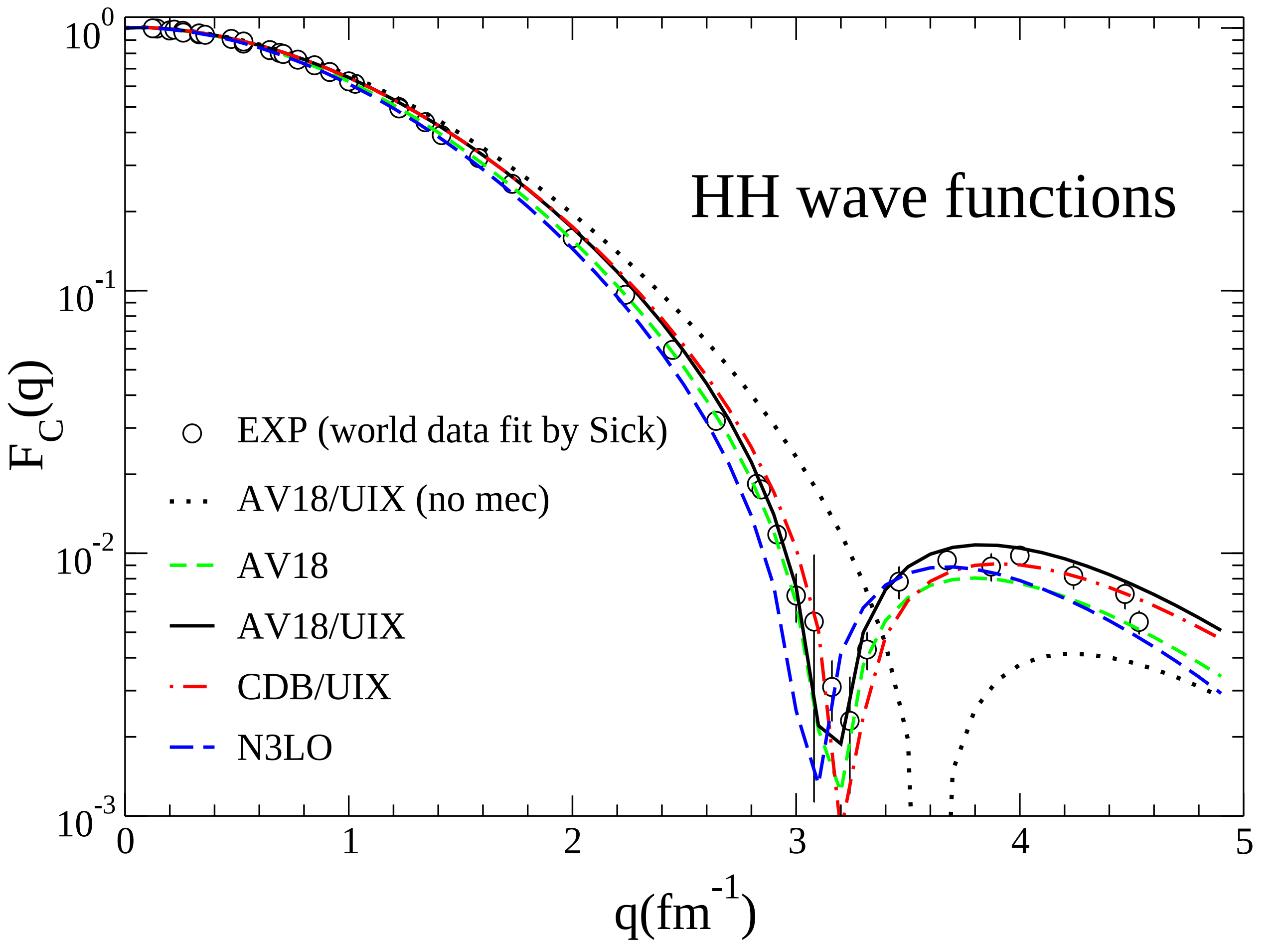


$$\text{diagram (a)} = v^{\pi}_{ij} \frac{1}{E_i - E} \frac{F_1^S + F_1^V \tau_{i,z}}{2} \rightarrow \text{included in IA}$$

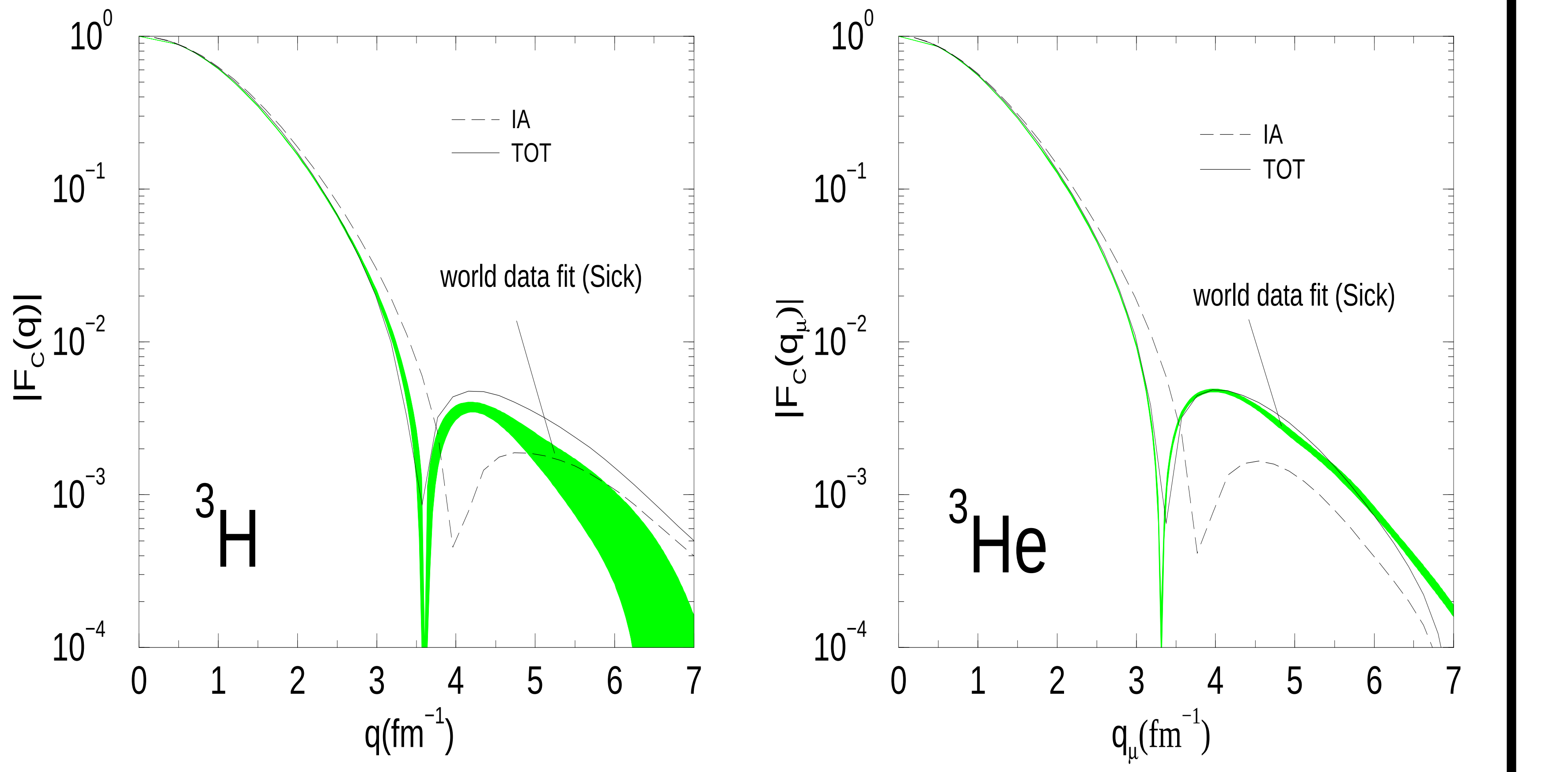
$$+ \frac{f^2}{4\, m\, m_\pi^2} \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q}\, \boldsymbol{\sigma}_j \mathbf{k}_j}{k_j^2 + m_\pi^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \frac{F_1^S + F_1^V \tau_{i,z}}{2} + \mathcal{O}(E_i - E)$$

- Essential for predicting the charge f.f.'s of ^{2}H, ^{3}H, ^{3}He, and ^{4}He
- Additional (small) contributions from vector exchanges as well as transition mechanisms like $\rho\pi\gamma$ and $\omega\pi\gamma$

^{4}He Charge Form Factor



A=3 Charge Form Factors



Summary (I)

- Energy spectra of light nuclei well described by two- and three-nucleon interactions (AV18/IL2)
- $3N$ and $4N$ scattering as a crucial testing ground for three-nucleon interactions (tests of IL2 are in progress)
- Constructed a conserved current, which reproduces well light-nuclei EM observables with a few exceptions: ${}^2\mathrm{H}(n,\gamma)^3\mathrm{H}$, diffraction region in $F_M^V(q)$, ...

II. Tensor Correlations in Nuclei: New Opportunities

Preeminent features of v_{ij}:

- <u>short-range repulsion</u>
- <u>intermediate- to long-range tensor character</u>

These produce strongly anisotropic femtometer structures in T=0,S=1 channel in all nuclei:

$$\begin{aligned} \rho^{M_S}_{T=0,S=1}(\mathbf{r}) &\propto \rho^{M_S}_{d}(\mathbf{r}) \\ \rho^{M_S=0}_{T=0,S=1}(\mathbf{r}) &\neq \rho^{M_S=\pm 1}_{T=0,S=1}(\mathbf{r}) \end{aligned}$$

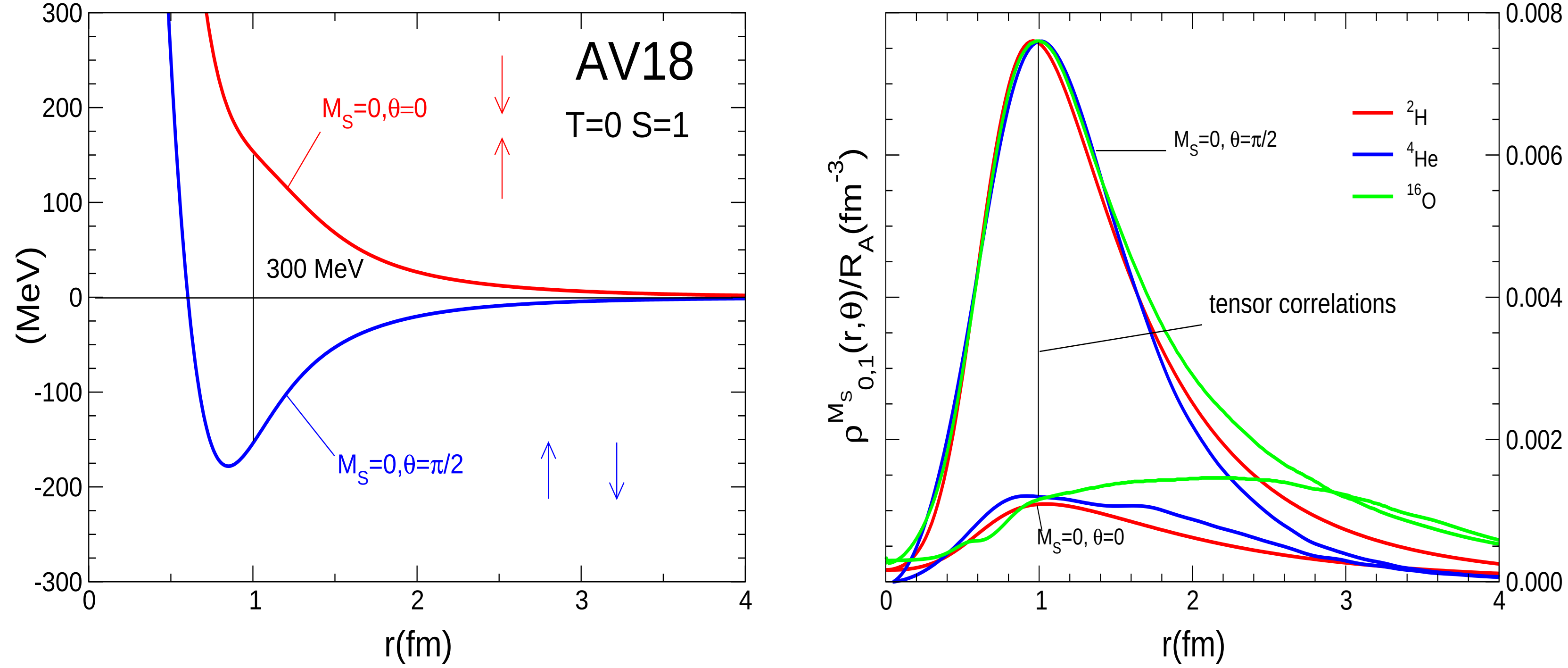
Two-nucleon density function:

$$\rho^{M_S}_{T,S}(\mathbf{r}) = \frac{1}{2J+1} \sum_{M_J} \langle JM_J | \sum_{i<j} P^{T,SM_S}_{ij}(\mathbf{r}) | JM_J \rangle$$

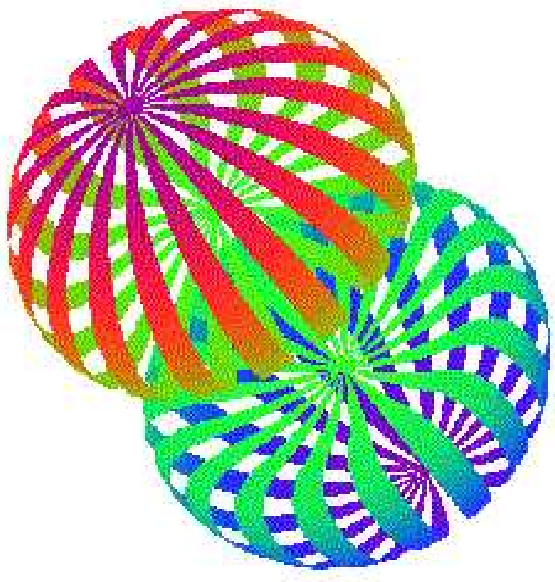
$$P^{T,SM_S}_{ij}(\mathbf{r}) \equiv \delta(\mathbf{r} - \mathbf{r}_{ij}) \, P^{T}_{ij} \, |SM_S, ij\rangle\langle SM_S, ij|$$

Coupling of Spatial and Spin Variables

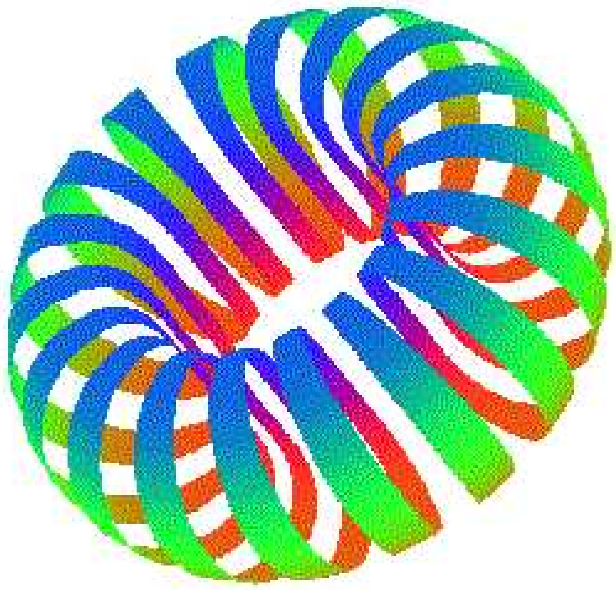
Forest *et al.*, PRC**54**, 646 (1996)



Two-Nucleon Density Profiles in T, S=0,1 States



$M_S = \pm 1$



$M_S = 0$

- Hole due to short-range repulsion
- Angular confinement due to tensor force
- Size of torus: $d \simeq 1.4$ fm, $t \simeq 0.9$ fm (at $\approx$ half-max density)

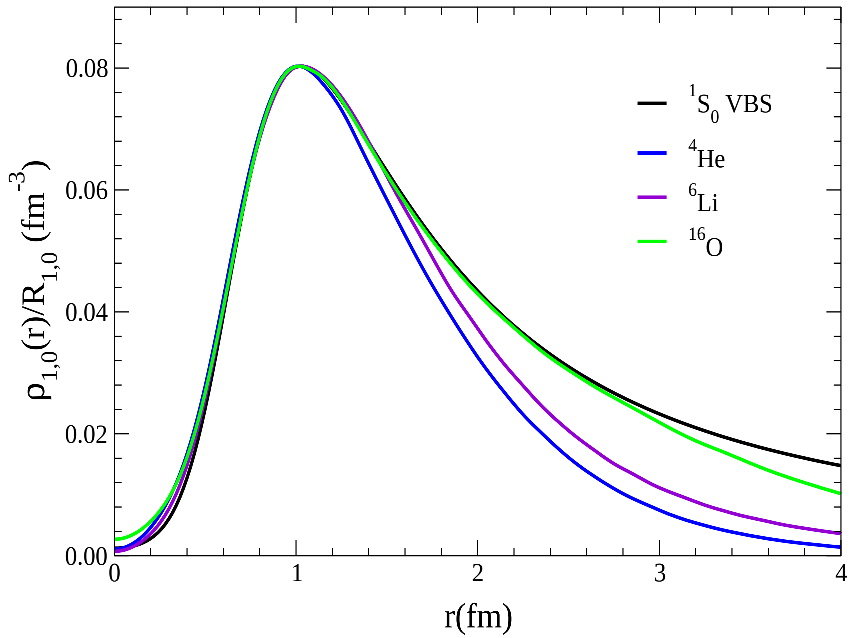
- At small separation, np relative w.f. in a nucleus $\propto$ deuteron w.f., but scaling factor $R_A >$ number of T, S=0,1 pairs
- $\langle O \rangle_A \simeq R_A \langle O \rangle_d$, where O is any short-range operator effective in the $T = 0, S = 1$ channel (*e.g.*, m.e. of axial two-body currents in pp weak capture and ^{3}H β-decay are proportional to each other $\rightarrow$ model independent prediction of pp cross section [Schiavilla *et al.*, PRC**58**, 1263 (1998)])

Scaling

	R_A	$\langle v^\pi \rangle_A / \langle v^\pi \rangle_d$	$\sigma^\pi_A / \sigma^\pi_d$	$\sigma^\gamma_A / \sigma^\gamma_d$
^{3}He	2.0	2.1	2.4(1)	$\simeq 2$
^{4}He	4.7	5.1	4.3(6)	$\simeq 4$
^{6}Li	6.3	6.3		
^{7}Li	7.2	7.8		$\simeq 6.5(5)$

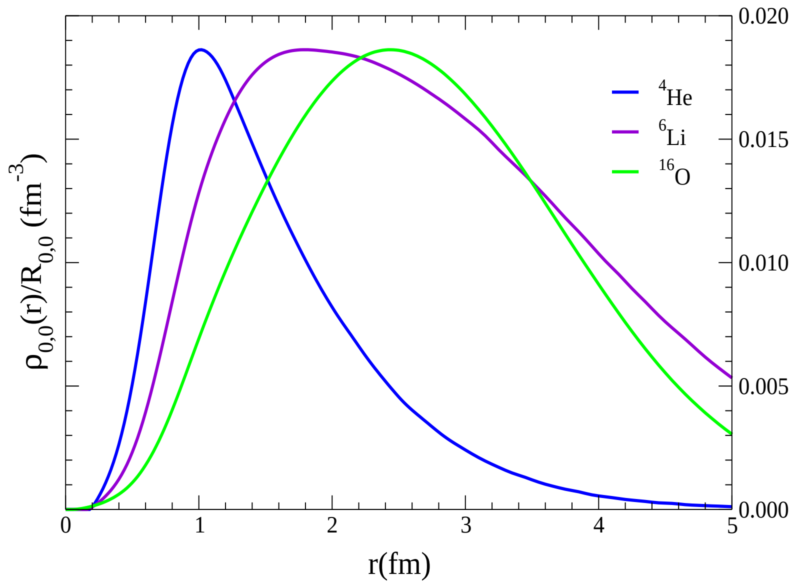
Two-Nucleon Density Profiles in other ($T, S \neq$0,1) States

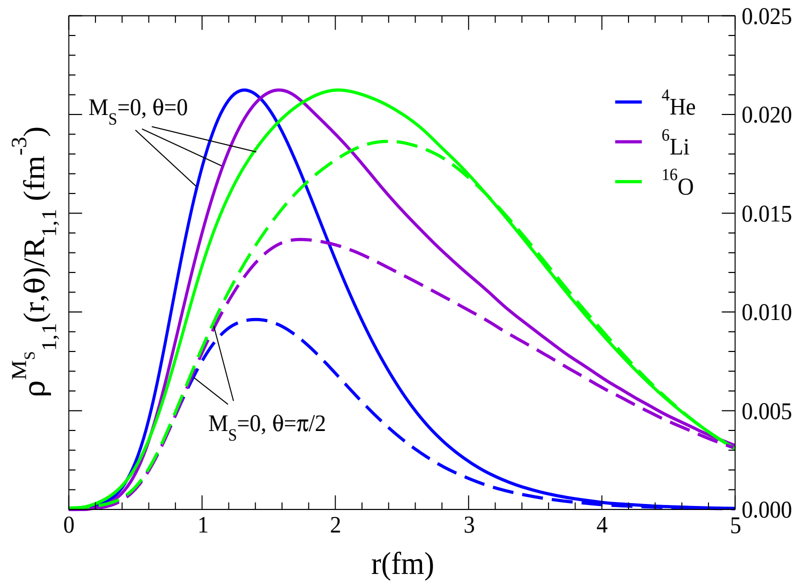
- Scaling occurs in T, S=1,0 channel (quasibound ^{1}S$_0$ state) for $r \leq 2$ fm



- But <u>no scaling</u> in remaining channels (interaction either repulsive or weakly attractive)

T, S=0,0 $\qquad\qquad$ T, S=1,1





Experimental Evidence for Tensor Correlations in $A > 2$ Nuclei

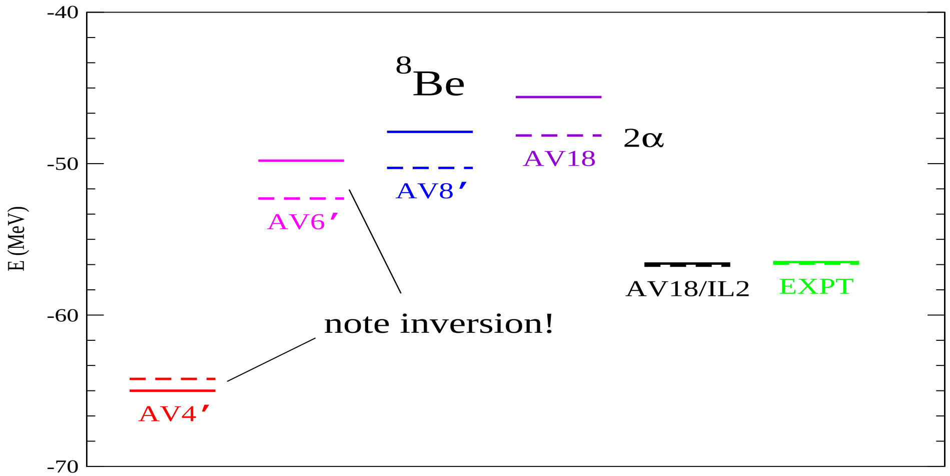
Several nuclear properties influenced by tensor correlations including:

- Ordering of levels in low-energy spectra of light nuclei and absence of stable A=8 nuclei
- Radiative (and weak) capture processes involving few-nucleon systems, *e.g.* ^{2}H$(n,\gamma)^3$H, ^{3}He$(n,\gamma)^4$He, ^{2}H$(d,\gamma)^4$He, ...
- Distribution of strength in response to electromagnetic and hadronic probes, such as (e,e') scattering and (p,n) reactions
- Momentum distributions $N(k)$ and spectral functions $S(k,E)$ at high k and E

However, effects of tensor correlations are generally subtle, and are not easily isolated in the experimental data

Build series of potentials designed to reproduce as many features of the deuteron and elastic NN scatttering as feasible at each stage:

1. $AV4' = [1\,,\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2]\otimes[1\,,\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]$
2. $AV6' = AV4'$+tensor
3. $AV8' = AV6'$+spin-orbit, …



Wiringa and Pieper, PRL**89**, 182501 (2002)

Tensor Correlations and Two-Nucleon Momentum Distributions

$$\rho^{NN}(\mathbf{q},\mathbf{Q}) = \frac{1}{2J+1}\sum_{M_J}\langle\psi_{JM_J} \mid \sum_{i<j} P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \mid \psi_{JM_J}\rangle$$

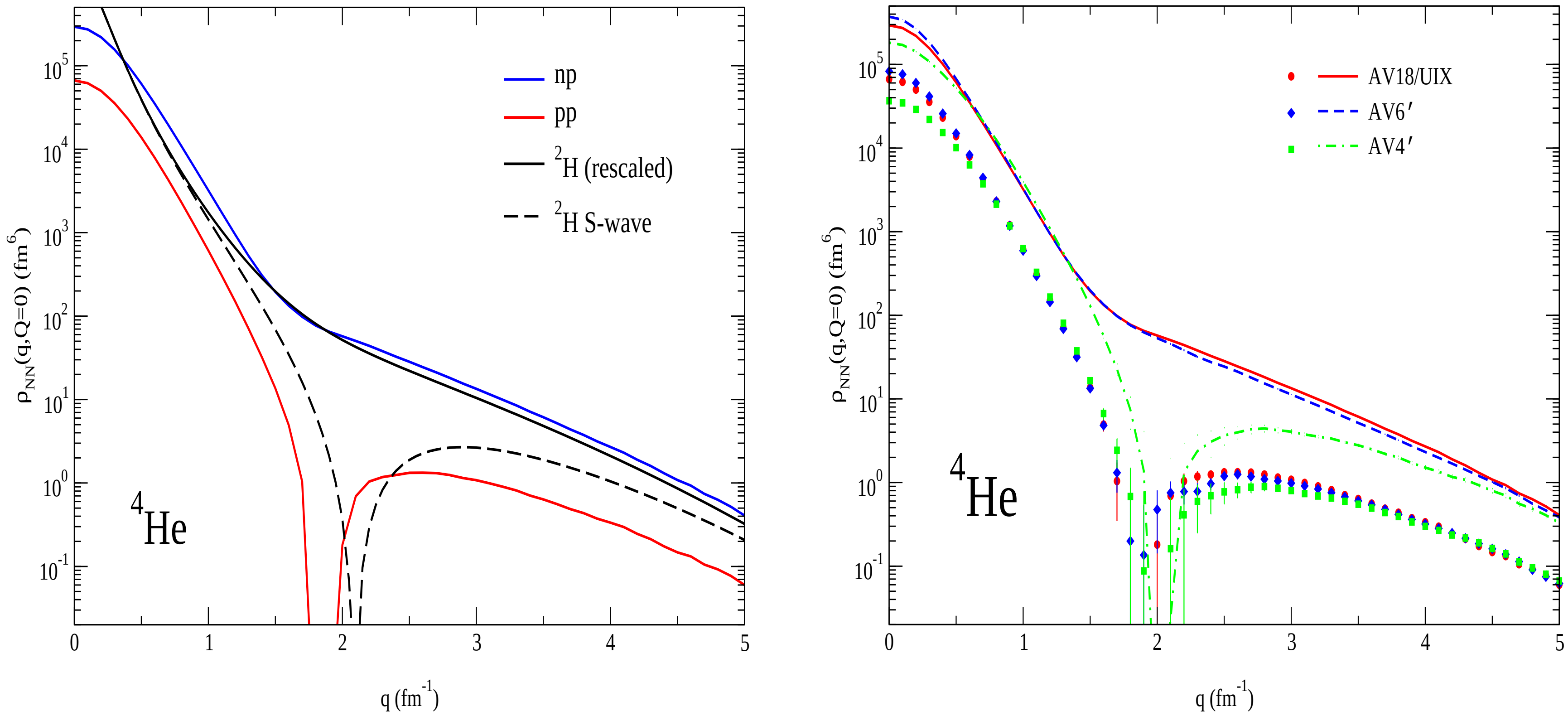
where $\mathbf{q}$ and $\mathbf{Q}$ are respectively the relative and total momenta of the NN pair, and

$$P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \equiv \delta(\mathbf{k}_{ij}-\mathbf{q})\delta(\mathbf{K}_{ij}-\mathbf{Q})\, P_{NN}(ij)$$

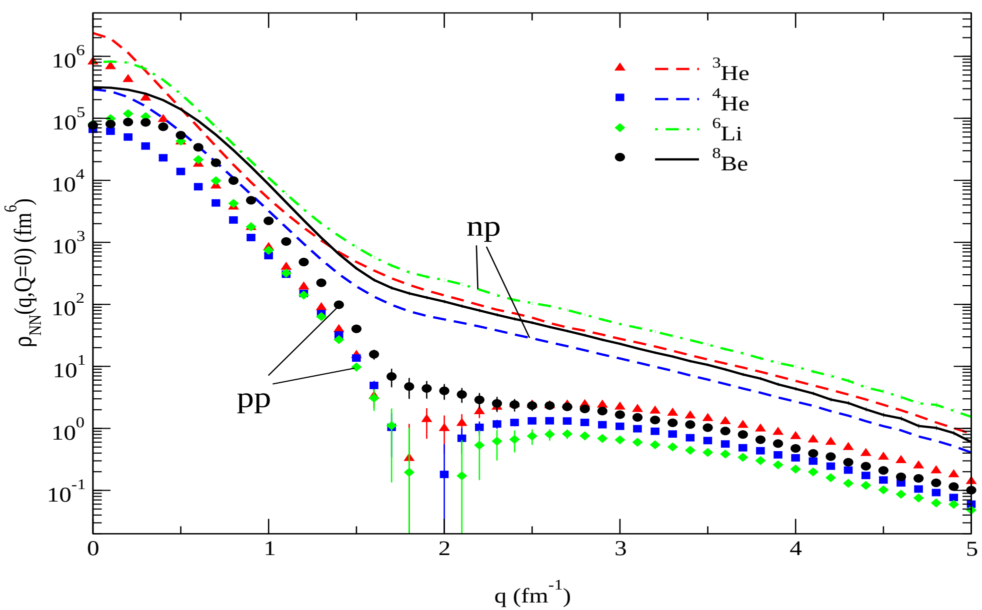
- np (pp) pairs predominantly in T=0 deuteron-like (T=1 quasi-bound) state→ large differences between ρ^{np} and ρ^{pp}
- These differences should be seen in $A(e,e'np)$ and $A(e,e'pp)$ (back-to-back kinematics)
- ρ^{NN} can be calculated exactly with QMC

NN momentum distributions at Q=0

Schiavilla, Wiringa, Pieper, and Carlson, PRL **98**, 132501 (2007)



- Universal feature
- First indications from: i) analysis of ^{12}C (p, pp) and (p, ppn) BNL data, and ii) JLab measurements of $^{12}\mathrm{C}(e, e'pp)$ and $^{12}\mathrm{C}(e, e'pn)$: $P_{pp}/P_{np} \lesssim 0.04^{+0.09}_{-0.04}$ [Piasetzky *et al.*, PRL**97**, 162504 (2006)]
- Possibly also seen in π-absorption: $\sigma(\pi^-, np)/\sigma(\pi^+, pp) \ll 1$



Summary (II)

- Tensor correlations affect a variety of nuclear properties ($\rho^{M_S}_{T=0,S=1}(\mathbf{r})$, spectra, ...), but hard to isolate in $A > 2$ nuclei
- They also lead to order of magnitude differences between the (back-to-back) *np*- and *pp*-pair momentum distributions
- This isospin dependence should be easily observable in *np*- or *pp*-knockout processes (already "seen" in BNL and JLab data)

III. Isospin Symmetry Breaking and G_E^s

$^4\text{He}(\vec{e}, e')^4\text{He}$ Scattering

$$A_{\text{PV}} = -\frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \frac{\langle ^4\text{He} \mid j^{\mu=0}_{\text{NC}} \mid ^4\text{He}\rangle}{\langle ^4\text{He} \mid j^{\mu=0}_{\text{EM}} \mid ^4\text{He}\rangle} \rightarrow \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} 4\, s^2_W$$

where

$$\begin{aligned} j^{\mu=0}_{\text{EM}} &= j^{(0)} + j^{(1)} \\ j^{\mu=0}_{\text{NC}} &= -4\, s^2_W j^{(0)} + (2 - 4\, s^2_W) j^{(1)} - j^{(s)} \end{aligned}$$

- A_{PV} sensitive to $G^s_E(Q^2)$, provided negligible:
  1. relativistic corrections (RC) and MEC contributions
  2. isospin symmetry breaking (ISB) in the nucleon and ^{4}He
- At low Q^2, RC+MEC contributions calculated to be tiny[a]

[a] Musolf, Schiavilla, and Donnelly, PRC**50**, 2173 (1994)

Parameterizing ISB in the nucleon

Dmitrasinović and Pollock, PRC**52**, 1061 (1995); Kubis and Lewis, PRC**74**, 015204 (2006)

In terms of the measured $G_E^{p/n} = \langle p/n | j_{\rm EM}^{\mu=0} | p/n \rangle$:

$$(G_E^p + G_E^n)/2 = G_E^0 + G_E^{\not{1}} \qquad (G_E^p - G_E^n)/2 = G_E^1 + G_E^{\not{0}}$$

from which

$$\begin{aligned} G_E^{p,Z} &= (1 - 4s_W^2) G_E^p - G_E^n + 2(G_E^{\not{1}} - G_E^{\not{0}}) - G_E^s \\ G_E^{n,Z} &= (1 - 4s_W^2) G_E^n - G_E^p + 2(G_E^{\not{1}} + G_E^{\not{0}}) - G_E^s \end{aligned}$$

where ISB in G_E^s are ignored: $\langle p | j^{(s)} | p \rangle = \langle n | j^{(s)} | n \rangle \to G_E^s(Q^2)$

Nuclear EM and NC (Vector) Charge Operators

$$\rho^{(\mathrm{EM})}(\mathbf{q}) = G_E^p \sum_{k=1}^{Z} e^{i\mathbf{q}\cdot\mathbf{r}_k} + G_E^n \sum_{k=Z+1}^{A} e^{i\mathbf{q}\cdot\mathbf{r}_k} \equiv \rho^{(0)}(\mathbf{q}) + \rho^{(1)}(\mathbf{q})$$

$$\rho^{(0)}(\mathbf{q}) = \frac{G_E^p + G_E^n}{2} \sum_{k=1}^{A} e^{i\mathbf{q}\cdot\mathbf{r}_k}$$

$$\rho^{(1)}(\mathbf{q}) = \frac{G_E^p - G_E^n}{2} \left(\sum_{k=1}^{Z} e^{i\mathbf{q}\cdot\mathbf{r}_k} - \sum_{k=Z+1}^{A} e^{i\mathbf{q}\cdot\mathbf{r}_k} \right)$$

With $G_E^{p/n} \rightarrow G_E^{p/n,Z}$, $\rho^{(\mathrm{NC})}(\mathbf{q})$ can be written as

$$\rho^{(\mathrm{NC})}(\mathbf{q}) = -4s_W^2 \rho^{(\mathrm{EM})}(\mathbf{q}) + \frac{2\, G_E^{\not{1}} - G_E^s}{(G_E^p + G_E^n)/2} \rho^{(0)}(\mathbf{q})$$

$$+2\rho^{(1)}(\mathbf{q}) - \frac{2\, G_E^{\not{0}}}{(G_E^p - G_E^n)/2} \rho^{(1)}(\mathbf{q})$$

Up to linear terms in ISB corrections:

$$A_{\rm PV} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left[4\, s_W^2 - 2\, \frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2\, G_E^{\not{1}} - G_E^s}{(G_E^p + G_E^n)/2} + \text{RC/MEC} \right]$$

where

$$\langle ^4\text{He} | \rho^{(a)}(\mathbf{q}) | ^4\text{He} \rangle / Z \equiv F^{(a)}(q) \;, \quad a = \text{EM}, 0, 1$$

The HAPPEX collaboration [PRL**98**, 032301 (2007)] reports:

$A_{\rm PV}[Q^2 = 0.077\ (\text{GeV/c})^2] = [+6.40 \pm 0.23\ (\text{stat}) \pm 0.12\ (\text{syst})]\text{ppm}$

from which, using G_μ=1.16637×10^{-5} GeV^{-2}, α=1/137.036, and s_W^2=0.2286 (with radiative corrections),

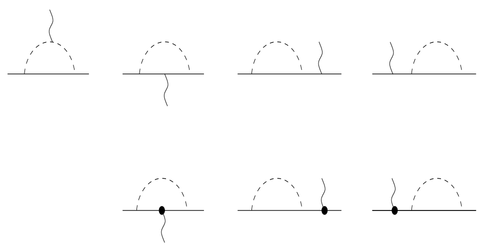
$$\Gamma \equiv -2 \frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2\, G_E^{\not{1}} - G_E^s}{(G_E^p + G_E^n)/2} = 0.010 \pm 0.038$$

ISB Corrections (I): Nucleon

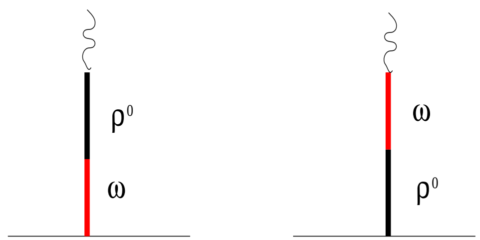
Kubis and Lewis, PRC**74**, 015204 (2006)

Up to NLO in ChPT:

1. Loop effects due $\Delta m = m_n - m_p$



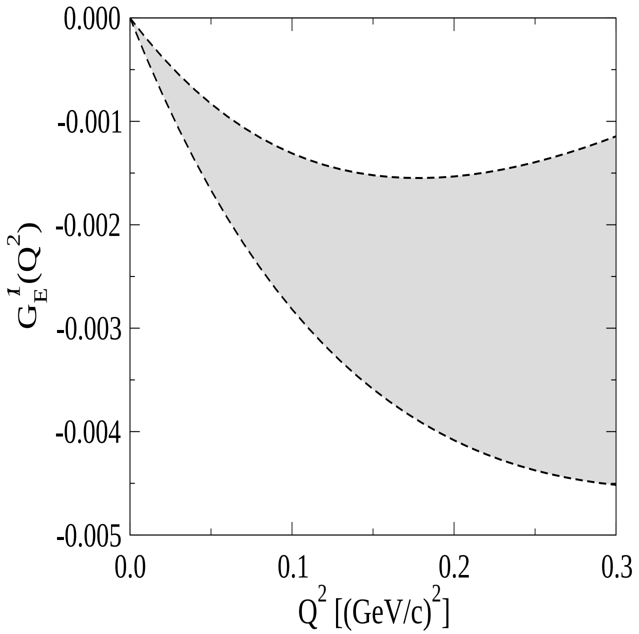
2. A single counterterm, fixed by resonance saturation

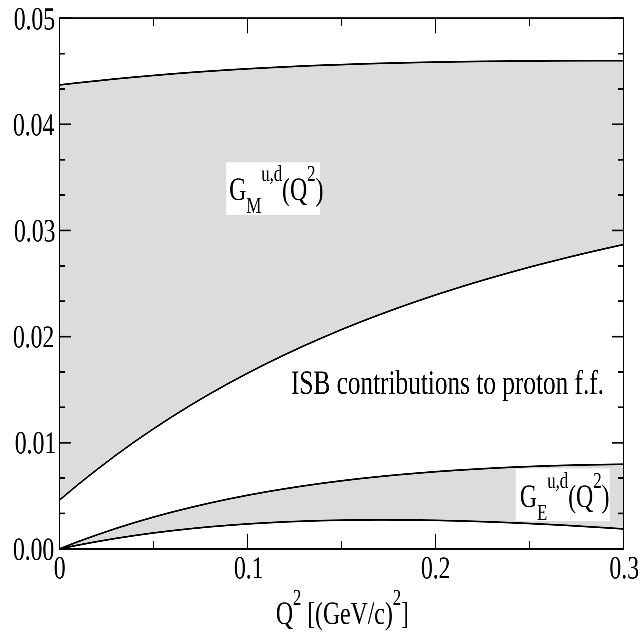


Kubis and Lewis, PRC**74**, 015204 (2006)

$$
\begin{aligned}
G_E^{\not{I}}(Q^2) &= -\frac{g_A^2 m_N \Delta m}{F_\pi^2}\Bigg\{\frac{M_\pi}{m_N}\left[\overline{\gamma}_0(-Q^2) - 4\overline{\gamma}_3(-Q^2)\right] \\
&- \frac{Q^2}{2m_N^2}\Bigg[\xi(-Q^2) - \frac{M_\pi}{m_N}\left[\overline{\gamma}_0(-Q^2) - 5\overline{\gamma}_3(-Q^2)\right] \\
&\quad -\frac{1}{16\pi^2}\left(1 + 2\log\frac{M_\pi}{M_V} - \frac{\pi(\kappa^v + 6)M_\pi}{2m_N}\right)\Bigg]\Bigg\} \\
&+ \frac{g_\omega F_\rho \Theta_{\rho\omega} Q^2}{2M_V(M_V^2 + Q^2)^2}\left(1 + \frac{\kappa_\omega M_V^2}{4m_N^2}\right)
\end{aligned}
$$

- $\overline{\gamma}_0$, $\overline{\gamma}_3$, and ξ are loop functions: $\propto Q^2$ as $Q^2 \to 0$
- Largest uncertainty in ω tensor coupling κ_ω





- Band provides an estimate of higher order ChPT corrections as well as of uncertainties in vector-meson couplings
- At $Q^2 = 0.077$ (GeV/c)^2:

$$-\frac{2\,G_E^{1}}{(G_E^p + G_E^n)/2} = 0.008 \pm 0.003$$

ISB Corrections (II): ^{4}He Nucleus

Nuclear ISB Hamiltonian: $H_{\rm ISB} = H_{\rm C} + H_{\rm CD/CA} + H_{\rm EM} + K_\Delta$

- $H_{\rm C}$ from (point) Coulomb interaction
- $H_{\rm CD/CA}$ from CD and CA strong-interactions
- $H_{\rm EM}$ from remaining EM interactions (magnetic moments, ...)
- K_Δ from n-p mass difference in kinetic energy

Viviani, Kievsky, and Rosati, PRC**71**, 024006 (2005)

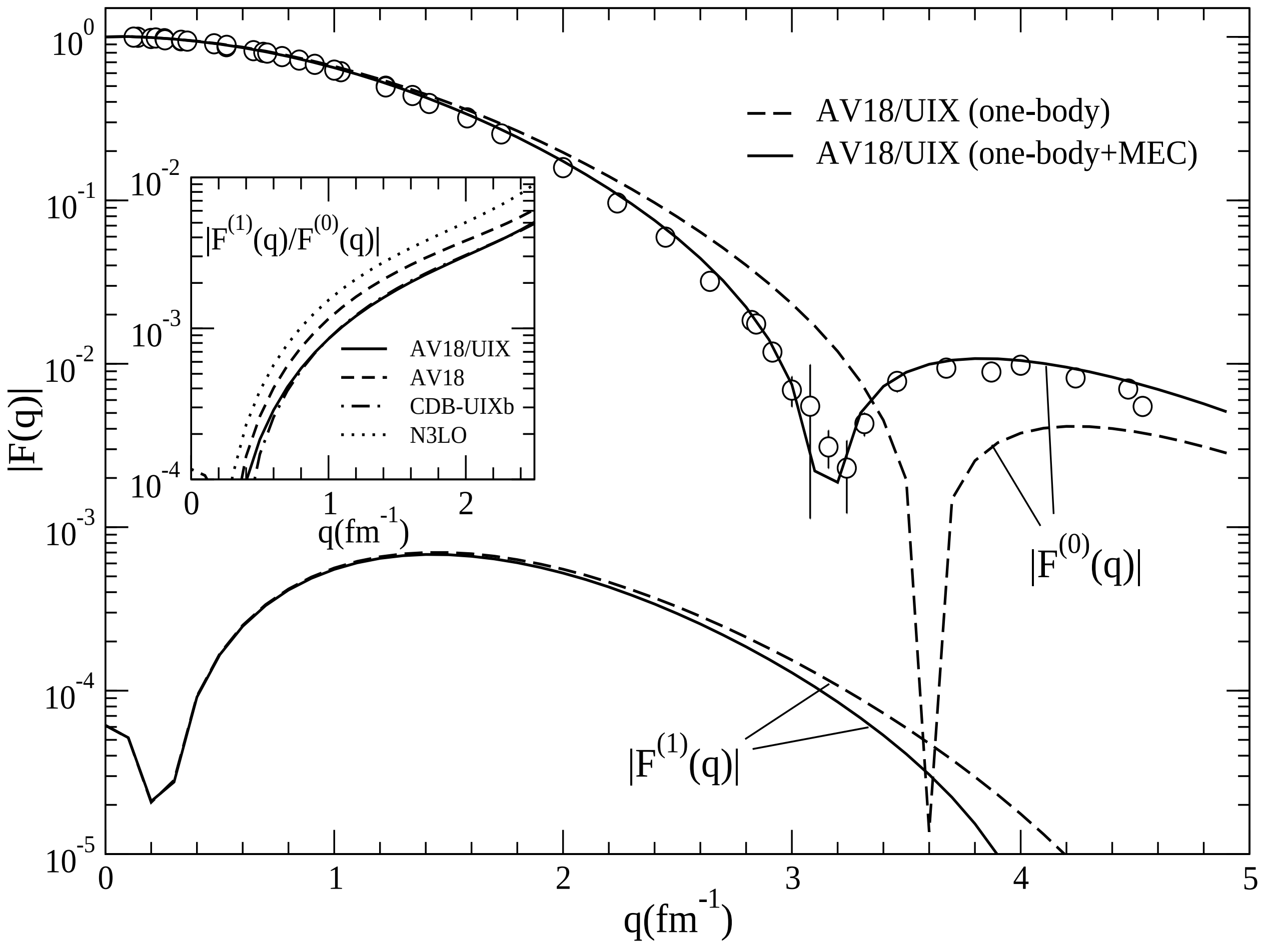
ISB term (AV18)	$P^{(1)}$ %	$P^{(2)}$ %
$H_{\rm C}$	1.5×10^{-3}	0.1×10^{-3}
$H_C + H_{\rm CD/CA}$	3.0×10^{-3}	4.9×10^{-3}
$H_C + H_{\rm CD/CA} + H_{\rm EM}$	2.8×10^{-3}	5.2×10^{-3}

Contributions of ISB terms to isomultiplet energies (keV)

Pieper, Pandharipande, Wiringa, and Carlson, PRC**64**, 014001 (2001)

A	T	n	K_{Δ}	$H_{\rm C}$	$H_{\rm EM}$	$H_{\rm CD/CA}$	TOT	EXP
3	1/2	1	14(0)	649(1)	29(0)	64(0)	757(1)	764
6	1	1	16(0)	1091(5)	18(0)	47(1)	1172(6)	1173
8	1	1	23(0)	1686(5)	24(0)	76(1)	1810(6)	1770
6	1	2		166(1)	19(0)	107(13)	293(13)	223
8	1	2		141(1)	4(0)	–3(8)	143(8)	145

- Good overall agreement between theory and experiment



- Weak model dependence
- $F^{(1)}$ scales as $\approx \sqrt{P^{(1)}}$; RC/MEC small at low q (≤ 1.5 fm^{-1})
- $F^{(1)}/F^{(0)} \approx -0.00157$ from AV18/UIX and CDB/UIXb

Summary (III)

Using: i) $-2\, G_E^{\not 1}/[(G_E^p + G_E^n)/2] \approx 0.008$ for hadronic ISB

ii) $-2\, F^{(1)}(q)/F^{(0)}(q) \approx 0.00314$ for nuclear ISB

in

$$\Gamma \equiv -2\frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2\, G_E^{\not 1} - G_E^s}{(G_E^p + G_E^n)/2} = 0.010 \pm 0.038$$

gives $G_E^s \left[Q^2 = 0.077\,(\mathrm{GeV/c})^2\right] = -0.001 \pm 0.016$

- Measuring ISB admixtures? (arguably … error on Γ too large!)
- $G_E^s \left[Q^2 = 0.1\,(\mathrm{GeV/c})^2\right] = +0.001 \pm 0.004 \pm 0.003$ estimated by using LQCD input [Leinweber *et al.*, PRL **97**, 022001 (2006)]
- If the LQCD-based analysis above is confirmed, ISB at the hadronic and/or nuclear level are the leading correction to A_{PV}