Two-Photon Physics

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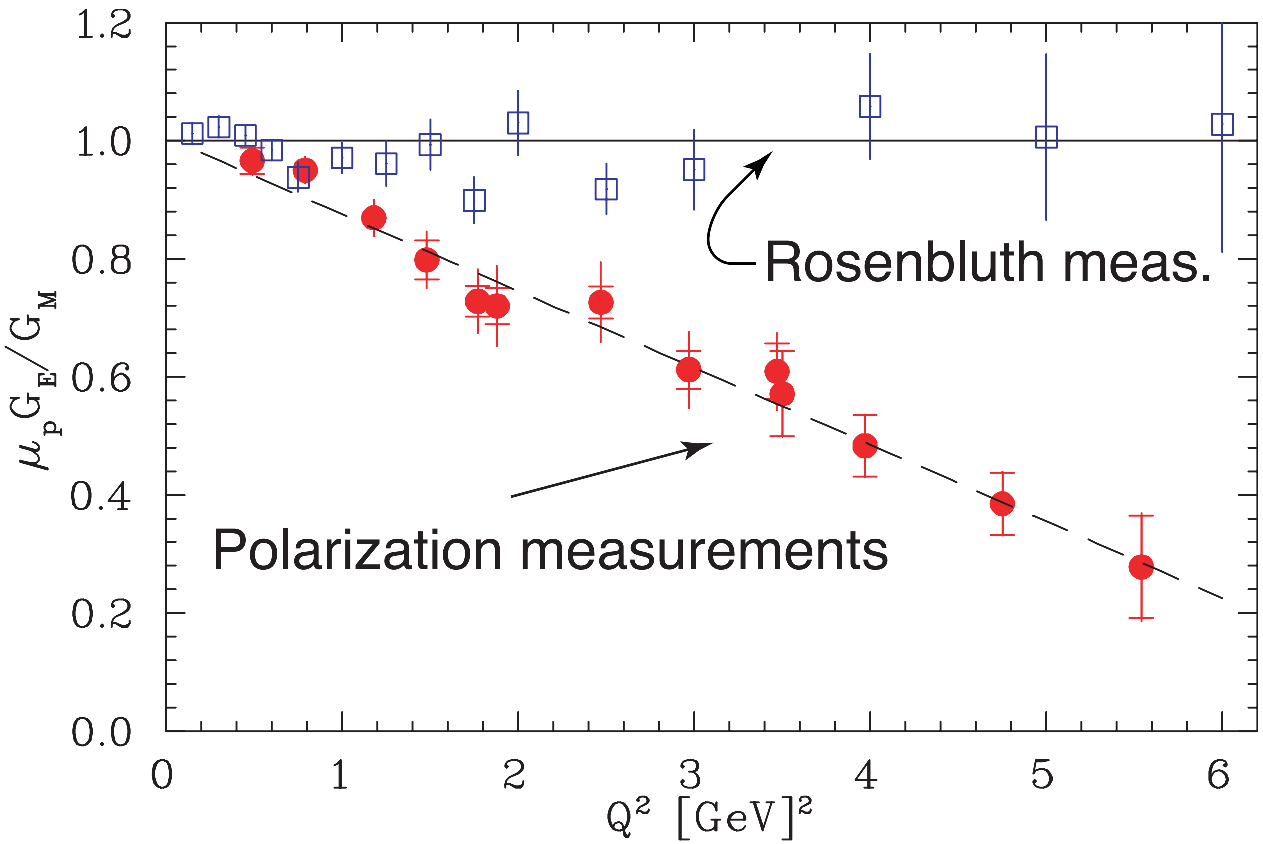
Goals for talk

- Re-present the theory
  - the problem
  - what was absent in the "old days"
  - three attempts to complete the box calculations
    - single bound hadrons
    - partonic with GPDs
    - pQCD
- Analysis of manifestations of two photon processes
  - Rosenbluth corrected
  - polarizations
  - positron/electron ratio
- Appreciation of new experimental results

The problem

- There were two ways of measuring G_E/G_M (proton), and they gave different answers
- As we saw it in about 2003,



The problem

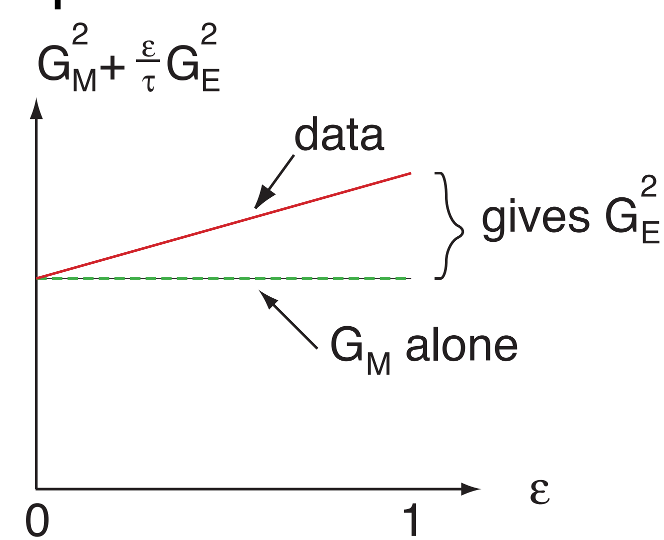
- Rosenbluth means measure the differential cross section
- In a one-photon exchange approximation,

$$\frac{d\sigma}{d\Omega_{Lab}} = \frac{d\sigma_{NS}}{d\Omega_{Lab}} \times \frac{\tau}{\epsilon(1+\tau)}\left(|G_M(Q^2)|^2 + \frac{\epsilon}{\tau}|G_E(Q^2)|^2\right)$$

where

$$\tau \equiv \frac{Q^2}{4M^2}\,,\ \frac{1}{\epsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$

- distinguish G_M and G_E by different angular dependences
- Typically plot $|G_M|^2 + \frac{\epsilon}{\tau}|G_E|^2$ vs. ϵ , at fixed Q2



The problem

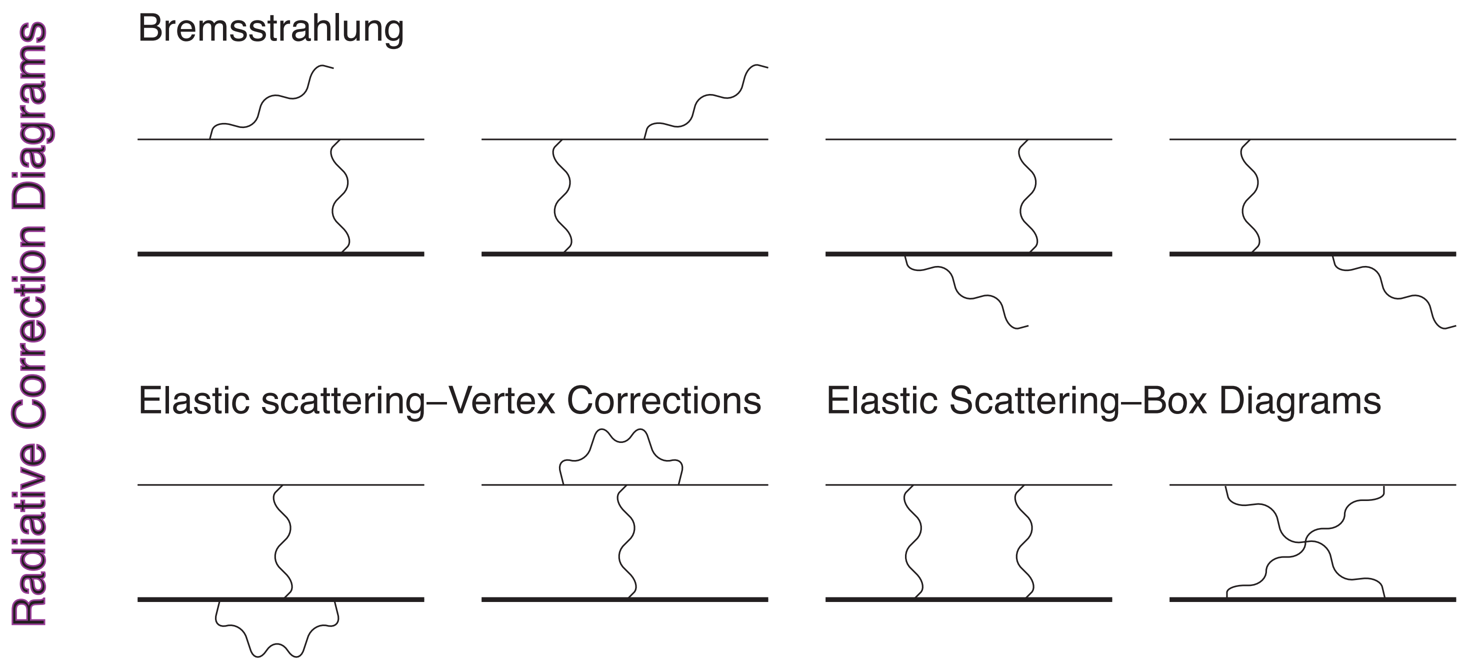
- Problem with the Rosenbluth separation for high Q^2 is that the G_E contribution is small compared to the G_M contribution. Hence small corrections to the G_M term can seriously affect the G_E term.
- Alternative is polarization transfer in $\overrightarrow{e} + p \rightarrow e + \overrightarrow{p}$
- Ratio of transverse-in-plane polarization and longitudinal polarization is

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\,\frac{G_E}{G_M}$$

for one-photon exchange, and gives polarization ratio directly

The problem

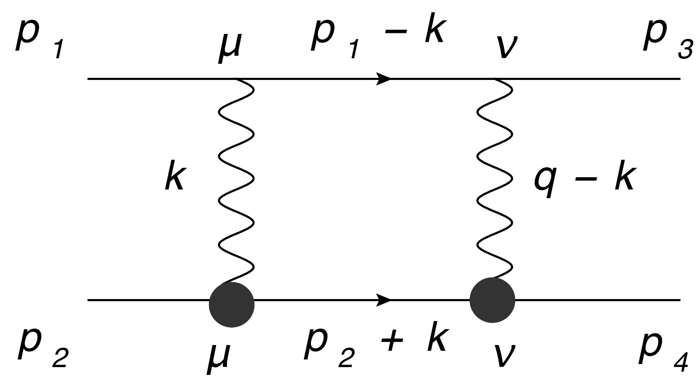
- Since the Rosenbluth separation involves a small term, need to consider the corrections, specifically radiative corrections



- Mostly well done in past,
  - Meister and Yennie (1963)
  - Mo and Tsai (1961 and 1969)
  - Maximon and Tjon (2000)
- But clear incompleteness in box or 2-photon exchange diagrams

Box diagrams in "old days"

- Couldn't have been neglected: they have IR divergences that cancel corresponding divergences from bremsstrahlung
- For elastic intermediate state,



(+ cross)

$$
\begin{aligned}
\mathcal{M}_{Box} &= (Ze^2)^2 \int (d^4k)(k^2-\lambda^2+i\epsilon)^{-1}((k-q)^2-\lambda^2+i\epsilon)^{-1} \\
&\times \bar{u}(p_3)\gamma_\nu \frac{\not{p}_1 - \not{k} + m}{(p_1-k)^2-m^2+i\epsilon}\gamma_\mu u(p_1) \\
&\times \bar{u}(p_4)\Gamma^\nu(q-k)\frac{\not{p}_2+\not{k}+M}{(p_2+k)^2-M^2+i\epsilon}\Gamma_\mu(k)u(p_2)
\end{aligned}
$$

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

IR divergence comes from photons almost on shell at $k = 0$ or $k = q$.

“Old” boxes

- Approximation: Leave propagators untouched, but
- set k = 0 or k = q (two separate possibilities) in numerator
- For k = 0, $\Gamma^\mu(k=0) \;=\; \gamma^\mu$

$$\begin{aligned}\mathcal{M}_{Box} \;&=\; (Ze^2)^2 \int (d^4k)(k^2-\lambda^2+i\epsilon)^{-1}((k-q)^2-\lambda^2+i\epsilon)^{-1} \\ &\times\; \bar{u}(p_3)\gamma_\nu \frac{\not{p}_1+m}{(p_1-k)^2-m^2+i\epsilon}\gamma_\mu u(p_1) \\ &\times\; \bar{u}(p_4)\Gamma^\nu(q)\frac{\not{p}_2+M}{(p_2+k)^2-M^2+i\epsilon}\gamma^\mu u(p_2)\end{aligned}$$

and

$$(\not{p}_2+M)\gamma^\mu u(p_2) = \left[\gamma^\mu(-\not{p}_2+M)+2p_2^\mu\right]u(p_2) = 2p_2^\mu\, u(p_2)$$

"Old" boxes

- Get

$$\mathcal{M}_{Box} = (Ze^2)^2 \; 2p_{1\mu} \cdot 2p_2^\mu \; \bar{u}(p_3)\gamma_\nu u(p_1) \; \bar{u}(p_4)\Gamma^\nu(q)u(p_2)$$

$$\times \int (d^4k) \, (k^2 - \lambda^2 + i\epsilon)^{-1} \, ((k-q)^2 - \lambda^2 + i\epsilon)^{-1}$$

$$\times \quad ((p_1 - k)^2 - m^2 + i\epsilon)^{-1} \; ((p_2 + k)^2 - M^2 + i\epsilon)^{-1}$$

$$= Ze^2 \; 4p_1 \cdot p_2 \; \mathcal{M}_{Lowest\ order} \times q^2 \times \text{Integral}$$

- Virtues
  - IR divergences gotten exactly
  - integral doable w/o further proton structure information

"Old" boxes

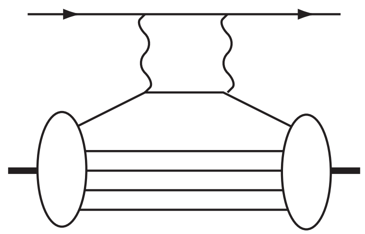
- Vices:
  - Wrong away from $k = 0$ (or $k = q$)
  - Approximation tossed $O(k^2)$ terms in numerator --- could have larger effect than early workers would like
  - Ignores non-elastic intermediate states
- Fixes:
  - Keep k in numerator, and do elastic terms completely
  - Treat intermediate hadron state as collection of quarks

2-photon calculations

- hadronic method: include full integrand in box diagrams—including form factors [Blunden et al. 2003]
- Later included other resonances.
- Explain half or more of discrepancy

2-photon calculations

- Partonic calculations
  - Chen et al. 2004
  - Afanasev et al. 2005



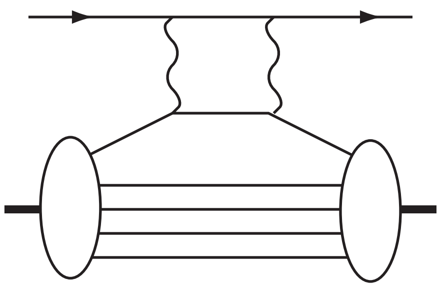
- General result: beyond one-photon exchange, there are extra terms in the e-p elastic scattering amplitude

$$\mathcal{M} = \frac{e^2}{Q^2}\,\bar{u}(k',h)\gamma_\mu u(k,h) \times \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2\frac{P^\mu}{M} + \tilde{F}_3\frac{\gamma\cdot K P^\mu}{M^2}\right)u(p,\lambda_N)$$

- In general, $\tilde{G}_M$, $\tilde{F}_2$, $\tilde{F}_3$ are complex and depend on energy as well as Q^2.
- For one-photon exchange $\tilde{G}_M = G_M$, $\tilde{F}_2 = F_2$, and $\tilde{F}_3 = 0$.

2-photon calculations

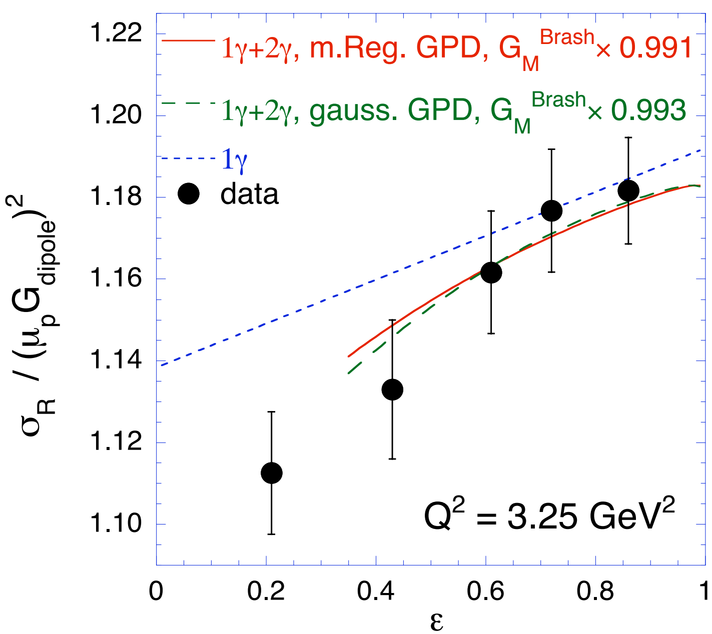
- Hadronic part of diagram described using generalized parton distributions
- Note: Calculation not possible without modern knowledge of GPDs and nucleon structure



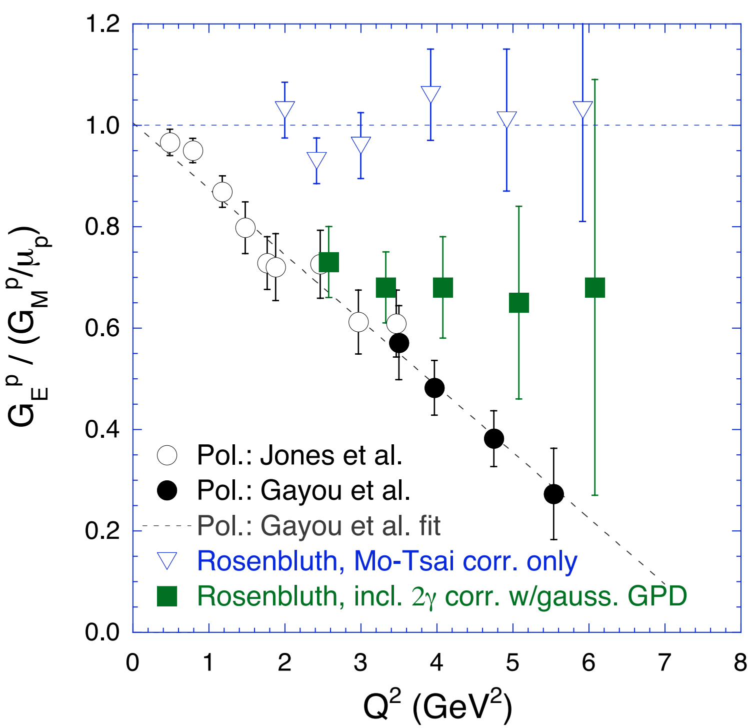
"Handbag"

- Sample results

Cross section for ep elastic scattering

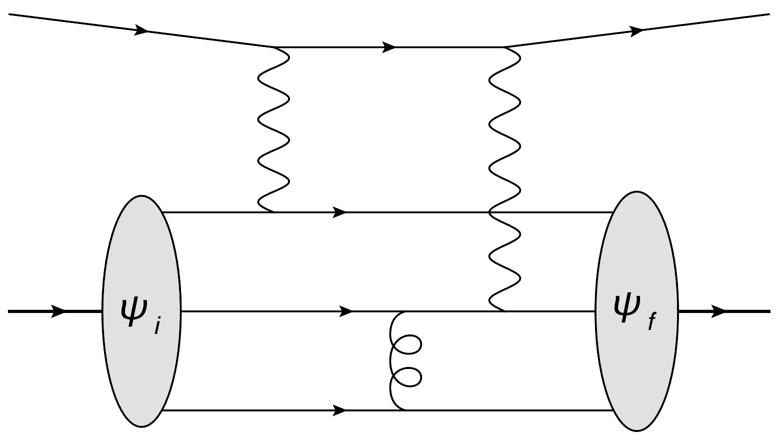


Rosenbluth w/2-γ corrections vs. Polarization data



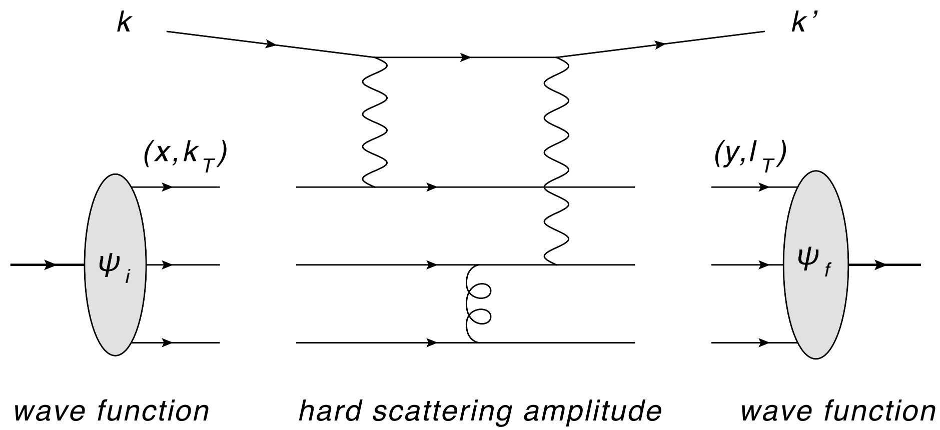
2-photon calculations

- 3rd calculation
- Kivel and Vanderhaeghen (2009): 2-photon contributions to *e*-*p* elastic scattering from perturbative QCD
- Sample diagram (24 total):



- Lowest order diagrams to convert three parallel moving quarks into three quarks moving parallel in different direction

2γX in pQCD



- Result is convolution of process specific hard scattering amplitude and general wave function for quarks in proton
- High enough momentum transfer, neglect transverse momentum of quarks, defining distribution amplitude,

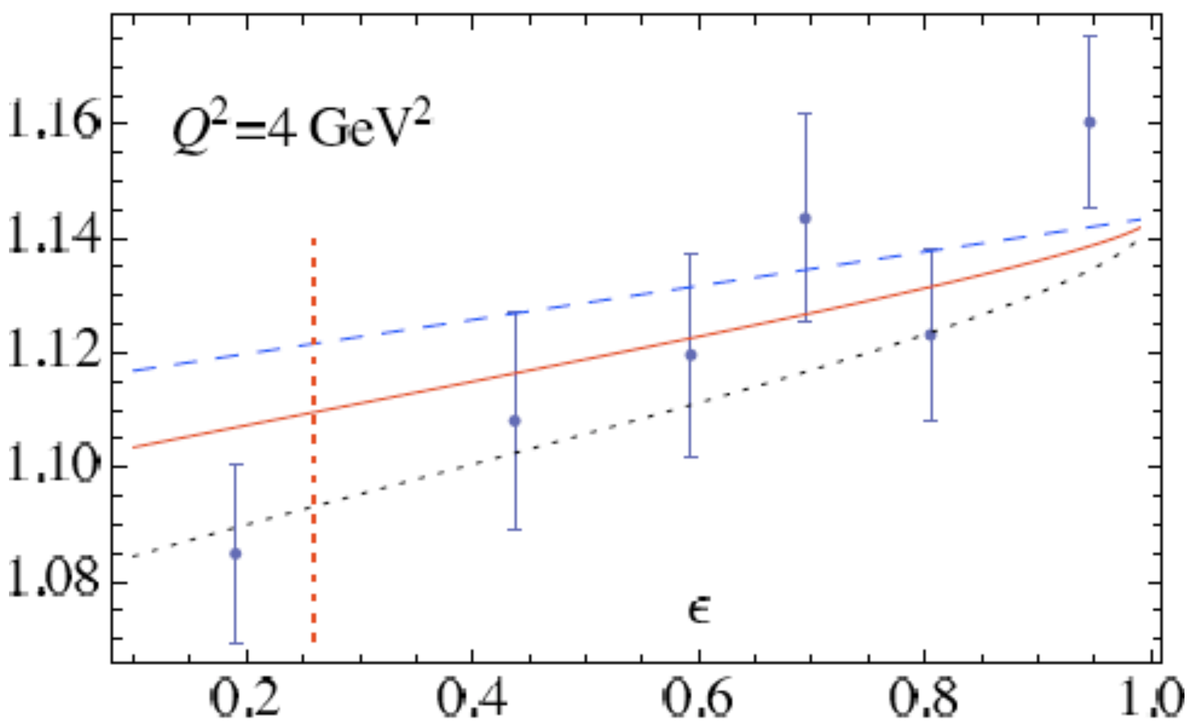
$$\phi(x) = \int [d^2k_\perp]\, \psi(x, k_\perp)$$

- Whence two-photon contribution to FF is (generically),

$$\delta\tilde{F}_i = \frac{1}{Q^4} \int [dx][dy] \phi^*(y)\, T_H(x, y, k, k')\, \phi(x)$$

2γX in pQCD

- The $1/Q^4$ factored out of the hard scattering amplitude
- Same falloff as one-photon exchange terms
- Leading twist. GPD contribution is higher twist. Hence pQCD dominates GPD at high enough momentum transfer.
- Sample result,



- blue dash -- 1 photon
- solid red -- BLW
- dotted black -- COZ

Other 2-γ exchange observables

- ε-dependence of polarizations
- Normal polarization, P_y
- positron/electron ratio (e^+p/e^-p)
  - No modern data yet, experiments at VEPP, Olympus(DESY), CLAS
- Curvature in Rosenbluth plot
  - Not seen in present data [Tvaskis et al., 2006]
  - Dedicated Hall C experiment
  - Theoretically quantified by Abidin et al.

Other 2-γ exchange observables

- Different observables measure different 2γ contributions

Recall three generalized form factors

$$\begin{aligned}\tilde{G}_M &= G_M(Q^2) + \delta\tilde{G}_M(\varepsilon, Q^2) \\ \tilde{G}_E &= G_E(Q^2) + \delta\tilde{G}_E(\varepsilon, Q^2) \\ \tilde{F}_3 &= 0 \qquad\quad + \delta\tilde{F}_3(\varepsilon, Q^2)\end{aligned}$$

$\Uparrow$ ordinary FF $\qquad$ $\Uparrow$ TPE

Form factors G_M and G_E are defined from matrix elements of the electromagnetic current,

the "δ" quantities come from two-photon exchange.

Sometimes (esp. Guichon-Vdh, 2003) replace F_3 by

$$Y_{2\gamma} \equiv \text{Re}\left(\frac{\nu\tilde{F}_3}{m_N^2 G_M}\right)$$

Other 2-γ exchange observables

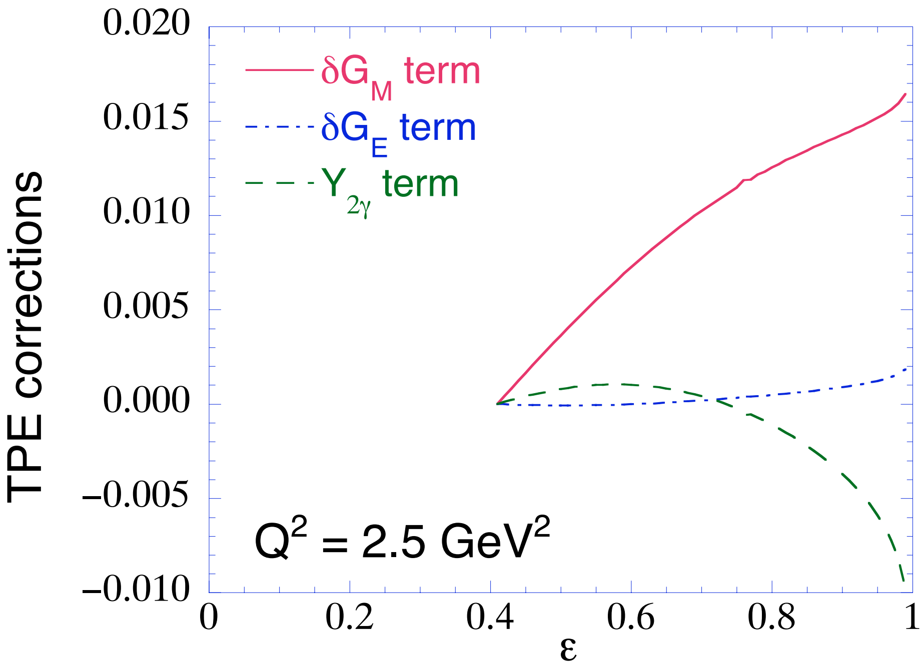
Cross section with two-photon corrections

Experimenters usually apply the Mo-Tsai corrections, so work with

$$R \equiv \frac{\sigma_R^{MTcorr}}{\mu_p^2 G_{\rm dipole}^2} = R^{(1\gamma)}(1+\pi\alpha) + \frac{2\tau G_M \Re\delta\tilde{G}_M^{hard} + 2\varepsilon G_E \Re\delta\tilde{G}_E^{hard} + 2\varepsilon G_M^2\left(\tau + \frac{G_E}{G_M}\right)Y_{2\gamma}}{\tau\mu_p^2 G_{\rm dipole}^2}$$

The extra terms change the slope of *R* vs. ε.

Check term-by-term contributions to ε dependence using the GPD model



(Started all corrections at same point ε, to make slope clear)

- δG_M dominates total for Rosenbluth
- δG_E term is small
- $Y_{2\gamma}$ by itself has the wrong sign

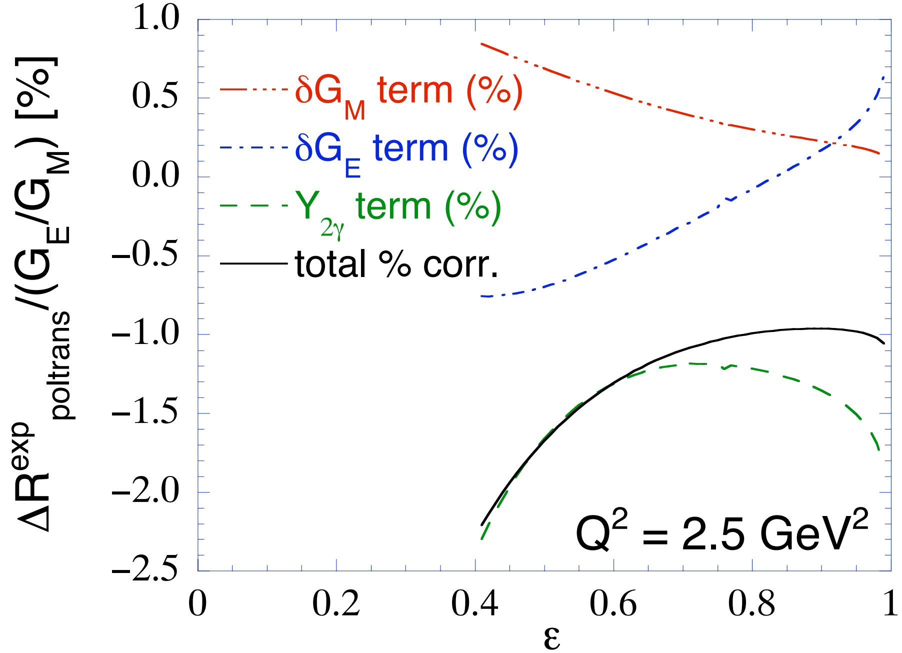
Other 2-γ exchange observables

Polarization transfer and two-photon corrections

One measures

$$R^{exp}_{poltrans} \equiv -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\,\frac{\mathcal{P}_s}{\mathcal{P}_l} = \frac{G_E}{G_M}\left\{1 - \frac{\Re\delta\tilde{G}_M^{hard}}{G_M} + \frac{\Re\delta\tilde{G}_E^{hard}}{G_E} + \left(\frac{G_M}{G_E} - \frac{2\varepsilon}{1+\varepsilon}\right) Y_{2\gamma}\right\}$$

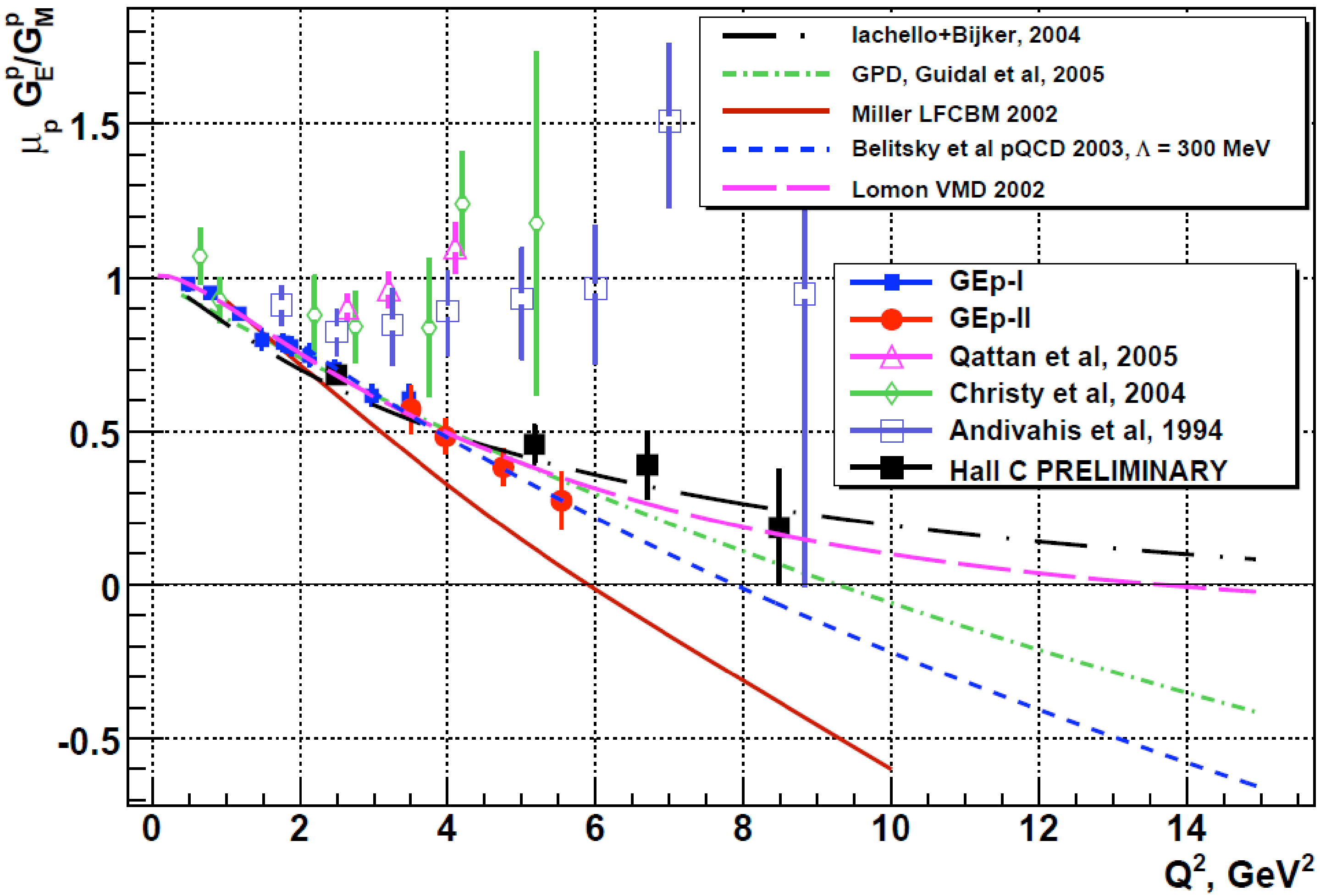
Using the GPD calculation, the corrections are



For polarization transfer, net corrections small, –1 to –2% at this Q^2, and come mainly from F_3 (or $Y_{2\gamma}$). BTW, $Y_{2\gamma}$ is ε dependent and about –(1/2)%

New data

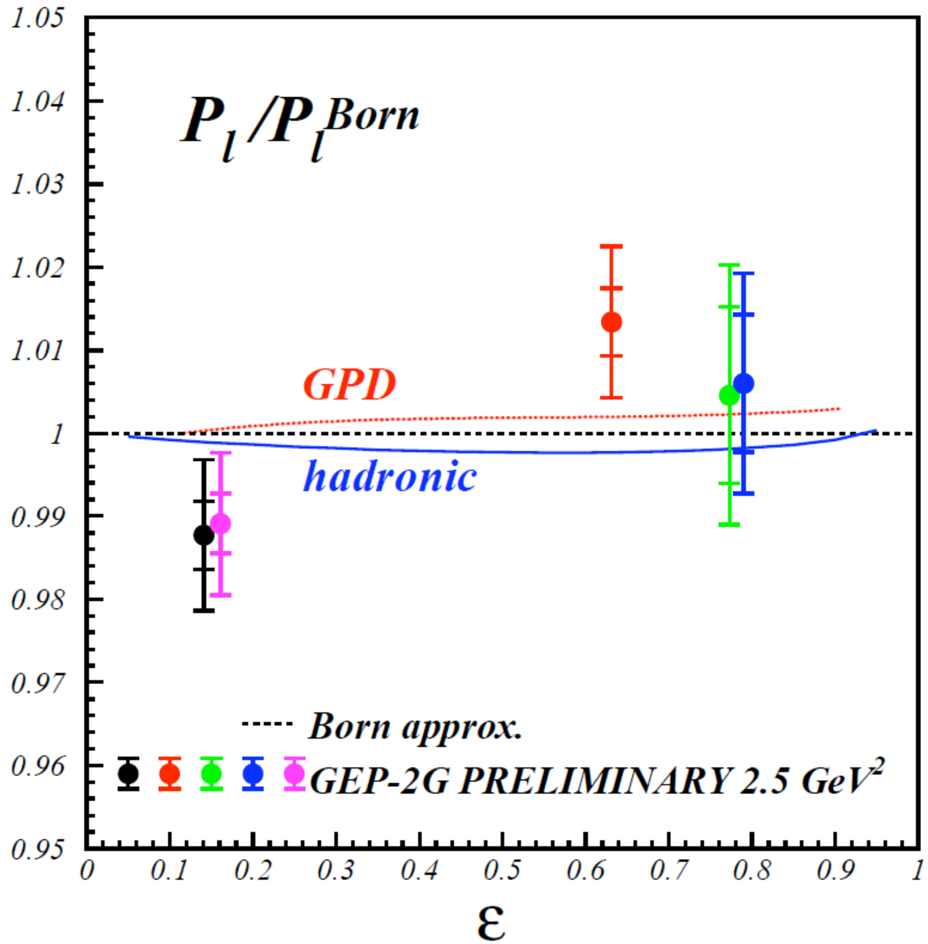
- New data was presented at this meeting and User's meeting
- From GEp-3,



Lubomar Pentchev, User's meeting

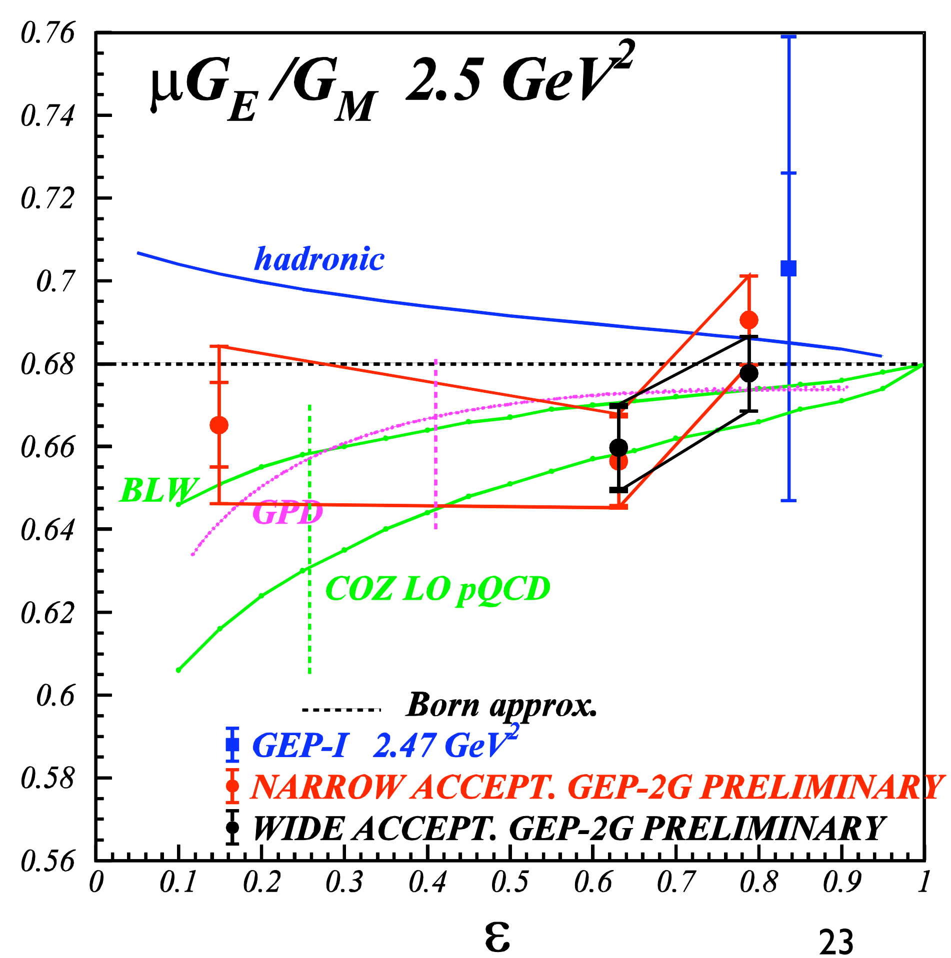
New data

- from GEp-2γ, longitudinal polarization
- predicted effect of 2γ is small for this observable



New data

- from GEp-2γ, ratio P_t/P_l polarizations at varying ε
- with kinematic factor removed, would be $\mu G_E/G_M$ and flat for one-photon exchange
- Famously opposite slope for hadronic and partonic calculations

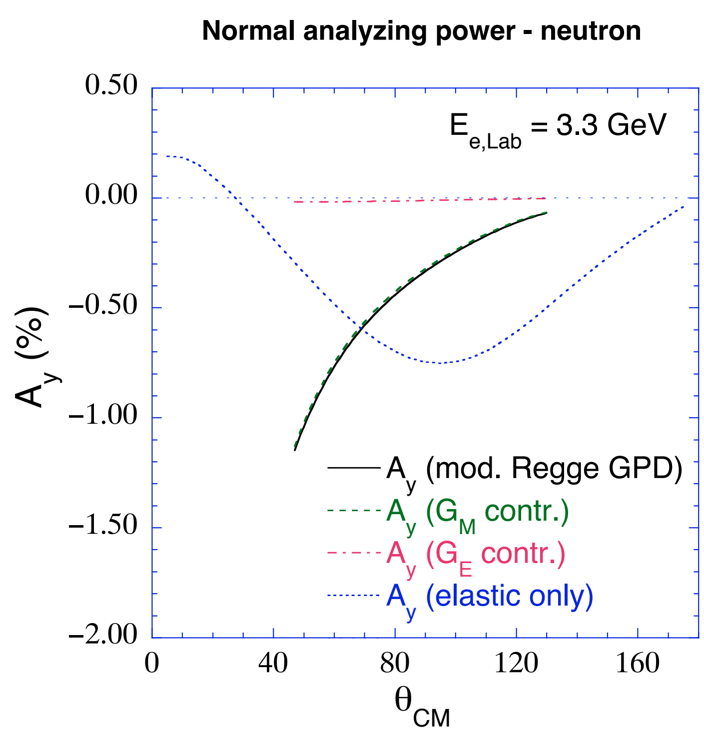


New data

- Single spin asymmetry (P_y or P_n) experiments
- zero if only one-photon exchange — any non-zero result means multiple photons
- Depends on imaginary part of new form factor functions

$$P_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}}\,\frac{1}{\sigma_R}\left\{-G_M\,\mathrm{Im}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E\,\mathrm{Im}\left(\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right)\right\}$$

- There exist "on-line" results showing P_n 10σ from 0 for neutron (YaWei Zhang, this morning).
- Calculated result shown



Final remarks (1/2)

- Clear evidence that 2-photon processes exist
    - Original Rosenbluth vs. polarization conflict
    - Observation of SSA in e^{-}-n scattering
    - no apparent evidence from polarization vs. ε
    - Other experiments expected
        - curvature in Rosenbluth plot
        - $e^{+}p$ vs. $e^{-}p$ comparison (VEPP, Olympus@DESY, CLAS)
- Reverse: measuring nucleon structure
    - Different observables sensitive to different quantities, as $\mathrm{Re}(\delta G_M)$, $\mathrm{Re}(F_3)$, and Imaginary parts of extended FF

Final remarks (2/2)

- Theory still not complete
  - Partonic calculation explains about half discrepancy at $Q^2 = 5.6\ \mathrm{GeV}^2$
  - Hadronic calculation perhaps a bit better in this regard
  - Questions of applicability at experimental Q^2
  - Do note summability of GPD and pQCD evaluations (no double counting)