Box Diagram Corrections

Carl Carlson
work with Ben Rislow
The College of William and Mary in Virginia
Hall A Collaboration Meeting
14 December 2011

Synopsis

- Theory panic over QWeak corrections
- Need for settling problem: resonance region structure functions — for the γZ case
- Relate to PVDIS

QWeak (reminder)

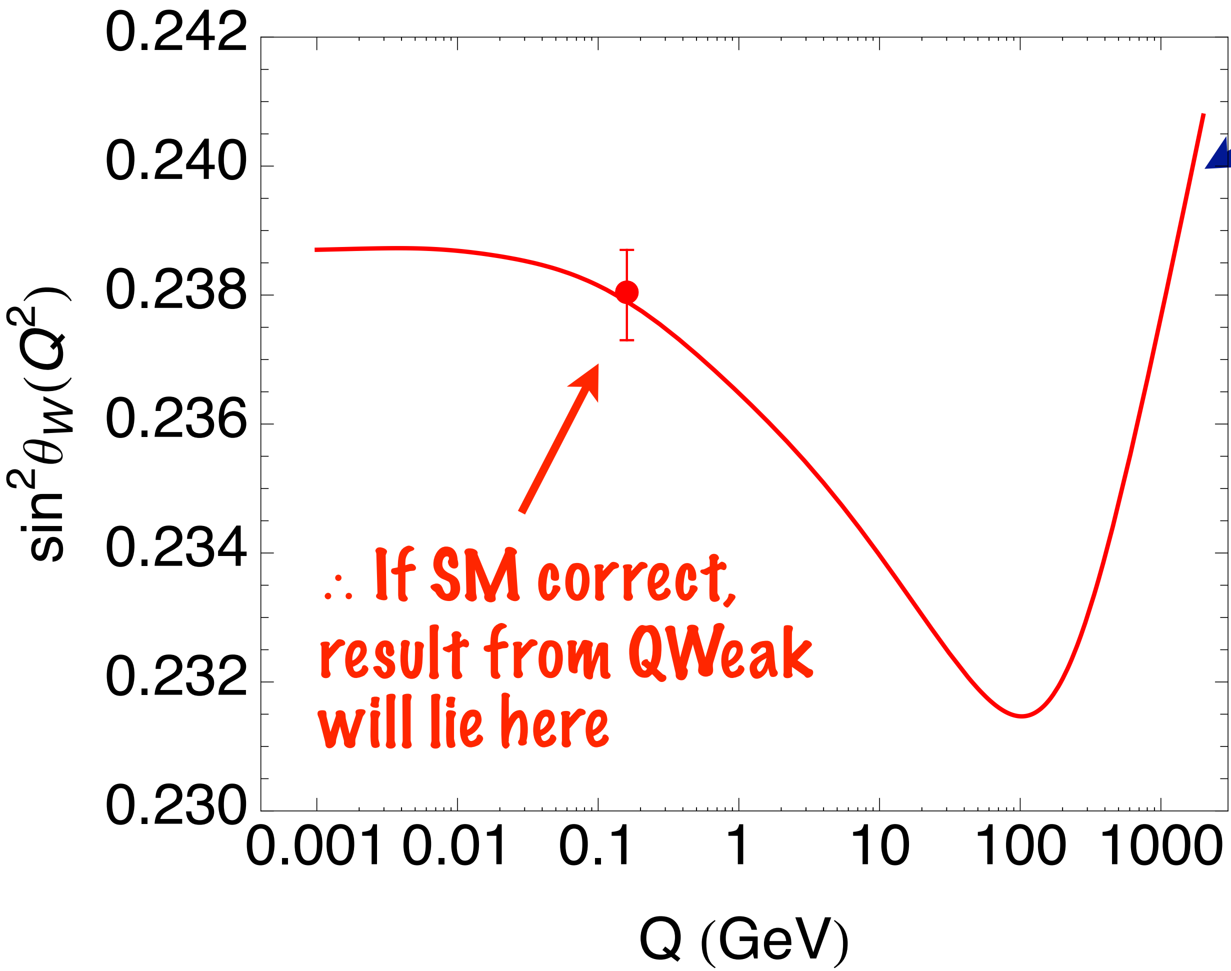
- ep elastic scattering, w/polarized electron
- measure asymmetry

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F}{4\pi\alpha\sqrt{2}} Q^2 \left[Q_W^p + BQ^2 + \ldots \right]$$

- LO: $Q_W^p = 1 - 4\sin^2\theta_W$
- Expt'l parameters:
  - $E_{electron}$ = 1.165 GeV
  - Q^2 = 0.026 GeV^2
  - 4% measurement of Q_W^p

Goal

- Test standard model



If SM is correct, $\sin^2\theta_W$ will evolve with Q^2 along this curve (norm fixed from HE expts. at Z-mass)

But there are radiative corrections!

Correction to ρ

$1 - 4\sin^2\theta_W(0)$

Troublesome box

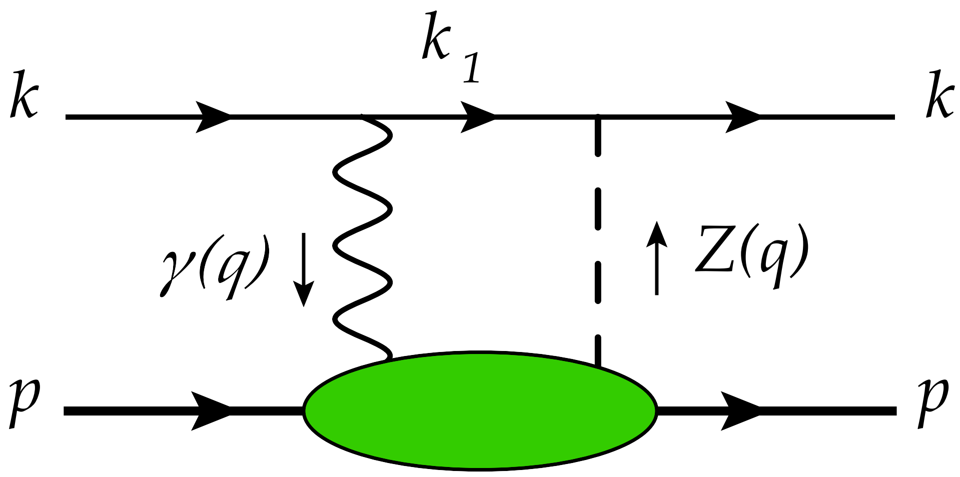
$$Q_W^p = (1 + \Delta\rho + \Delta_e)\left(Q_W^{p,LO} + \Delta_e'\right) + \Box_{WW} + \Box_{ZZ} + \mathrm{Re}\,\Box_{\gamma Z}$$

Corrections to the Z-boson and photon vertices

Well understood box corrections

- Summary as of 2003 in Erler et al. PRD 68, 016006

Troublesome box



- Troublesome: γ massless, so γ propagator (and diagram) big when momenta are small, whence cannot use perturbation theory on hadrons

- Idea (Gorchtein & Horowitz): Im part of loop comes when electron and blob on shell, hence like amplitude^2 or cross section in DIS. Then get Re part of loop from dispersion theory.

- Only works for $k_{in} = k_{out}$ (i.e., $Q^2 = 0$)

History

- Pre-2009, estimated box using perturbative calculation and putting in cutoff at hadronic scale.

$$\Box_{\gamma Z} = \frac{5\hat{\alpha}}{2\pi}(1 - 4\sin^2\theta_W(m_Z^2))\left(\ln\frac{m_Z^2}{m_\rho^2} + \frac{3}{2} \pm 1\right)$$

- Numerically this is 0.0051±0.0004 ; the error limit was about 0.6% of Q_W^p
- G&H, with a calculation that was different and arguably better in principle got a result for the vector (on the hadronic side) of 0.0027, but with unknown robustness.

Still trouble

- With γ and Z in and out, blob described by

$$W_{\mu\nu}^{\gamma Z} = \frac{1}{4\pi}\int d^4\eta e^{iq\eta}\langle ps\left|J_{Z\mu}(\eta)J_{\gamma\nu}(0)+J_{\gamma\mu}(\eta)J_{Z\nu}(0)\right|ps\rangle$$

and

$$W_{\mu\nu}^{\gamma Z} = \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)F_1^{\gamma Z}(x,Q^2)+\frac{p_\mu p_\nu}{p\cdot q}F_2^{\gamma Z}(x,Q^2)-i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha p^\beta}{2p\cdot q}F_3^{\gamma Z}(x,Q^2)$$

- "Wrong" structure functions: not $F_i^{\gamma\gamma}$ measured in purely EM deep inelastic scattering.

How to get

- Scaling region: no problem
- Resonance region
  - Use CQM, constituent quark model, to modify fits to $F_i^{\gamma\gamma}$ on proton target. (Rislow and me)
  - Use weak isospin and data on $F_i^{\gamma\gamma}$ for both proton and neutron targets to get $F_i^{\gamma Z}$ (G., H., and Ramsey-Musolf; Rislow and me, unpublished).
  - PVDIS measures $F_i^{\gamma Z}$ directly!

Workers (theory)

- Gorchtein & Horowitz (PRL 102, 091806 (2009)).
  Small distinction between $\gamma\gamma$ and γZ struc. fcns.
- Sibirtsev et al. (PRD 82, 013011 (2010)).
  Analytic result greater than above by a factor 2.
  Small distinction between $\gamma\gamma$ and γZ struc. fcns.
  Had uncertainty estimates (as do those below).
- Rislow & I (PRD 83, 113007 (2011))
  Confirmed factor 2.
  Quark model converting $\gamma\gamma$ to γZ structure fcns.
- Ramsey-Musolf joins G&H (PRC 84, 015502 (2011))
  Agrees on factor of 2.
  Different analysis of "background" contributions.

Results

- Results for γZ_V box,

Sibirtsev *et al.*	Rislow and Carlson	Gorchtein *et al.* (2011)
$(4.7^{+1.1}_{-0.4}) \times 10^{-3}$	$(5.7 \pm 0.9) \times 10^{-3}$	$(5.4 \pm 2.0) \times 10^{-3}$

- Agree within error limits
- Target experimental error on QWeak = 0.0028
- $\therefore$ above accuracy o.k.
- Opinion: G. et al. overly generous in error estimate of their "background" contributions
- Would like more security (for now and for future) on γZ structure functions

PVDIS in resonance region

- PVDIS asymmetry also depends on $F_i^{\gamma Z}$

$$A_{PVDIS} = g_A^e \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{xy^2 F_1^{\gamma Z} + \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}\left(y - \frac{y^2}{2}\right) x F_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + \left(1 - y - \frac{x^2 y^2 M^2}{Q^2}\right) F_2^{\gamma\gamma}}$$

- $x = Q^2/2m\nu$; $y = \nu/E$; $g_A^e = -½$; $g_V^e = -½ + 2\sin^2\theta_W$

Reminder of scaling region

- Current expt. has kinematics in scaling region.
  Formula simplifies and uses pdf's.
  For general target "A",

$$A_{PVDIS} = \frac{3G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{2C_{1u}(u_A + \bar{u}_A) - C_{1d}(d_A + \bar{d}_A + s_A + \bar{s}_A) + Y(2C_{2u}u_{VA} - C_{2d}d_{VA})}{4(u_A + \bar{u}_A) + d_A + \bar{d}_A + s_A + \bar{s}_A}$$

- where $Y(y) = \frac{1-(1-y)^2}{1+(1-y)^2}$ $\quad u_{VA} = u_A - \bar{u}_A$

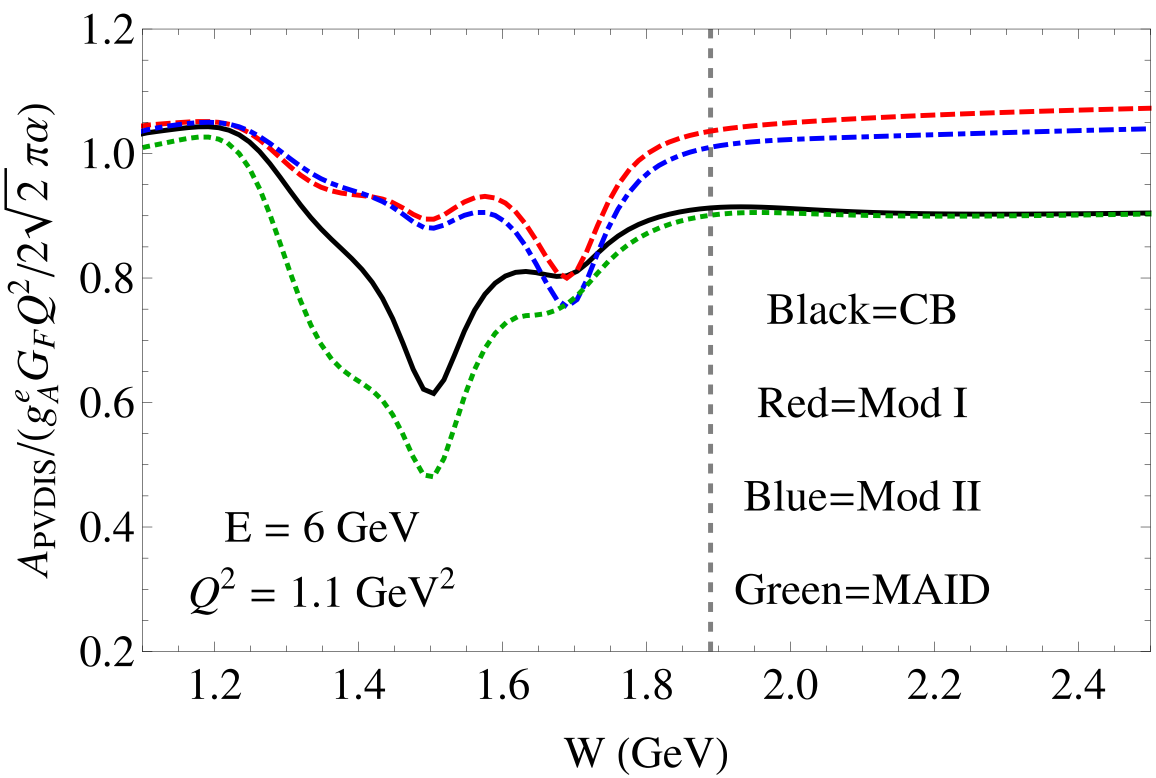
(in SM)

$$C_{1u} = 2g_A^e g_V^u = -\tfrac{1}{2} + \tfrac{4}{3}\sin^2\theta_W\,, \quad C_{1d} = 2g_A^e g_V^d = \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\,,$$
$$C_{2u} = 2g_V^e g_A^u = -\tfrac{1}{2} + 2\sin^2\theta_W\,, \quad C_{2d} = 2g_V^e g_A^d = \tfrac{1}{2} - 2\sin^2\theta_W\,.$$

- Proposal goal is to measure (small) Y term, and test standard model for axial quark coupling. Work in scaling region, where rest of terms better known, especially for isoscalar target (e.g., d).

PVDIS in resonance region

- Can measure $F_i^{\gamma Z}$ with wide coverage in energy, angle, Q^2.
- For sparser data, have predictions from existing models.



- proton target
- CB = CQM modified Christy-Bosted $F_{1,2}^{\gamma\gamma}$ fit
- Model I, II = GHRM based results
- MAID from isospin rotated MAID p & n EM fits
- Vertical dashed line = 6 GeV PVDIS expt. point

Where from? : CQM for resonance excitation

- Quark model with usual EM current:

$$J^\mu_\gamma = \sum_q e_q \bar{q}\, \gamma^\mu q$$

- Quark model with Z-boson current:

$$J^\mu_Z = \sum_q \bar{q}\, \gamma^\mu \left(g^q_V - g^q_A \gamma_5\right) q$$

- Get vector part of Z-boson matrix elements by substitution $e^q \rightarrow g_V^q$.

- In EM notation, current matrix elements for positive helicity photon gotten from effective operator

$$H_{eff} = \sum_q \left(A\, e_q L_{q+} + B\, e_q S_{q+}\right)$$

- A = electric, B = magnetic

Procedure CQM

- Use proton and resonance SU(6) quark model states, calculate helicity matrix elements A_λ for Z and γ
- Form ratios

$$C_R = \frac{2\sum A_\lambda(\gamma p \to R) A_\lambda(Z_V p \to R)}{\sum |A_\lambda(\gamma p \to R)|^2}$$

- Multiply contribution for each (of 7) resonances in Christy-Bosted fit by C_R
- Some cases simple, like p to Roper. Only B-term, and

  $C_R = 2/3 + Q_W^p$ (Q^2 indep.)

- Some, like $D_{13}(1520)$ and $S_{11}(1535)$, more complicated, but belong to same SU(6) multiplet, so can separate information on A and B terms, and get Z-boson matrix elements. C_R now Q^2 dependent.
- Do something related for smooth background

Where from : Weak isospin relations

- Basic relation

$$2\langle R^+|J_\mu^{Z_V}|p\rangle = (1-4\sin^2\theta_W)\langle R^+|J_\mu^\gamma|p\rangle - \langle R^0|J_\mu^\gamma|n\rangle - \langle R^+|\bar{s}\gamma_\mu s|p\rangle$$

- Neglect contribution of strange quark (A4, G0, HAPPEX)
- GHRM obtain matrix elements at $Q^2 = 0$ from PDG, form ratios C_R and neglect further Q^2 dependence in C_R. Use Christy-Bosted for resonances as we did.
- For background they extrapolate scaling region fits, modified for γZ case. [Model I = Color Dipole Model, G. Cvetic et al (2001), Model II = Generalized Vector Dominance, Alwall & Ingelman (2004)]

Where from : MAID

- Same basic relation

$$2\langle R^+|J_\mu^{Z_V}|p\rangle = (1-4\sin^2\theta_W)\langle R^+|J_\mu^\gamma|p\rangle - \langle R^0|J_\mu^\gamma|n\rangle$$

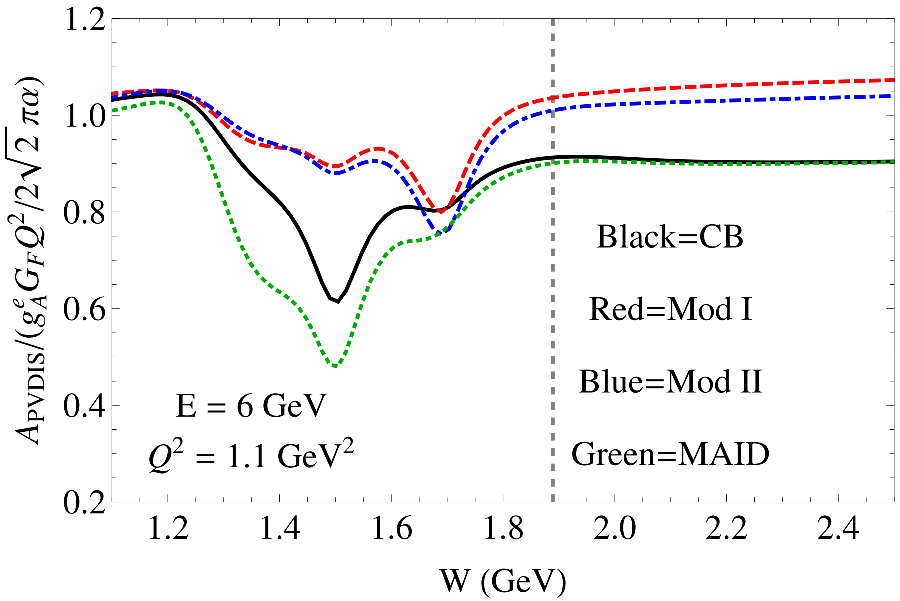
- MAID gives Q^2 dependent fits to both neutron and proton resonance electroproduction amplitudes [Tiator (2011)].

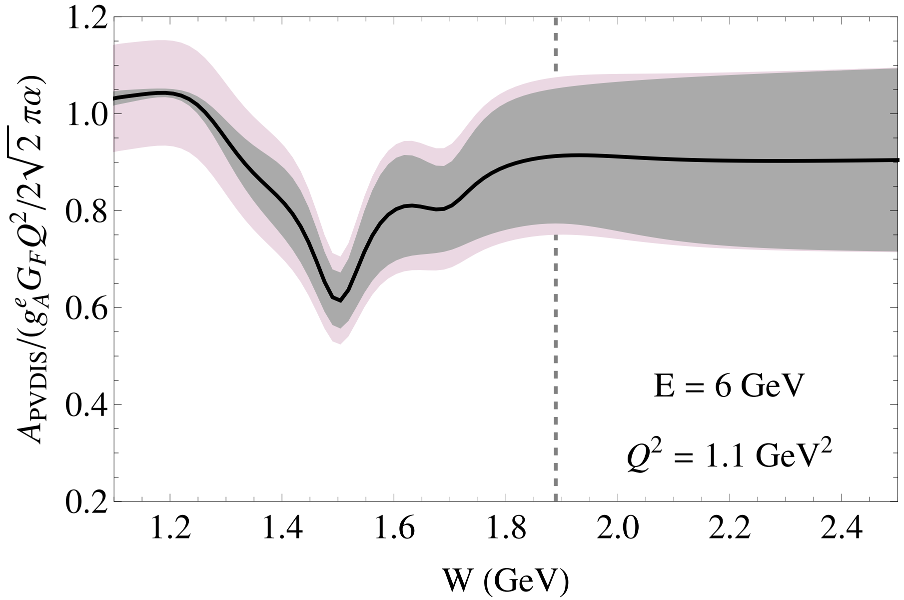
- C_R Q^2 dependent for all resonances.
See effect on Roper

$F_3^{\gamma Z}$

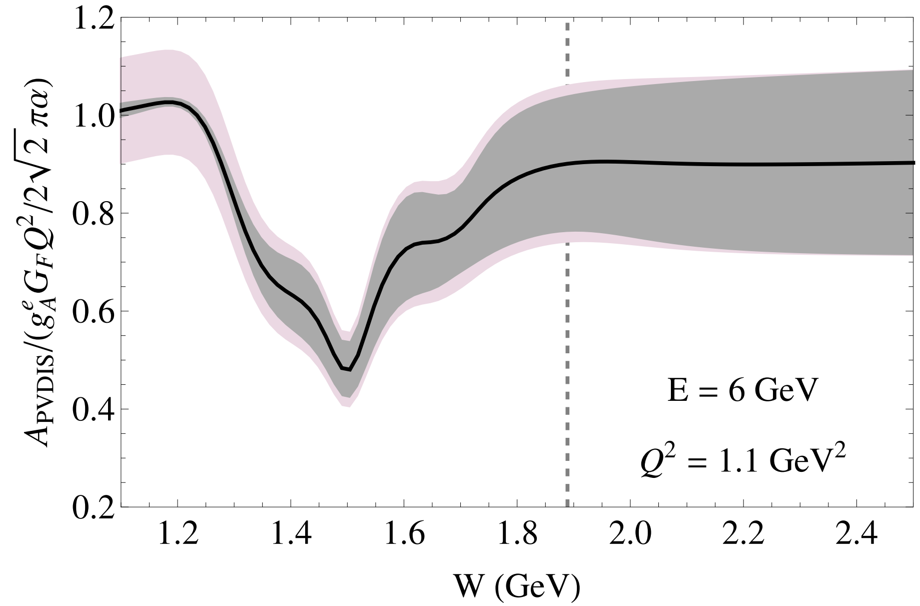
- Foremost: small contribution
  ($g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$)
  Less than 5% of PVDIS asymmetry in resonance region comes from $F_3^{\gamma Z}$
- CQM: nonrelativistically, axial term is kinematic factor times magnetic term
- Or, simple isospin rotation from charged current matrix elements. Fits by Lalakulich et al (2006). On the other hand there is almost no resonance neutrino data to fit to.

Proton plots with error estimates





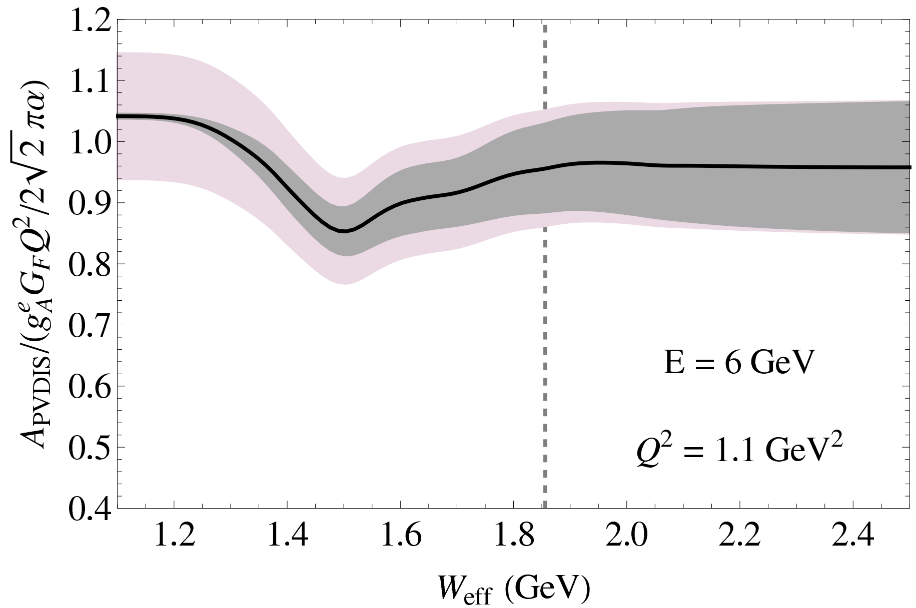
CQM ("CB")

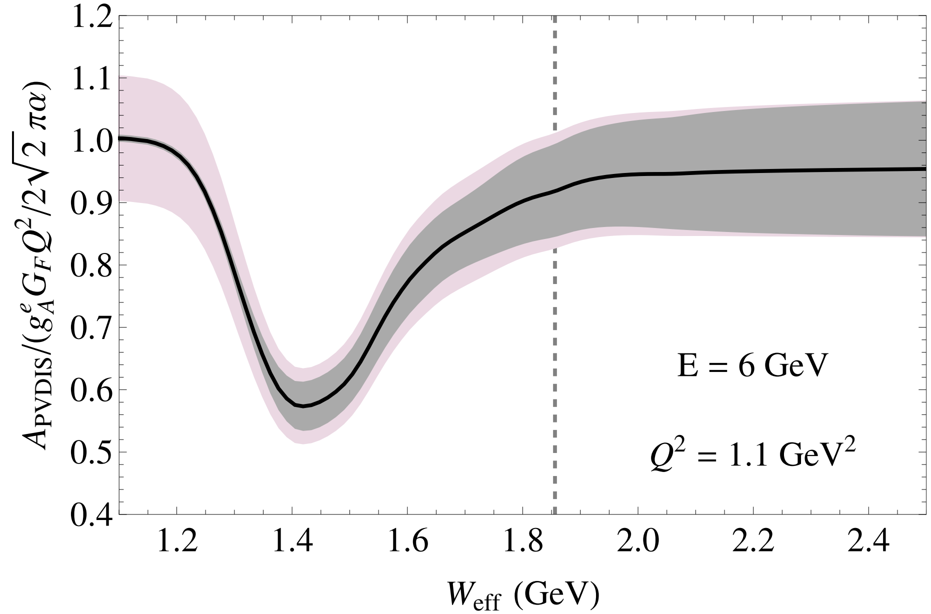


MAID

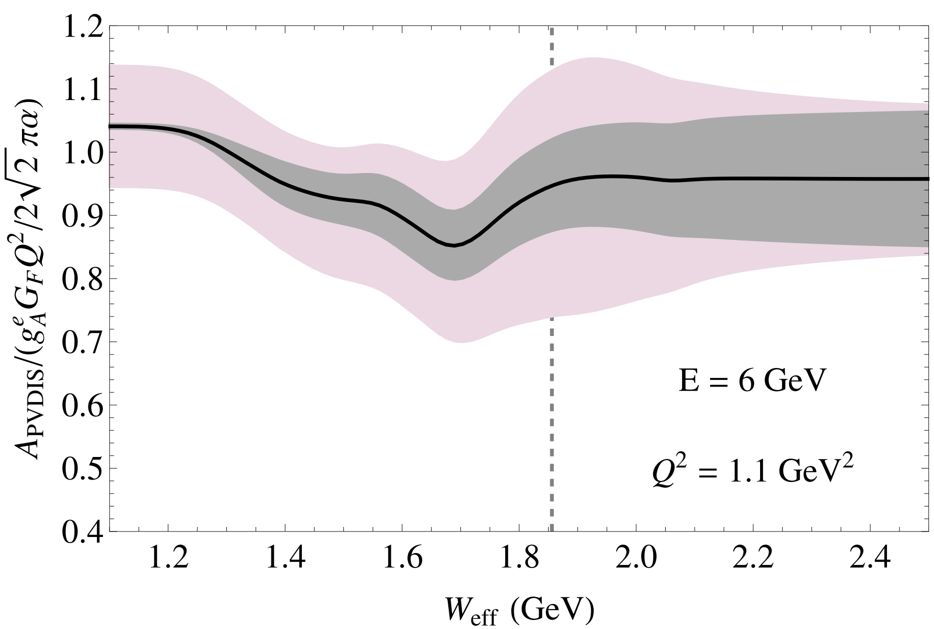
Deuteron plots

CQM ("CB")





MAID



GHRM

Ending

- Weak neutral current structure functions in resonance region are usable quatities
- Models notably different
- Can measure in PVDIS
- Would like on proton target, as well as deuteron
- Would also be interested in lower Q^2

Many thanks