FPP analysis: Extraction of target polarizations Maximum-likelihood method

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Outline

- Target polarization
- Maximum-likelihood technique
- FPP code

Target polarization

● Polarization of recoil proton = prefered orientation of its spin after reaction with electron (or maybe photon)

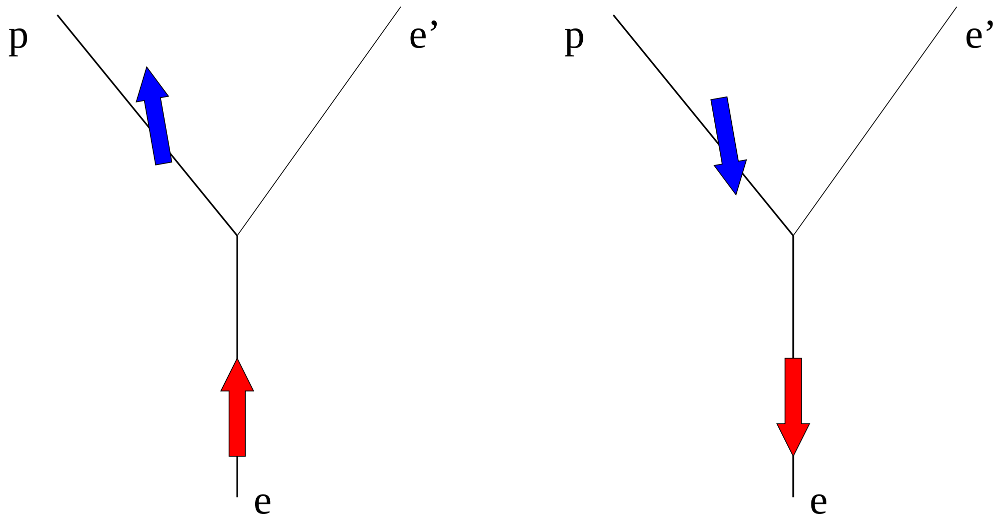
● If unpolarized target, two types of polarizations:

$\longrightarrow$ induced polarization, results from final state interaction

$\Rightarrow$ beam helicity independant $\mathbf{P}$

$\longrightarrow$ transferred polarization, from the electron (or photon)

$\Rightarrow$ beam helicity dependant $h\mathbf{P}'$



Separation of polarization types

- Overall polarization:

$$\Pi = \mathbf{P} + h\mathbf{P}'$$

- Can separate two types by flipping the sign of beam helicity, and measuring polarization for both states

$$\begin{aligned} \mathbf{P}^+ &= \mathbf{P}^- \\ h^+\mathbf{P}'^+ &= -h^-\mathbf{P}'^- \end{aligned}$$

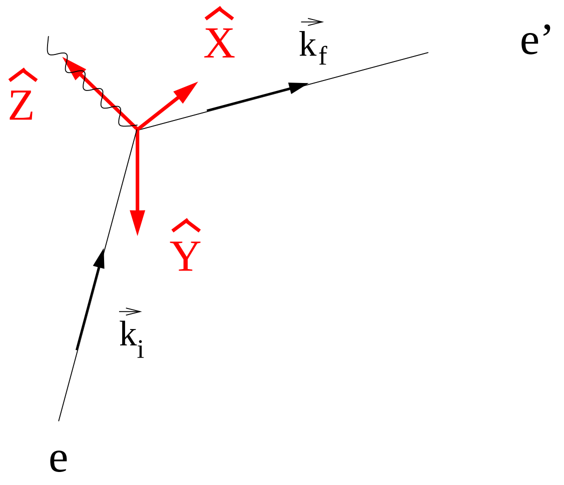
if $h^+ = h^- = h$ (achieved by accelerator at 0.3% level, as measured by Compton)

- Use sum and difference:

$$\begin{aligned} \mathbf{P} &= \frac{1}{2}\left(\Pi^+ + \Pi^-\right) \\ h\mathbf{P}' &= \frac{1}{2}\left(\Pi^+ - \Pi^-\right) \end{aligned}$$

Coordinate system

- Scattering plane



$$\hat{Z} = \frac{\vec{k}_i - \vec{k}_f}{|\vec{k}_i - \vec{k}_f|} \to \ell$$

$$\hat{Y} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|} \to n$$

$$\hat{X} = \hat{Y} \times \hat{Z} \to t$$

- 6 components to measure at target: $\mathbf{P}$ and $h\mathbf{P}'$

$$
\begin{aligned}
\Pi_x &= P_x + hP'_x \\
\Pi_y &= P_y + hP'_y \\
\Pi_z &= P_z + hP'_z
\end{aligned}
$$

- Only 4 things measured at the focal plane:

$$
\begin{aligned}
\Pi_x^{fpp} &= P_x^{fpp} + hP_x'^{fpp} \\
\Pi_y^{fpp} &= P_y^{fpp} + hP_y'^{fpp}
\end{aligned}
$$

$\rightarrow$ can't solve analytically $\Rightarrow$ parameter estimates

- Relation between the two:

$$
\begin{pmatrix} \Pi_x^{fpp} \\ \Pi_y^{fpp} \\ \Pi_z^{fpp} \end{pmatrix} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix} \begin{pmatrix} \Pi_x \\ \Pi_y \\ \Pi_z \end{pmatrix}
$$

Maximum likelihood method

- Probability that a proton i scatters in analyzer with angle φ_i:

$$P_i(\varphi_i) = \frac{1}{2\pi}\left[1 + (\epsilon_x + a_0)\sin\varphi_i + (\epsilon_y + b_0)\cos\varphi_i\right]$$

where

$$\begin{aligned}
\epsilon_x = -A_C.[\quad & S_{xx}(P_x \pm |h|P'_x) \\
+ \quad & S_{xy}(P_y \pm |h|P'_y) \\
+ \quad & S_{xz}(P_z \pm |h|P'_z)] \\
\epsilon_y = A_C.[\quad & S_{yx}(P_x \pm |h|P'_x) \\
+ \quad & S_{yy}(P_y \pm |h|P'_y) \\
+ \quad & S_{yz}(P_z \pm |h|P'_z)]
\end{aligned}$$

Likelihood function

- Probability that all protons have scattered the way they have (= prob of the actual distribution)

$$
\begin{aligned}
L &= \prod_{i=1}^{N} P_i \\
&= \prod_{i=1}^{N} \frac{1}{2\pi} \left(1 + \lambda_{0,i} + \sum_k \lambda_{k,i} P_k + \sum_k \lambda'_{k,i} P'_k \right)
\end{aligned}
$$

with

$$
\begin{aligned}
\lambda_{0,i} &= a_{0,i} \cos\varphi_i + b_{0,i} \sin\varphi_i \\
\lambda_{k,i} &= A_{Ci} \left(S_{yk,i} \cos\varphi_i - S_{xk,i} \sin\varphi_i \right) \\
\lambda'_{k,i} &= h \lambda_{k,i}
\end{aligned}
$$

$$k = (x, y, z)$$

- Maximize this function with parameters P_k and P'_k

$$\frac{\partial \ln L}{\partial P_k} = 0$$
$$\frac{\partial \ln L}{\partial P'_k} = 0$$

- Coupled nonlinear equations $\longrightarrow$ hard to solve
- Linearized by D. Besset et al., NIM A166, 515 (1979)

(expansion of $\ln L = \ln(1+x) = x + \frac{x^2}{2} + o(x^3)$)

$$\begin{pmatrix} \sum \lambda_x(1-\lambda_0) \\ \sum \lambda_y(1-\lambda_0) \\ \vdots \\ \sum \lambda'_z(1-\lambda_0) \end{pmatrix} = \begin{pmatrix} \sum \lambda_x\lambda_x & \cdots & \sum \lambda_x\lambda'_z \\ \sum \lambda_y\lambda_x & \cdots & \sum \lambda_y\lambda'_z \\ \vdots & \ddots & \vdots \\ \sum \lambda'_z\lambda_x & \cdots & \sum \lambda'_z\lambda'_z \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ \vdots \\ P'_z \end{pmatrix}$$

$$B = M \cdot (P_x, P_y, \cdots, P'_z)^T$$
$$(P_x, P_y, \cdots, P'_z)^T = M^{-1} \cdot B$$

- Statistical error is given by

$$\Delta P_k = \sqrt{M^{-1}_{kk}}$$

Constrained fit

- Elastic ep:

$$P_x = P_z = P'_y = 0$$

$\rightarrow$ possibility to constrain the fit to only 3 parameters

- Good agreement with other code for elastic ep

FPP code

- Code developped by Steffen Strauch
- FORTRAN-based
- Read data from ESPACE ntuple
- Ntuple contains:
  - target coordinates of proton and scattered electron
  - focal plane coordinates of proton
  - scattering in FPP
  - other ...
- COSY spin transport file (possibility of simple dipole)
- Output: summary file with a lot of information

Summary

- Measure at focal plane two polarization components transverse to the momentum, for each type of polarization (induced and transverse)

- Want to extract at the target three components

- Parameters (polarization components) estimated by maximum-likelihood method.

- Fortran-based code developped. Adaptation to C++ ?