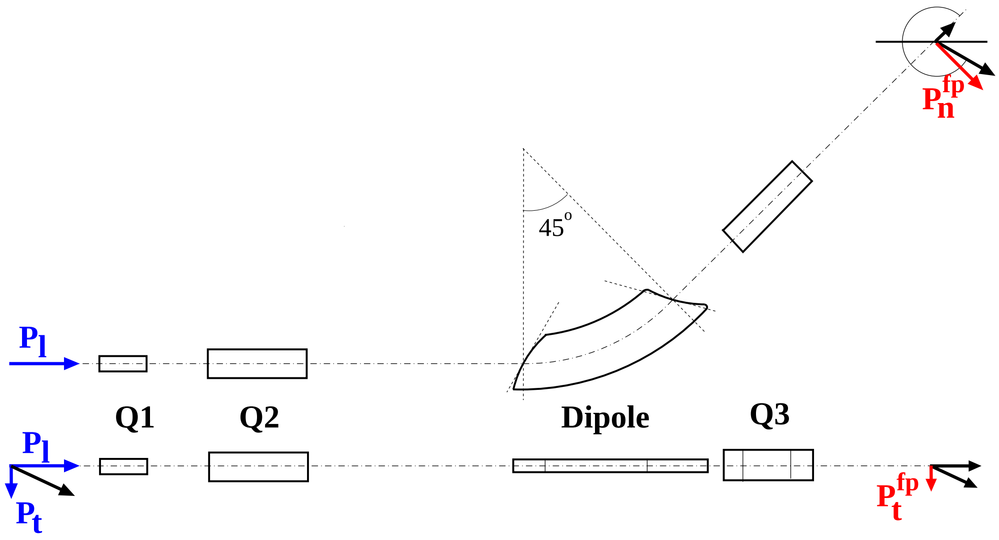
# SPIN TRANSPORT




$$\begin{pmatrix} P_n^{fp} \\ P_t^{fp} \\ P_l^{fp} \end{pmatrix} = \begin{pmatrix} S_{nn}\ S_{nt}\ S_{nl} \\ S_{tn}\ S_{tt}\ S_{tl} \\ S_{ln}\ S_{lt}\ S_{ll} \end{pmatrix} \begin{pmatrix} P_n \\ P_t \\ P_l \end{pmatrix}$$

$S_{ij}$ are functionals of trajectory:

$S_{ij}(\theta_{tg}, y_{tg}, \phi_{tg}, p)$ or $S_{ij}(x_{fp}, \theta_{fp}, y_{fp}, \phi_{fp}, p)$

- WHAT IS THE RELATION BETWEEN THE VELOCITY AND THE SPIN PROPAGATION IN THE SPECTROMETER?

- IF WITH A MODEL WE DESCRIBE THE OPTICS, DOES IT MEAN THAT WE DESCRIBE ALSO THE SPIN PRECESSION?

- WHICH ARE THE MODEL PARAMETERS, TO WHICH THE SPIN PRECESSION IS SENSITIVE?

## SPIN vs VELOCITY PROPAGATION

$$\frac{d\vec{S}}{dt} = \frac{e}{m\gamma}\vec{S} \times \left[\frac{g}{2}\vec{B}^{\parallel} + \left(1 + \frac{g-2}{2}\gamma\right)\vec{B}^{\perp}\right]$$

$$\frac{d\vec{v}}{dt} = \frac{e}{m\gamma}\vec{v} \times \vec{B}^{\perp}$$

• $\vec{B}^{\parallel} = 0$, planar trajectory (dipole approximation)

• $\vec{B}^{\parallel} = 0$, non-planar trajectory

• $\vec{B}^{\parallel} \neq 0$, no fringe field

• $\vec{B}^{\parallel} \neq 0$, with fringe field

• $\vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$

• $\vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$, linear change of $\phi$ and $\theta$

## SPIN vs VELOCITY PROPAGATION

- $\vec{B}^{\parallel} = 0$, planar trajectory (dipole approximation)

Relation between spin precession $\chi_\theta$ and bending angle $\Delta\theta$:

$$\chi_\theta = \gamma \left(\frac{g}{2} - 1\right) \Delta\theta = f \cdot \Delta\theta$$

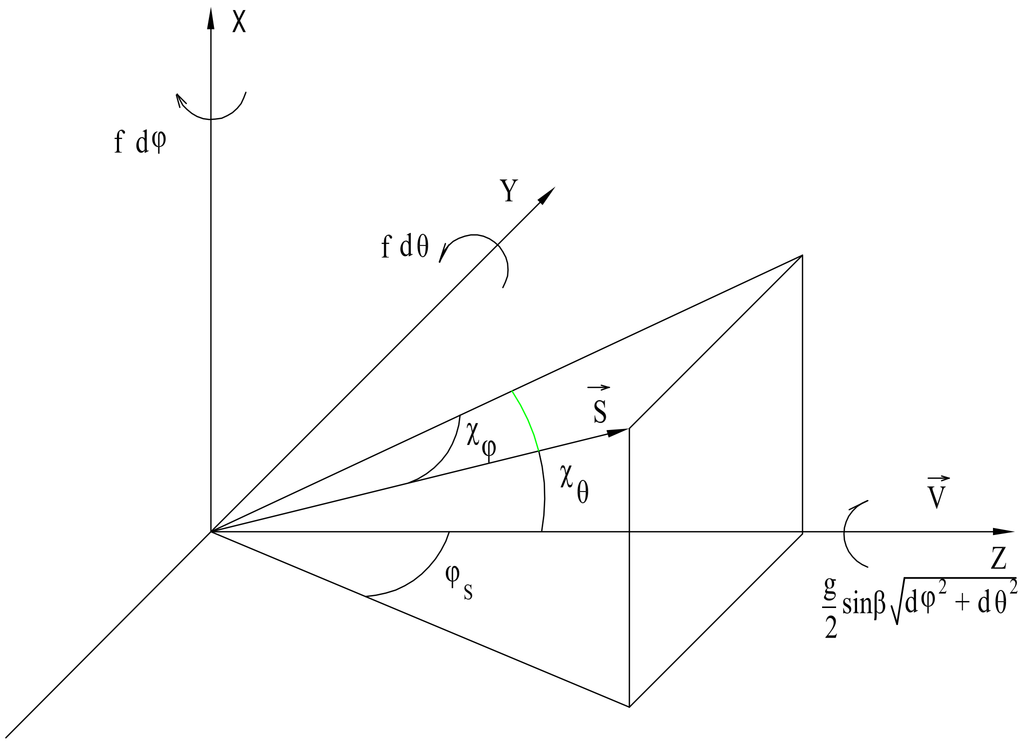
Is there a relation between the spin matrix and the bending angles under weaker assumption about the magnetic field?

## SPIN vs VELOCITY PROPAGATION

$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory

Using trajectory angles $\theta(s)$ and $\phi(s)$ one can calculate the spin matrix.

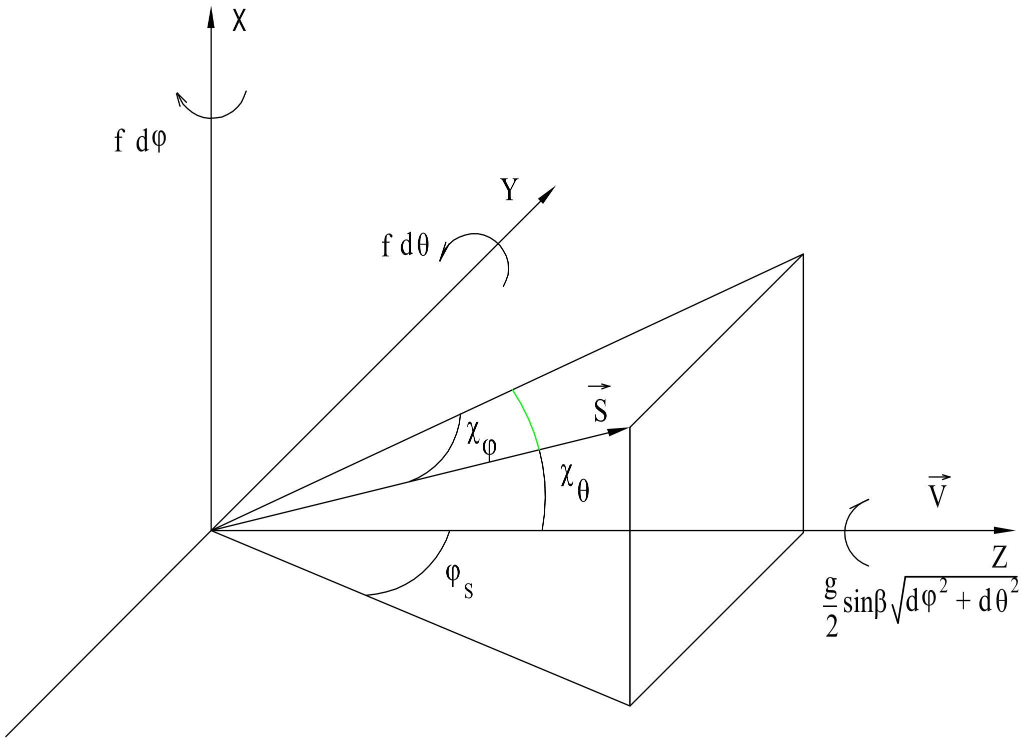
No need to know the angles w.r.t. spectrometer axis!



## SPIN vs VELOCITY PROPAGATION

• $\vec{B}^{\parallel} \neq 0$, no fringe field

$\beta$ - angle between $\vec{v}$ and $\vec{B}$, $\beta = \phi$




• $\vec{B}^{\parallel} \neq 0$, with fringe field

COSY calculations

## SPIN vs VELOCITY PROPAGATION

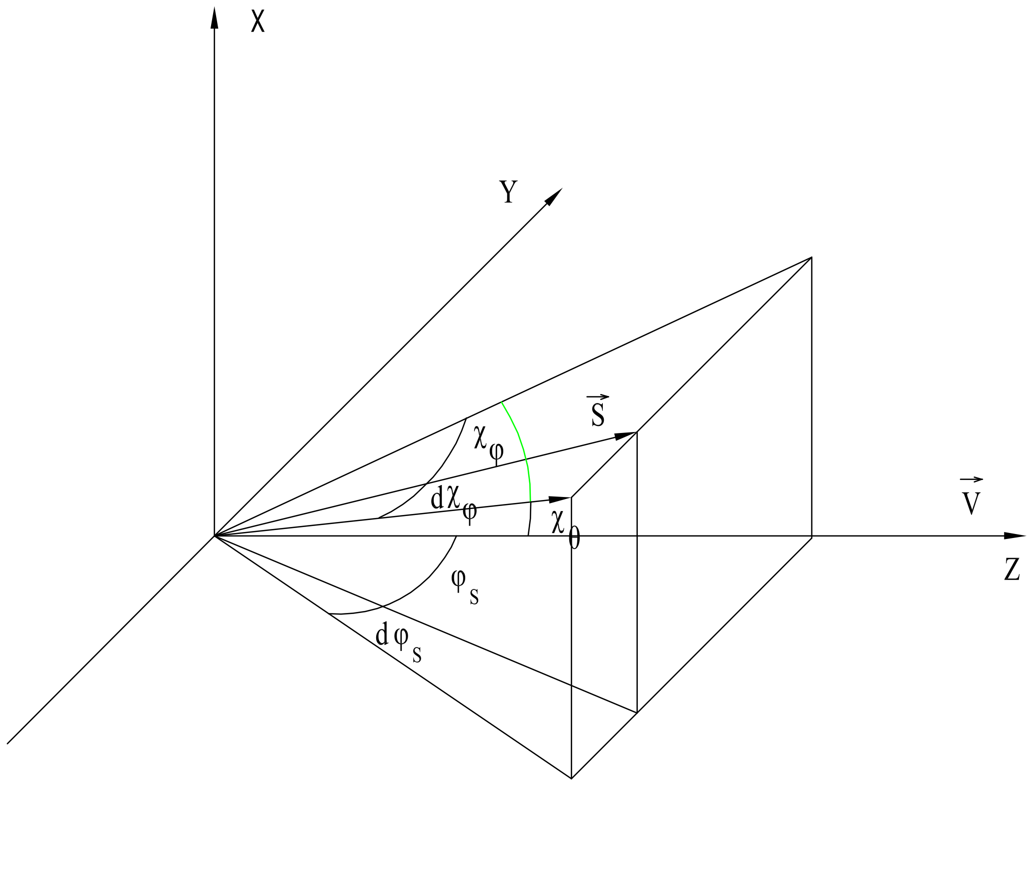
$\bullet \vec{B}^{\|} = 0$, non-planar trajectory, small $\phi_s$

$$d\chi_\phi = cos\chi_\theta(s)d\phi_s = cos\chi_\theta(s) f d\phi(s), \quad d\chi_\theta = f d\theta$$

$$\chi_\phi(s) = \int_0^s cos\chi_\theta(s) f d\phi(s), \qquad \chi_\theta = f\Delta\theta$$

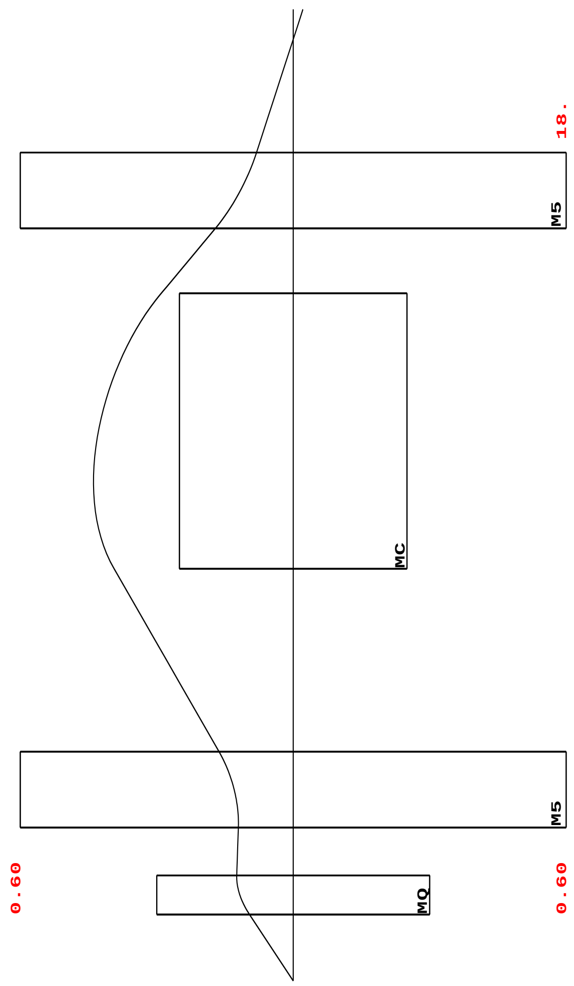
$$S_{nt} = -sin\chi_\theta cos\chi_\phi \qquad S_{nl} = sin\chi_\theta$$

$$S_{tt} = sin\chi_\phi \qquad S_{tl} = cos\chi_\phi$$



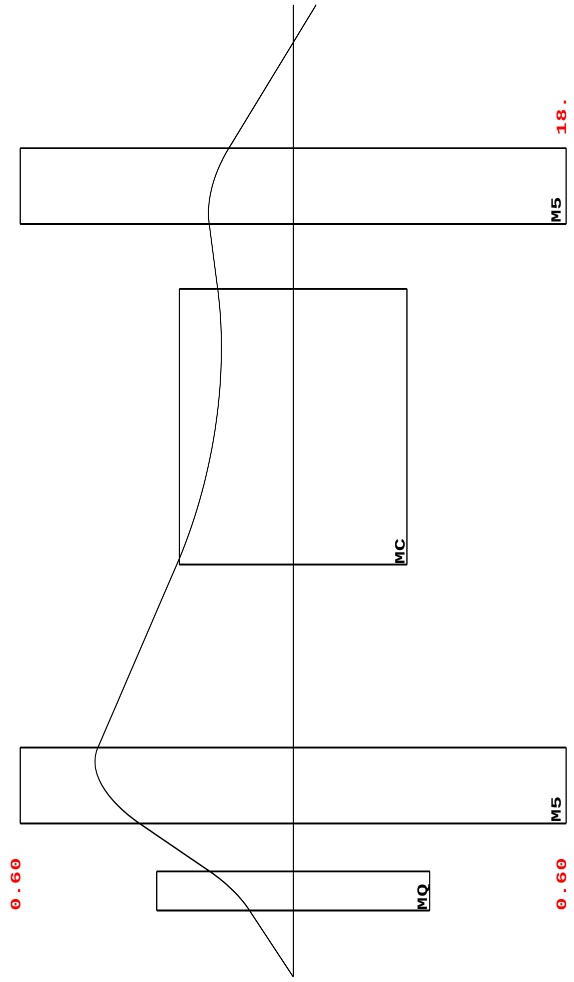
$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$

dispersive:



$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$

non-dispersive:



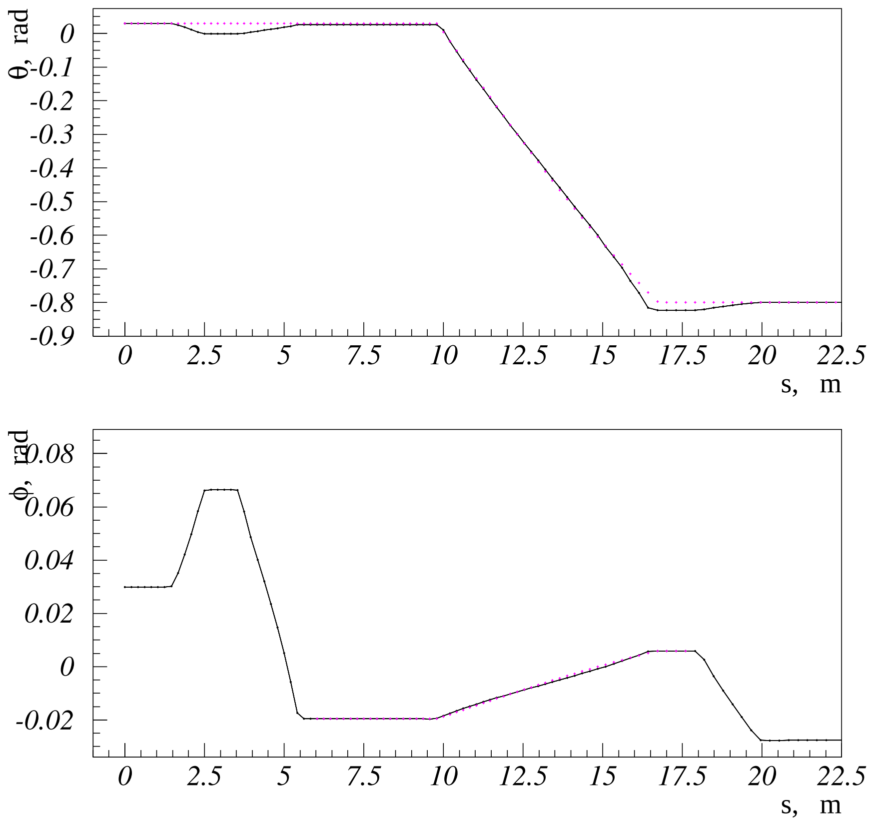
## SPIN vs VELOCITY PROPAGATION

$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$, linear change of $\phi$ and $\theta$

$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$

$$\chi_\phi(s) = \int_0^s cos\chi_\theta(s) f d\phi(s)$$

$$\chi_\phi = f \cdot \left[(\phi_{d1} - \phi_{tgt}) + (\phi_{d2} - \phi_{d1})\frac{sin\chi_\theta}{\chi_\theta} + (\phi_{fp} - \phi_{d2})cos\chi_\theta\right]$$



## SPIN vs VELOCITY PROPAGATION - SUMMARY

$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$

One can define precession angles in the non-dispersive and dispersive plane as:

$$\chi_\phi = \int_0^s cos\chi_\theta(s) f d\phi(s), \qquad \chi_\theta = f\Delta\theta$$

$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$, linear change of $\phi$ and $\theta$

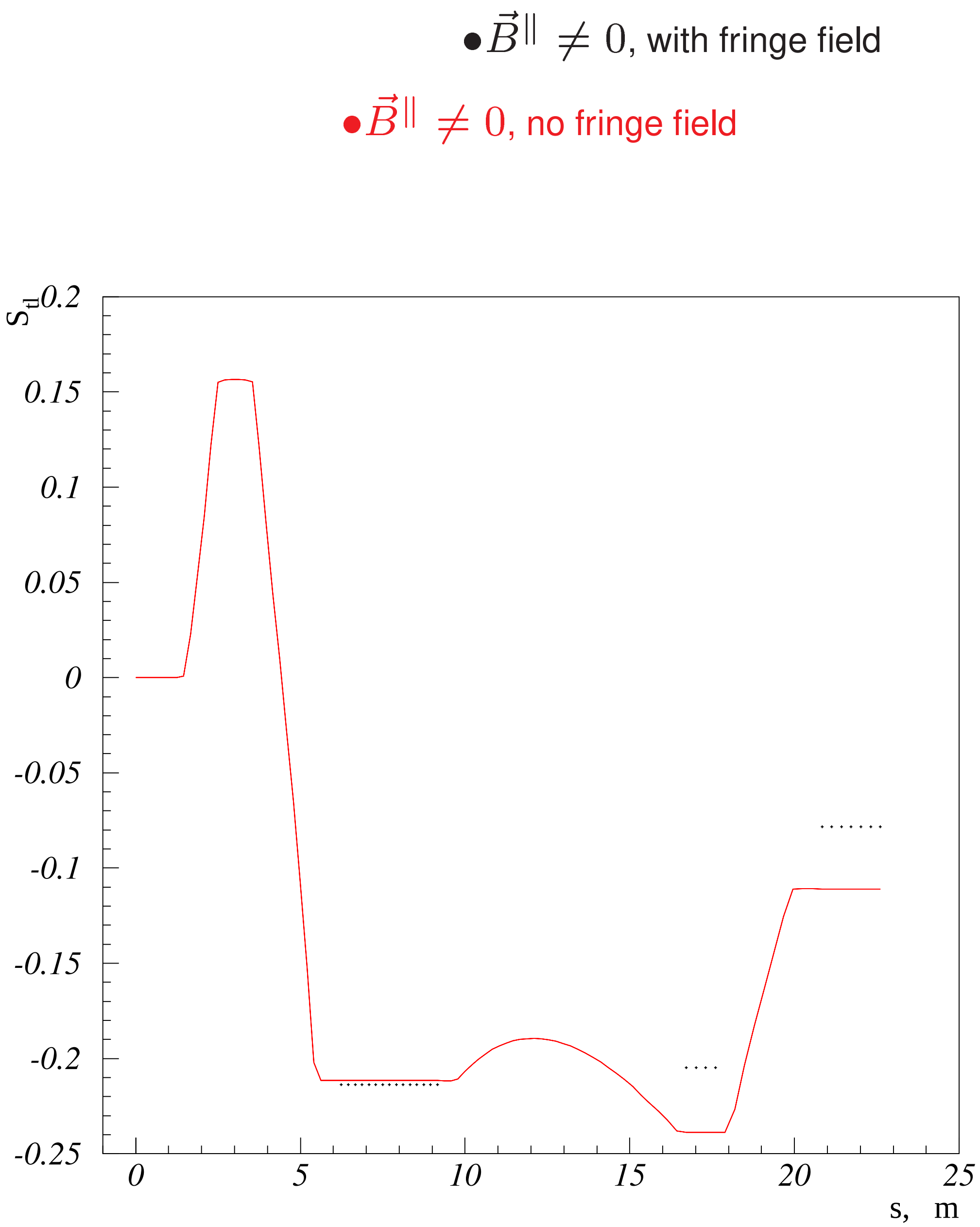
$$\chi_\phi = f \cdot \left[ (\phi_{d1} - \phi_{tgt}) + (\phi_{d2} - \phi_{d1}) \frac{sin\chi_\theta}{\chi_\theta} + (\phi_{fp} - \phi_{d2}) cos\chi_\theta \right]$$

$$S_{nt} = -sin\chi_\theta cos\chi_\phi \qquad S_{nl} = sin\chi_\theta$$

$$S_{tt} = sin\chi_\phi \qquad S_{tl} = cos\chi_\phi$$

There is a relation between bending angles and spin matrix in this approximation, which is much more general than the dipole approximation and can be used for spin transport in HRS

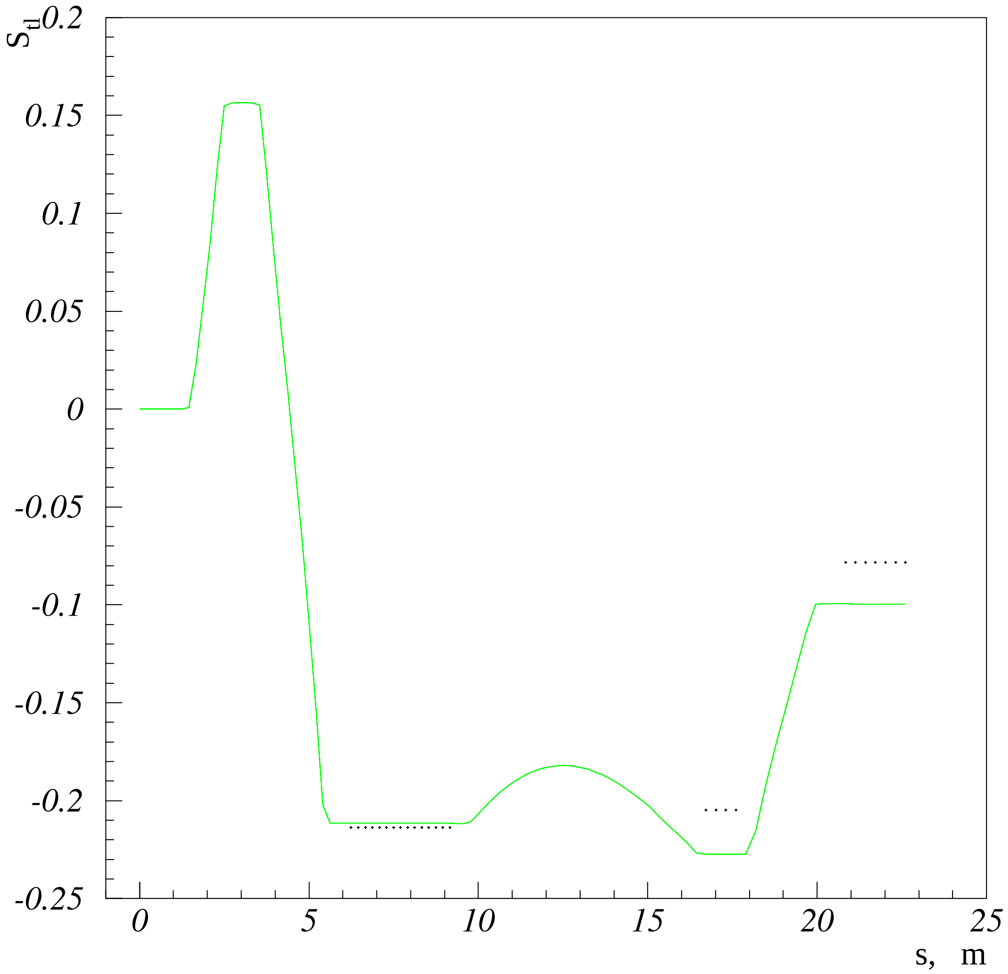
$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$



$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$

$\bullet \vec{B}^{\|} \neq 0$, with fringe field

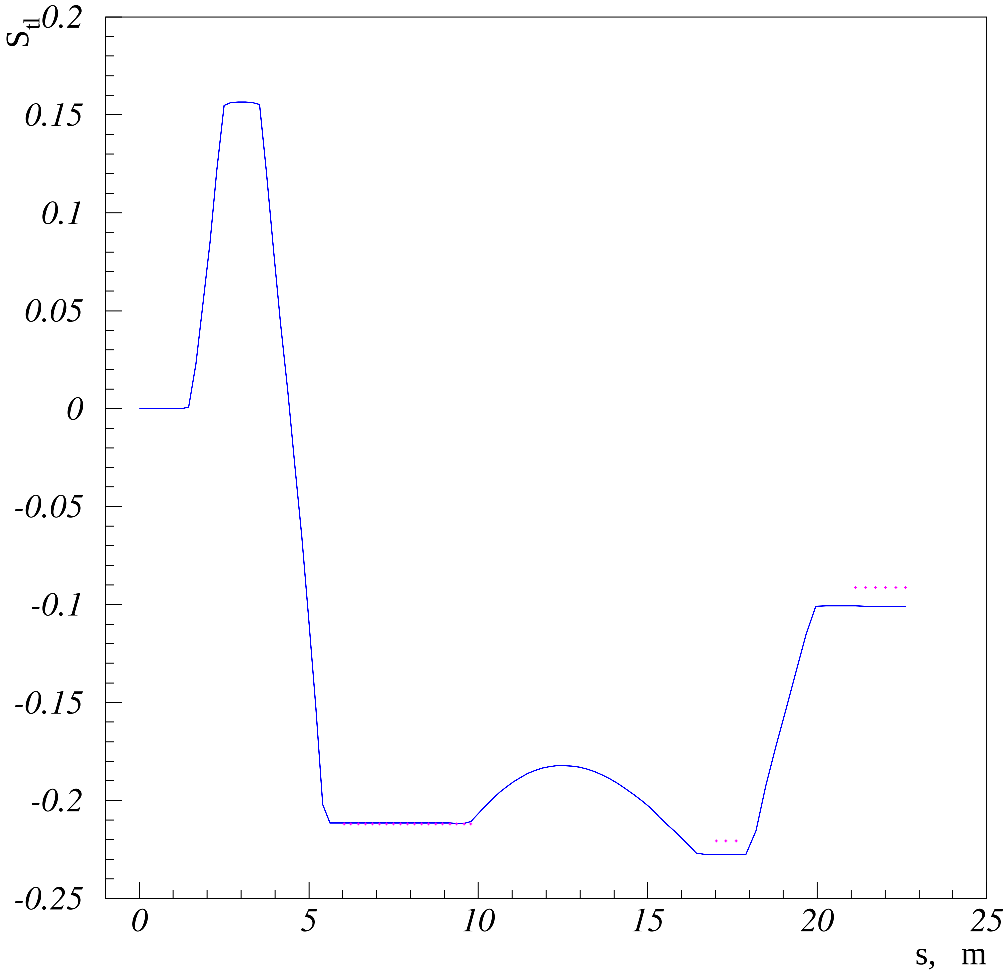
$\bullet \vec{B}^{\|} = 0$, non-planar trajectory



$\theta = 30mrad, p = p_0 = 2.053GeV/c, y_{tgt} = 0, \phi_{tgt} = 30mrad$

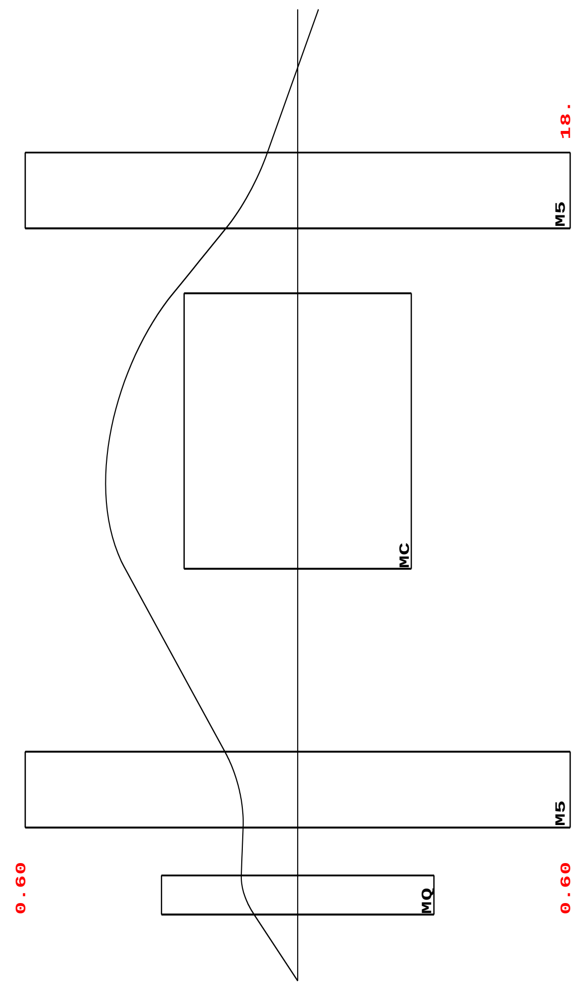
• $\vec{B}^{\|} = 0$, non-planar trajectory, small $\phi_s$

• $\vec{B}^{\|} = 0$, non-planar trajectory, small $\phi_s$, linear change



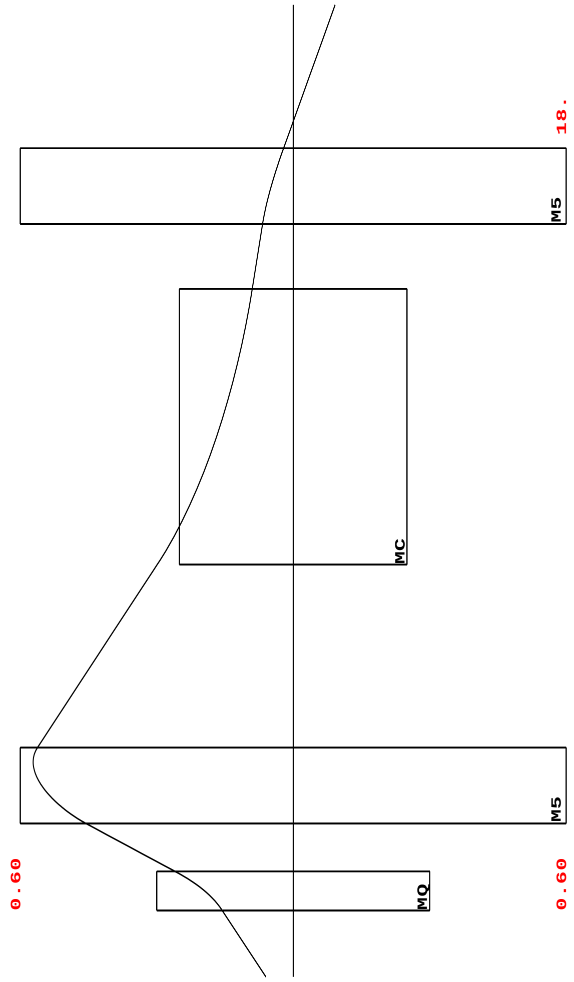
$\theta = 30mr, p = p_0 = 2.053GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$

dispersive:

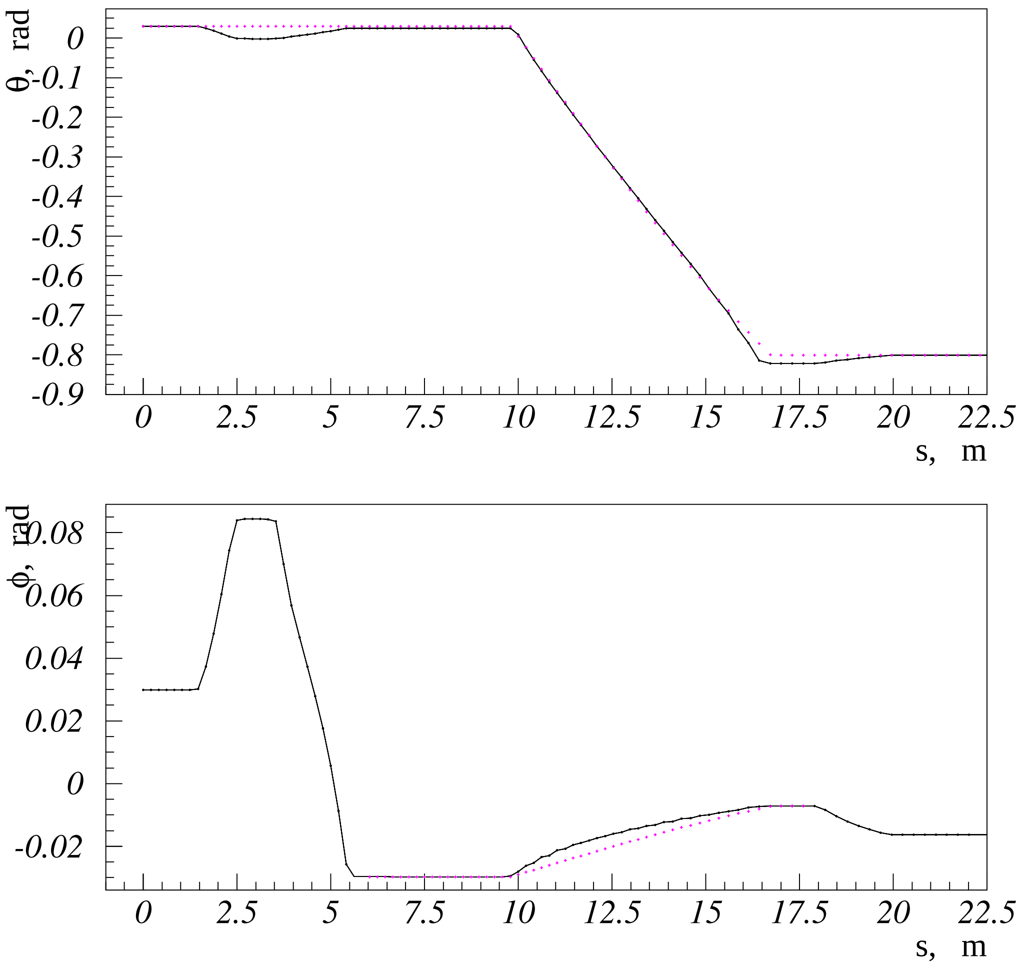


$\theta = 30mr, p = p_0 = 2.053GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$

non-dispersive:



$$\theta = 30mr, p = p_0 = 2.053GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$$



$$\theta = 30mr, p = p_0 = 2.053GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$$

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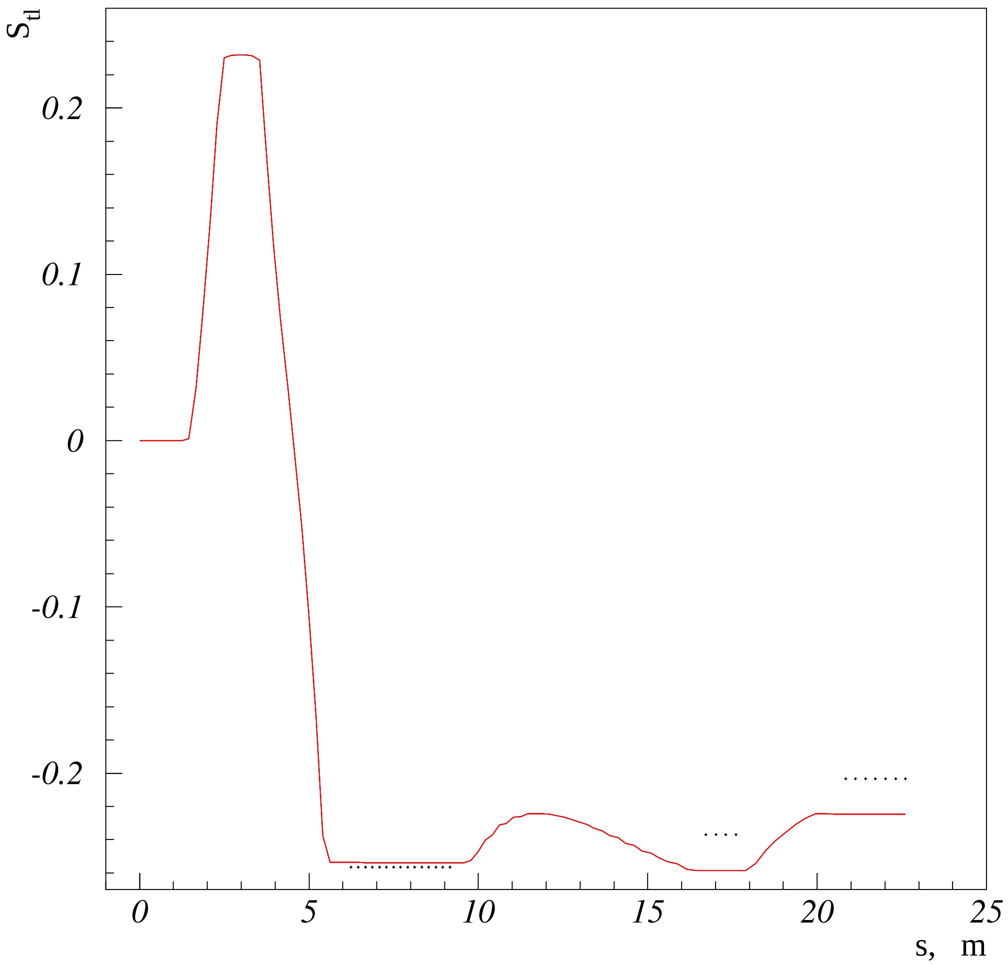

$\theta = 30mr, p = p_0 = 2.053 GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$

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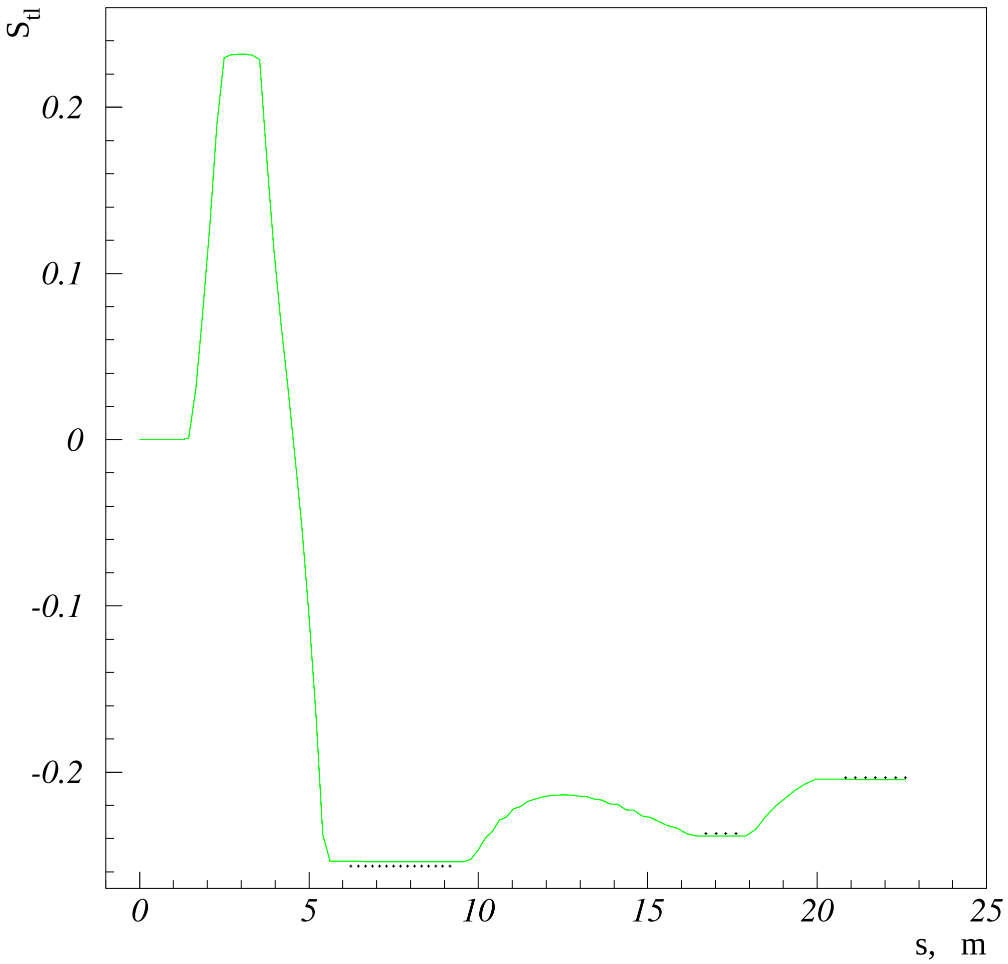
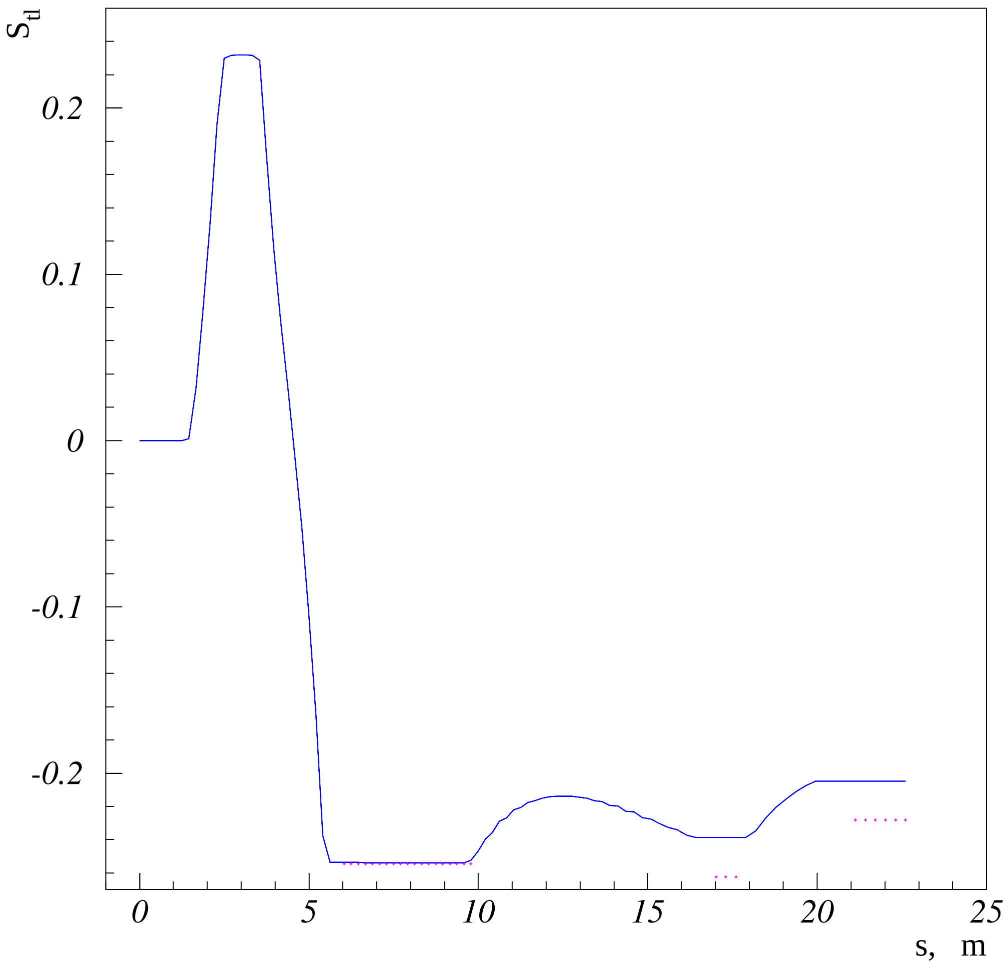
$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory



$\theta = 30mr, p = p_0 = 2.053 GeV/c, y_{tgt} = 30mm, \phi_{tgt} = 30mr$

• $\vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$

• $\vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$, linear change



## SPIN vs VELOCITY PROPAGATION

$\theta = 30mr, p = p_0 = 2.053 GeV/c$

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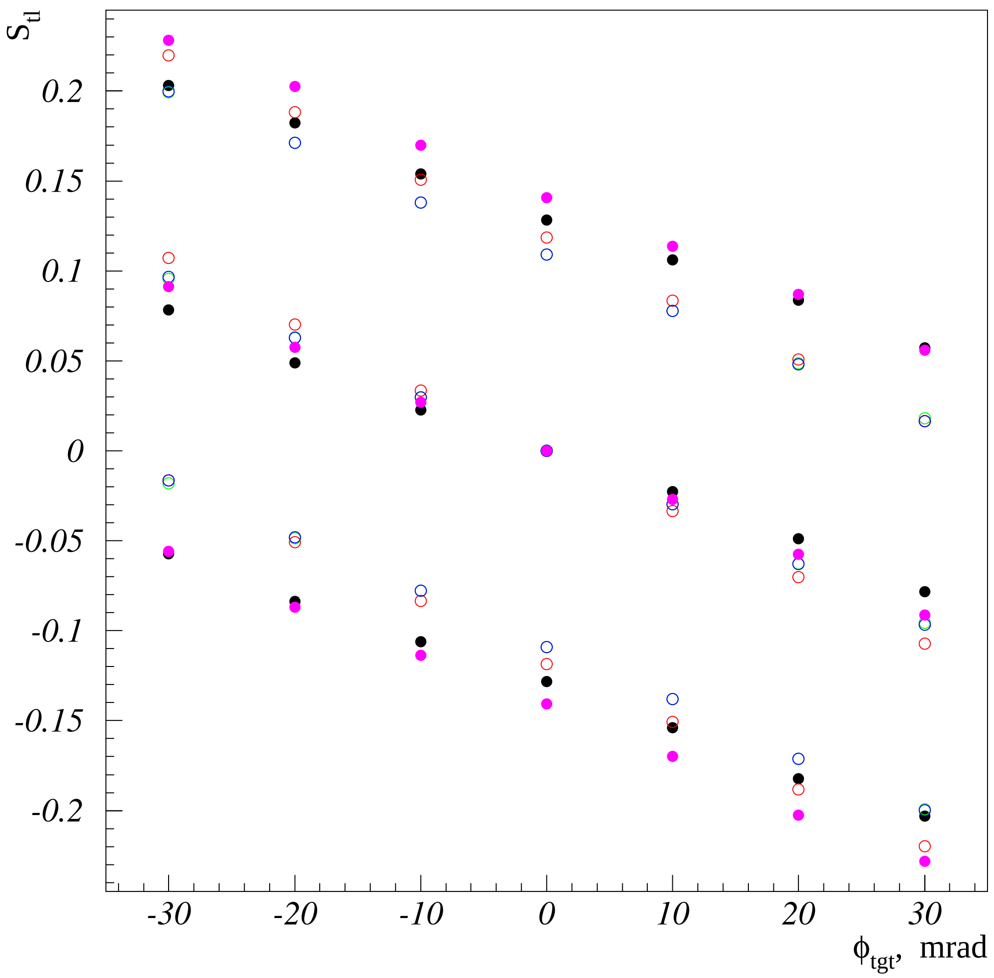
$\bullet \vec{B}^{\parallel} = 0$, non-planar trajectory, small $\phi_s$, linear change



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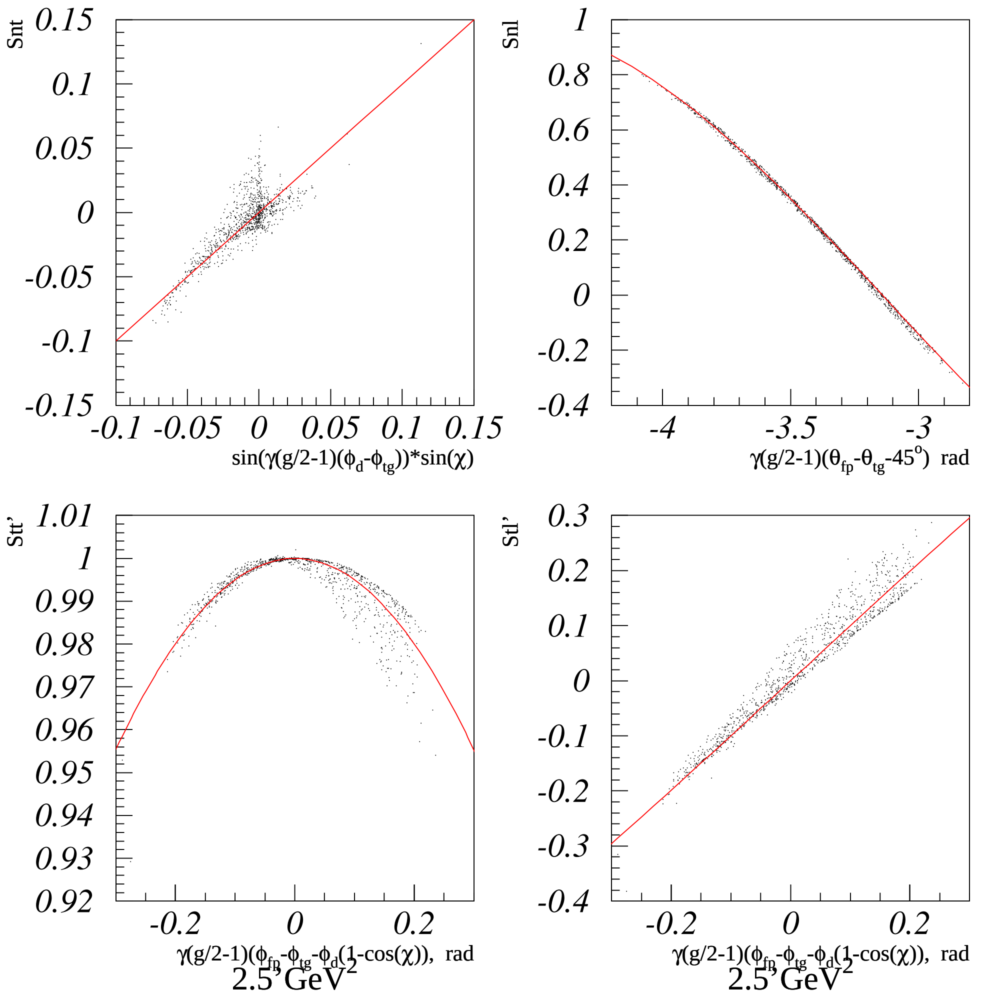
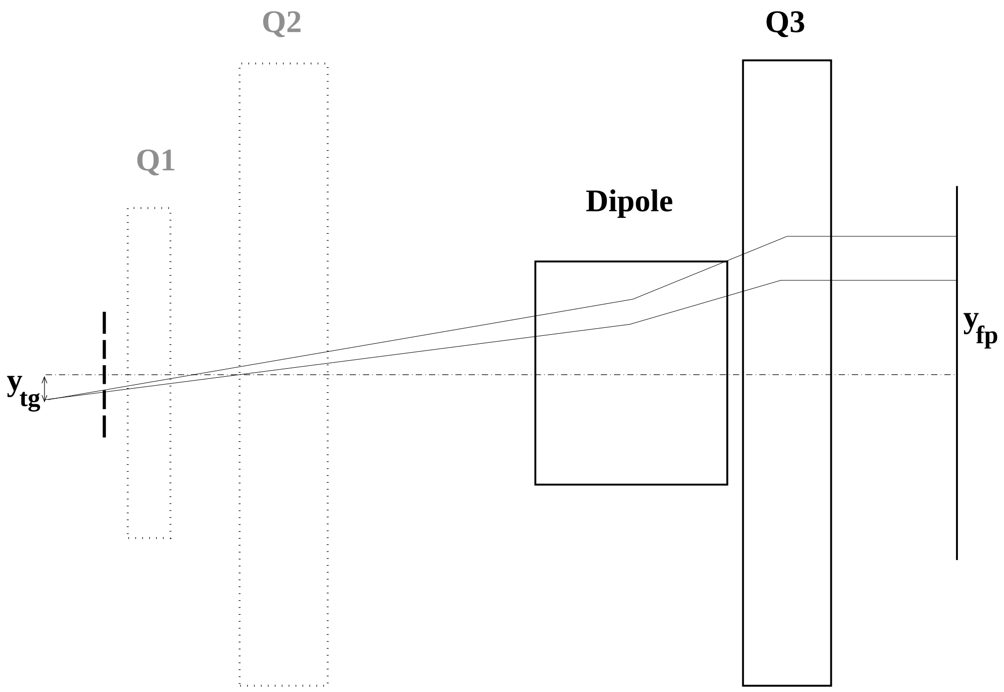


## PRECISE MEASUREMENT OF $\phi_{tg}$ and $y_{tg}$

$\Delta\phi \ \pm 0.3mrad$

$y_{tg} \ \pm 0.2mm$




$$y_{fp} = 2.75y_{tg} + 40.90[mm/mrad]\phi_{tg}$$

$$\phi_{fp} = -0.02[mrad/mm]y_{tg} + 0.00\phi_{tg}$$

*Quadrupole alignment studies in the HRSs,*

*L. Pentchev, J. LeRose, and the FPP collaboration,* **Jlab-TN-01-052**

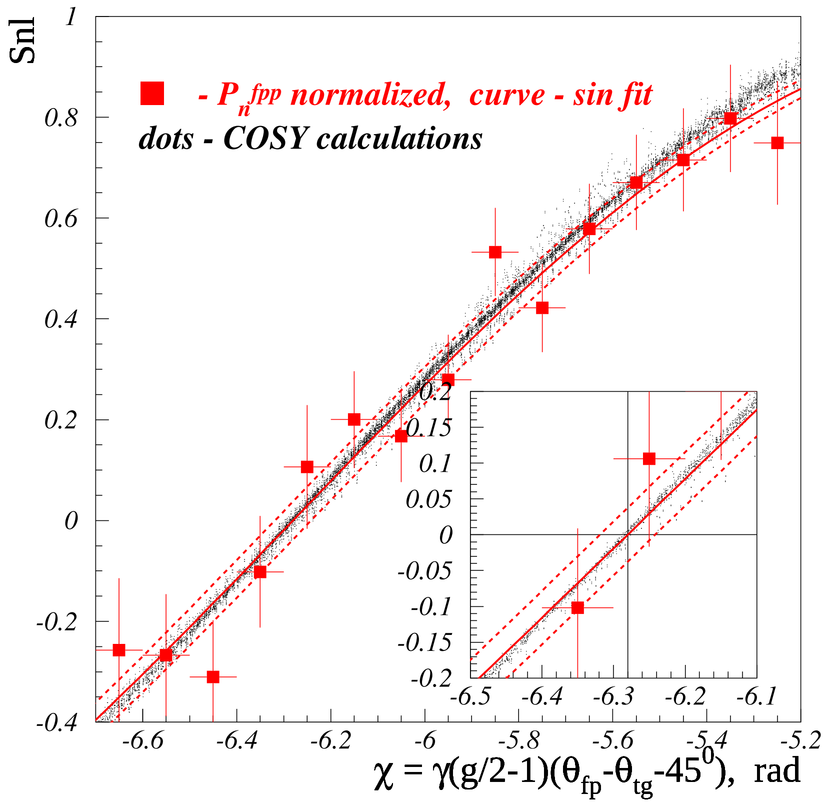
## Measuring the total bending angle in dispersive plane

$P_n^{fpp} \sim S_{nl}$

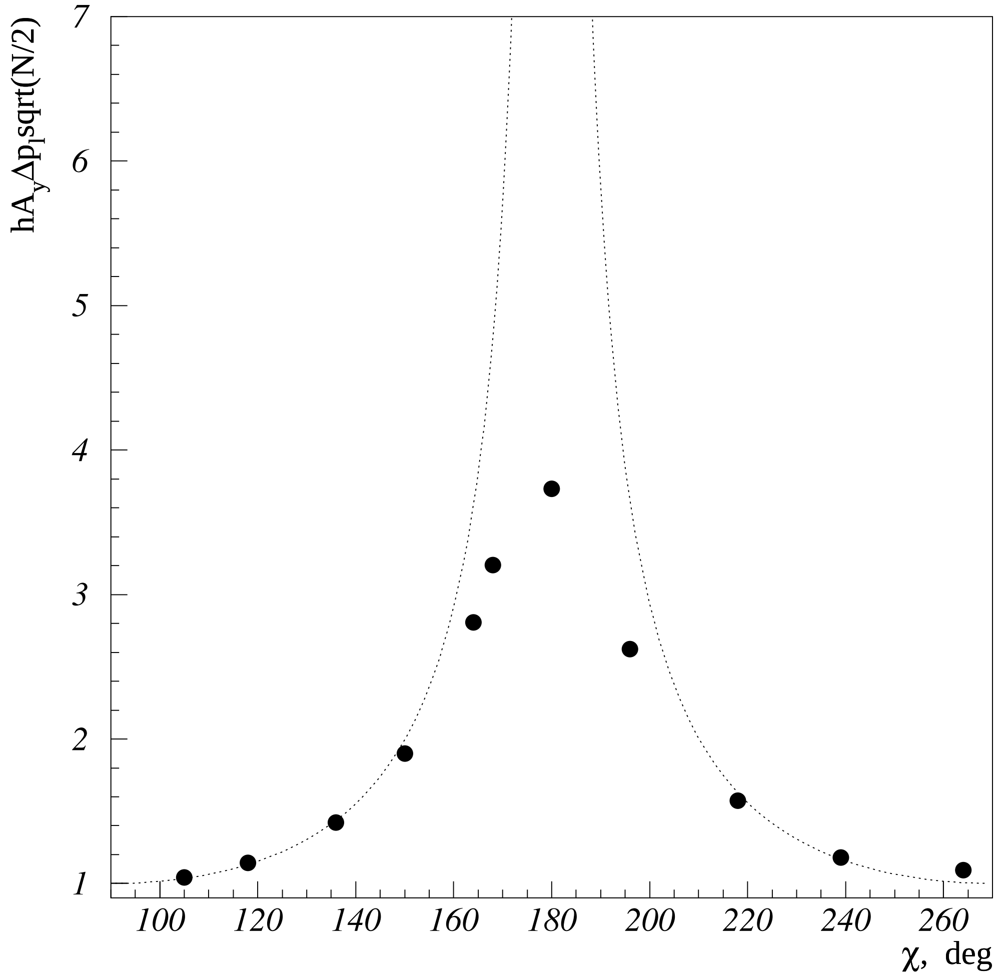
$\Delta\chi = 35mrad$

$\Delta\theta = 4.4mrad$

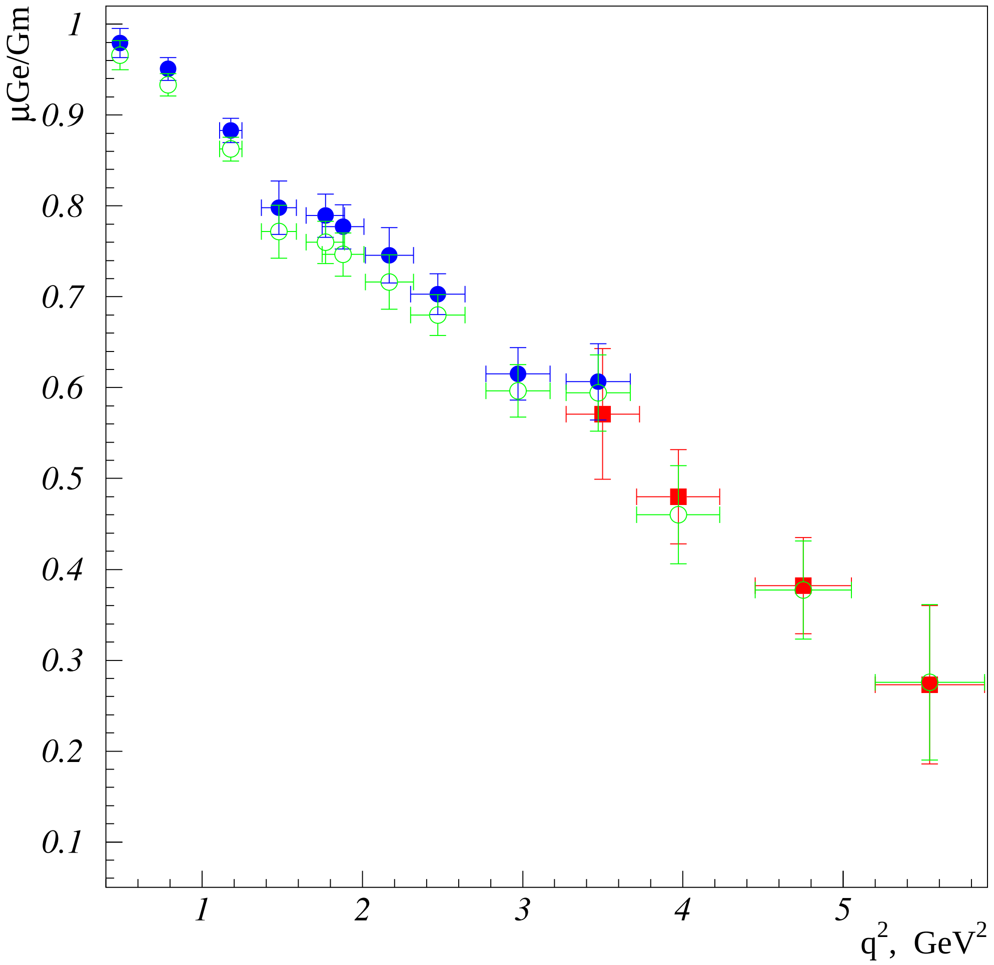
GEP-II experiment for $Q^2 = 5.6GeV^2$



## Reconstruction of $P_l$



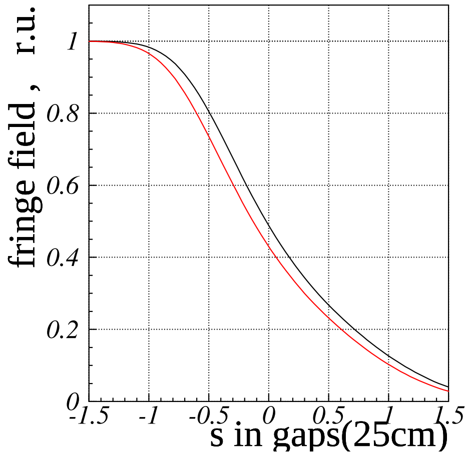
$\mu G_e/G_m$

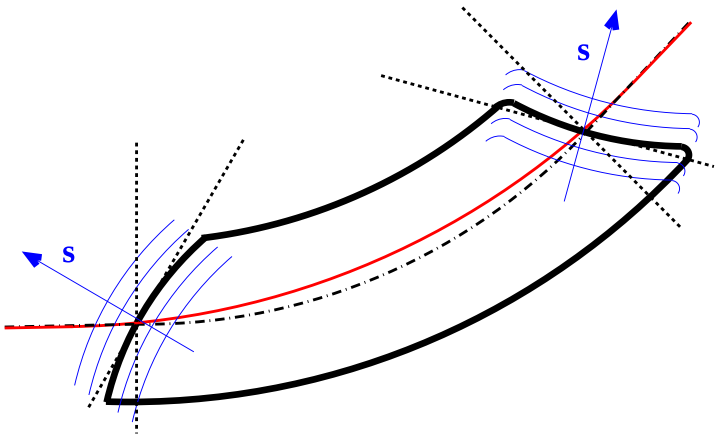


# FRINGE FIELDS

black - extended FF to take into account the extra effective field length (standard)

red - non-extended FF to be used with R=8.4m * 1.007




black axis line - R=8.4m

red - R=8.4m * 1.007

## SUMMARY

- The HRS spin matrix is in good approximation a function only of the total bending angles $\Delta\theta$, $\Delta\phi$, and the momentum $p$.

- The dipole fringe field is the main reason for the deviations from this approximation. (therefore, the approximation will not work for spectrometers with strong fringe fields)

- The total bending angles of the central trajectory were measured with a precisions of $\Delta\phi \pm 0.3 mrad$ and $\Delta\theta \pm 4.4 mrad$, that reduced significantly the systematic errors in the GEP experiments.