Determining the QE Neutron Asymmetry

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1 Removing False asymmetries

The asymmetry measured during the experiment (A) is a combination of the asymmetry of the observed particle yields (A_{obs}) , as well as instrumental effects. The raw asymmetry is assembled directly through the yields of events (N) with helicity + or -. The asymmetry A_{obs} can also contain other reactions that also need to be removed, eg: proton leakage into the neutron cuts. So even after getting A_{obs} , additional corrections are required.

$$A \equiv \frac{N_{+} - N_{-}}{N_{+} + N_{-}}$$
$$N_{\pm} = \mathcal{L}_{\pm}\sigma_{\pm}\eta_{\pm}d\Omega$$
$$\mathcal{L}_{\pm} = Q_{\pm}T$$
$$\eta_{\pm} = D_{\pm}\epsilon$$
$$\sigma_{\pm} = \Sigma \pm P_{B}P_{T}\Delta$$
$$\Delta' = P_{B}P_{T}\Delta$$
$$A_{obs} \equiv \frac{\Delta'}{\Sigma}$$

where \mathcal{L}_{\pm} is the helicity-dependent integrated luminosity, Q_{\pm} is the accumulated charge per helicity state, T is the target density in nucleons per cm², η_{\pm} is the helicity-dependent livetime constructed of the DAQ (D_{\pm}) and electronic (ϵ_{\pm}) livetime fractions, and σ_{\pm} is the helicity dependent cross-section that contains the actual asymmetry we wish to extract. The beam and target polarizations are given as P_B and P_T , respectively.

$$A = \frac{\mathcal{L}_{+}\sigma_{+}\eta_{+} - \mathcal{L}_{-}\sigma_{-}\eta_{-}}{\mathcal{L}_{+}\sigma_{+}\eta_{+} + \mathcal{L}_{-}\sigma_{-}\eta_{-}}$$

$$= \frac{Q_{+}\eta_{+}(\Sigma + \Delta') - Q_{-}\eta_{-}(\Sigma - \Delta')}{Q_{+}\eta_{+}(\Sigma + \Delta') + Q_{-}\eta_{-}(\Sigma - \Delta')}$$

$$= \frac{\Sigma(Q_{+}\eta_{+} - Q_{-}\eta_{-}) + \Delta'(Q_{+}\eta_{+} + Q_{-}\eta_{-})}{\Sigma(Q_{+}\eta_{+} + Q_{-}\eta_{-}) + \Delta'(Q_{+}\eta_{+} - Q_{-}\eta_{-})}$$

$$- \frac{Q_{-}\eta_{-}}{Q_{+}\eta_{+}}$$

for clarity, assign $f = 1 - \frac{Q_-\eta_-}{Q_+\eta_+}$

$$A = \frac{\Sigma f + \Delta'(2 - f)}{\Sigma(2 - f) + \Delta' f}$$
$$= \frac{f + \frac{\Delta'}{\Sigma}(2 - f)}{(2 - f) + \frac{\Delta'}{\Sigma} f}$$
$$\approx \frac{\Delta'}{\Sigma} + f, \text{ since } f \approx .01 \text{ and } \frac{\Delta'}{\Sigma} < .1.$$

So the false asymmetry contributions, encapulased in f, contribute directly to the observed asymmetry and must be removed. Rewriting, to first order then in the corrections

$$f = 1 - \frac{Q_{-}\eta_{-}}{Q_{+}\eta_{+}}$$

$$= 1 - (1 + (\frac{Q_{-}}{Q_{+}} - 1))(1 + (\frac{\eta_{-}}{\eta_{+}} - 1))$$

$$\approx (1 - (\frac{Q_{-}}{Q_{+}}) + (1 - \frac{\eta_{-}}{\eta_{+}})$$

$$A_{obs} = A - f$$

$$= A + (\frac{Q_{-}}{Q_{+}} - 1) + (\frac{\eta_{-}}{\eta_{+}} - 1)$$

Once A_{obs} is determined, then the physics related backgrounds have to be removed.

Table 1: Contributions to the observed asymmetry and their approximate uncertainties (when known) to kin4, the 1.7GeV^2 dataset.

term	value	uncertainty
$1 - Q_{-}/Q_{+}$	< 0.0002	
$1 - D_{-}/D_{+}$	0.001	
$1 - \epsilon/\epsilon_+$	0.01?	
A	0.04	0.005

Physics (background) asymmetries $\mathbf{2}$

As will be described elsewhere, the quasi-elastic neutrons and protons are defined by a series of cuts to select quasi-elastic events by matching hits in the ND to the virtual photon direction and momentum as determined by BigBite. To then differentiate the protons and neutrons, cuts are placed upon the veto detectors. This will be detailed elsewhere.

Each of the backgrounds to our desired neutron sample can have an asymmetry as well. Let us write our neutron sample as having contributions from three parts with the notation of the superscript is how the particle is identified by the ND, and the subscript is the identify upon leaving the target: $N_n^{(n)}$ is the number of 'true' neutrons, $N_p^{(n)}$ is the number of protons that are mis-identified as neutrons, and $N_{ACC}^{(n)}$ is number of accidental coincidences (of both neutrons and mis-identified protons) in the neutron sample. A similar set of variables can be defined for protoncandidates: $N_n^{(p)}$, $N_p^{(p)}$, $N_{ACC}^{(p)}$. If each has its own asymmetry of A_n , A_p , $A_{ACC}^{(n)}$ and $A_{ACC}^{(p)}$, then:

$$A_{obs}^{(n)} = \frac{N_n^{(n)}}{T_n} A_n + \frac{N_p^{(n)}}{T_n} A_p + \frac{N_{ACC}^{(n)}}{T_n} A_{ACC}^{(n)}$$
(1)

$$A_{obs}^{(p)} = \frac{N_n^{(p)}}{T_p} A_n + \frac{N_p^{(p)}}{T_p} A_p + \frac{N_{ACC}^{(p)}}{T_p} A_{ACC}^{(p)}$$
(2)

where $T_n = N_n^{(n)} + N_p^{(n)} + N_{ACC}^{(n)}$ and $T_p = N_p^{(p)} + N_n^{(p)} + N_{ACC}^{(p)}$. To separate the random accidentals from particle identification, lets assemble an

intermediate set of asymmetries for only coincident events.

$$A_{coinc}^{(n)} = \frac{N_n^{(n)}}{N_n^{(n)} + N_p^{(n)}} A_n + \frac{N_p^{(n)}}{N_n^{(n)} + N_p^{(n)}} A_p$$
(4)

$$= \frac{T_n}{N_n^{(n)} + N_p^{(n)}} \left(A_{obs}^{(n)} - \frac{N_{ACC}^{(n)}}{T_n} A_{ACC}^{(n)} \right)$$
(5)

$$A_{coinc}^{(p)} = \frac{N_n^{(p)}}{N_n^{(p)} + N_p^{(p)}} A_n + \frac{N_p^{(p)}}{N_n^{(p)} + N_p^{(p)}} A_p$$
(6)

$$= \frac{T_p}{N_n^{(p)} + N_p^{(p)}} \left(A_{obs}^{(p)} - \frac{N_{ACC}^{(p)}}{T_p} A_{ACC}^{(p)} \right)$$
(7)

Here we define the **dilution parameters** as the fraction of observed coincidenceevent neutrons or protons that are properly identified:

$$D_n = \frac{N_n^{(n)}}{N_n^{(n)} + N_p^{(n)}}$$
(8)

$$D_p = \frac{N_p^{(p)}}{N_n^{(p)} + N_p^{(p)}}$$
(9)

The dilution factors can be determined by comparing targets with different protonto-neutron ratios.

Rewriting eqns 4 and 6 we see a solution to extract the desired A_n and A_p .

$$A_{coinc}^{(n)} = D_n A_n + (1 - D_n) A_p \tag{10}$$

$$A_{coinc}^{(p)} = (1 - D_p)A_n + D_pA_p$$
, so (11)

$$A_n = \frac{D_p A_{coinc}^{(n)} - (1 - D_n) A_{coinc}^{(p)}}{D_p + D_n - 1}$$
(12)

$$A_p = \frac{D_n A_{coinc}^{(p)} - (1 - D_p) A_{coinc}^{(n)}}{D_p + D_n - 1}$$
(13)

Alternatively, assuming we know A_p , just go directly to A_n :

$$A_{n} = \left(A_{coinc}^{(n)} - (1 - D_{n})A_{p}\right) / D_{n}$$
(14)

(15)

3 Proton and Neutron mixing and ID

In the neutron detector, both "proton" and "neutron" candidates are present within the quasi-elastic cuts. However, due to materials before the neutron detector and its intrinsic performance, charged and neutral particles can be converted or leak into the particle ID cuts of the other category. For example, a proton leaving the target could under-go charge-exchange on a Pb nucleus in the shielding in front of the veto detectors resulting in a neutron and low-energy π^+ which might or might-not be detected in the veto planes. Similarly, a neutron could convert in the presence of a nucleus to a $p \pi^-$ pair and appear as a charged particle in the neutron detector. This effect can be studied with our data and Monte-Carlo techniques; here I will focus on extracting this from the data.

The observed number of coincident neutron $(N^{(n)})$ and proton $(N^{(p)})$ candidates is

$$N^{(n)} = N_n^{(n)} + N_n^{(n)} \tag{16}$$

$$N^{(p)} = N_n^{(p)} + N_p^{(p)}.$$
(17)

All the terms share common scale factors for the luminosity and effective angular acceptance, however they do have differing detection efficiencies and initial cross-sections. Using $\eta_a^{(b)}$ to denote the probability to detect an a as a b, and σ_a as the cross-section for quasi-elastically off an a,

$$N_n^{(n)} \propto (A-Z)\sigma_n \eta_n^{(n)} \tag{18}$$

$$N_p^{(p)} \propto Z\sigma_p \eta_p^{(p)} \tag{19}$$

where A(Z) is the atomic mass (atomic number) of the target nucleus. Taking the ratio of the number of observed neutrons to protons then yields:

$$R_{(A-Z)/Z} = \frac{N^{(n)}}{N^{(p)}} = \frac{(A-Z)\sigma_n\eta_n^{(n)} + Z\sigma_p\eta_p^{(n)}}{(A-Z)\sigma_n\eta_n^{(p)} + Z\sigma_p\eta_p^{(p)}}$$
(20)

$$R_{(A-Z)/Z} = \frac{\frac{(A-Z)}{Z} \frac{\sigma_n}{\sigma_p} \left(\eta_n^{(n)} / \eta_p^{(p)} \right) + \left(\eta_p^{(n)} / \eta_p^{(p)} \right)}{\frac{(A-Z)}{Z} \frac{\sigma_n}{\sigma_p} \left(\eta_n^{(p)} / \eta_p^{(p)} \right) + 1}$$
(21)

This ratio can be collected for targets with different ratios of nuclear neutrons to protons. The ratio σ_n/σ_p is primarily determined by the known magnetic form factors G_M^p and G_M^n at our kinematics. Assuming that nuclear effects affect the σ_n and σ_p similarly as the target is changed, which can be helped by using targets where the protons and neutrons fill out to the same shell, a collection of $R_{A-Z/Z}$ ratios can be used to determine the detection efficiency ratios.

3.1 Dilution factor

Note that the goal is to determine the dilution factors for 3 He, which are the fraction of observed neutrons (protons) that originated from the target as a neutron (proton). In this notation, they are given by:

$$D_{n} = \frac{\frac{\sigma_{n}}{\sigma_{p}} \left(\eta_{n}^{(n)}/\eta_{p}^{(p)}\right)}{\frac{\sigma_{n}}{\sigma_{p}} \left(\eta_{n}^{(n)}/\eta_{p}^{(p)}\right) + 2 \left(\eta_{p}^{(n)}/\eta_{p}^{(p)}\right)}$$
(22)

$$D_p = \frac{2}{\frac{\sigma_n}{\sigma_p} \left(\eta_n^{(p)}/\eta_p^{(p)}\right) + 2}.$$
(23)

3.2 Different nuclei

During E02-013, we took data on four targets: ³He, N₂, H₂, and a mixed BeO/C foil target. For our purposes, the N₂ and BeO/C targets have the same neutron-to-proton ratios.

$$R_{0/1} = R_H = -\eta_p^{(n)} / \eta_p^{(p)}$$
(24)

$$R_{1/1} = R_N = \frac{\frac{\sigma_n}{\sigma_p} \eta_n^{(r)} / \eta_p^{(p)} + \eta_p^{(n)} / \eta_p^{(p)}}{\frac{\sigma_n}{\sigma_p} \eta_n^{(p)} / \eta_p^{(p)} + 1}$$
(25)

$$R_{1/2} = R_{3He} = \frac{\frac{\sigma_n}{\sigma_p} \eta_n^{(n)} / \eta_p^{(p)} + 2\eta_p^{(n)} / \eta_p^{(p)}}{\frac{\sigma_n}{\sigma_p} \eta_n^{(p)} / \eta_p^{(p)} + 2}$$
(26)

This leads to

$$\frac{\eta_p^{(n)}}{\eta_p^{(p)}} = R_H \tag{27}$$

$$\frac{\eta_n^{(n)}}{\eta_p^{(p)}} = \frac{\sigma_p}{\sigma_n} \frac{R_{3He}(R_N + R_H) - 2R_N R_H}{R_N - R_{3He}}$$
(28)

$$\frac{\eta_n^{(p)}}{\eta_p^{(p)}} = \frac{\sigma_p}{\sigma_n} \left(\frac{R_{3He} - R_H}{R_N - R_{3He}} - 1 \right)$$
(29)

and subsequently:

$$D_n = \frac{R_{3He}(R_N + R_H) - 2R_N R_H}{R_{3He}(R_N - R_H)}$$
(30)

$$D_p = 2\frac{R_N - R_{3He}}{R_N - R_H}$$
(31)

3.3 **Approximate Dilution factor**

Seamus came up with a different approach to approximate the dilution factor. If one assumes that the neutron-to-proton rate is negligible, then D_n can be approximated by D_n where

$$\widetilde{D_n} = \frac{R_{3He} - R_H}{R_{3He}}, \text{ since}$$
(32)

$$R_{3He} \approx \widetilde{R_{3He}} = \frac{\frac{\sigma_n \eta_n^{(n)}}{\sigma_p \eta_p^{(p)}} + 2\frac{\eta_p^{(r)}}{\eta_p^{(p)}}}{2}.$$
 (33)

(34)

Worked out,

$$\widetilde{D_n} = \frac{\frac{\sigma_n \eta_n^{(n)}}{\sigma_p \eta_p^{(p)}} \left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}} \frac{\eta_p^{(n)}}{\eta_p^{(p)}}\right)}{2\frac{\eta_p^{(n)}}{\eta_p^{(p)}} + \frac{\sigma_n \eta_n^{(n)}}{\sigma_p \eta_p^{(p)}}}$$
(35)

$$= D_n \left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}} \frac{\eta_p^{(n)}}{\eta_p^{(p)}} \right)$$
(36)

In Sergey's studies for kin4, with NO accidentals on the veto detector, he found:

$$\frac{\eta_n^{(p)}}{\eta_n^{(n)}}_{\text{Sergey}} \approx \frac{1820}{3971} = 0.458$$

$$\frac{\eta_p^{(n)}}{\eta_p^{(p)}}_{\text{Sergey}} \approx \frac{191}{6233} = 0.031$$
(38)

$$\frac{\eta_p^{(n)}}{\eta_p^{(p)}}_{\text{Sergey}} \approx \frac{191}{6233} = 0.031$$
 (38)

$$\left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}} \frac{\eta_p^{(n)}}{\eta_p^{(p)}}\right) \approx 98.6\%$$
(39)

which suggests Seamus's approximation should be very good.

3.4 Rate Dependence

The identification of the protons and neutrons is dependent upon the rate in the veto detectors. Since the neutron ID is performed by requiring a LACK of hits in the veto planes within a reasonably large region and time, changes in the accidental rate can change the observed neutron yield. Thus it is necessary to evaluate the neutron-to-proton ratios for the different targets at the same veto rate.

4 Application to Kin4

*** Summary of Asym metry Total instrumental (BCA, LT) asymmetry -0.0006 Raw Neutron asymmetry for Pperp < 150 MeV/c: 0.0510 +/- 0.0026 with a total yield of 1.530250e+05 0.0144 +/- 0.0074 with a yield of 1.803900e+04 (11.7883 %) 0.0559 +/- 0.0031 Raw random Neutral Background asymmetry : Coincident Neutral-candidate Asymmetry : 0.0025 +/ - 0.0009 with a rel. contr. to neutrons (28.7129 %) 0.0032 +/- 0.0137 with a yield of 5.303000e+03 (0.4316 % rel. to protons) Raw Proton asymmetry for Pperp < 150 MeV/c: Raw random Proton Background asymmetry : 0.0025 +/- 0.0009 Coincident Proton-candidate Asymmetry : Extracting Neutron and Proton asymmetries coherently: : 0.0902 +/- 0.0181 (20.0851 %) Final observed neutron asymmetry is : -0.0153 +/- 0.0233 (-152.4666 %) Final observed proton asymmetry is sing 'known' value of proton asymmetry (FW) GEp/GMP (Q2-1.68606) = 0.292844 proton APphys = 0.2037 ... has obs asymm of -0.0022 +/- -0.0003 2nd final observed neutron asymmetry is : 0.0839 +/- 0.0108 (12.0051 %) Using 'known' value of proton asymmetry (FW) Effective neutron yield is 91048 (67.45 % of coinc. neutron candidates) Effective proton yield is 1016646 (83.10 % of coinc. proton candidates) Using a beam polarization of : 0.8500 +/- 0.0200 and a target polarization of : 0.4820 +/- 0.0200 and a neutron/target pol. of : 0.8600 +/- 0.0200 and a N2 dilution factor of : 0.9500 +/- 0.0200 Based upon First Final neutron asymmetry: *** Get Aphys = -0.2694 +/- 0.0563 Get GEn/GMn to be -0.2296 +/- 0.07223 Using a value of GMn(1.69 GeV²) = -0.1677 we find GEn to be 0.03851 +/- 0.01211 (+/- 31.45 %) OR, with 2nd Final neutron asymmetry (USING KNOWN PROTON ASYMMETRY) *** Get Aphys = -0.2506 +/- 0.0354 Apar contributes ~ -0.060 -0.2061 +/- 0.04347 Get GEn/GMn to be Using a value of $GMn(1.69 \text{ GeV}^2) = -0.1677$ we find GEn to be 0.03457 +/- 0.007289 (+/- 21.09 %)



Figure 1: Present understanding of E02-013