Determining the QE Neutron Asymmetry

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1 Removing False asymmetries

The asymmetry measured during the experiment (A) is a combination of the asymmetry of the observed particle yields (A_{obs}), as well as instrumental effects. The raw asymmetry is assembled directly through the yields of events (N) with helicity $+$ or $-$. The asymmetry A_{obs} *can also* contain other reactions that also need to be removed, eg: proton leakage into the neutron cuts. So even after getting A_{obs}, additional corrections are required.

$$
\begin{aligned}
A &\equiv \frac{N_+ - N_-}{N_+ + N_-} \\
N_\pm &= \mathcal{L}_\pm \sigma_\pm \eta_\pm d\Omega \\
\mathcal{L}_\pm &= Q_\pm T \\
\eta_\pm &= D_\pm \epsilon \\
\sigma_\pm &= \Sigma \pm P_B P_T \Delta \\
\Delta' &= P_B P_T \Delta \\
A_{obs} &\equiv \frac{\Delta'}{\Sigma}
\end{aligned}
$$

where $\mathcal{L}_\pm$ is the helicity-dependent integrated luminosity, $Q_\pm$ is the accumulated charge per helicity state, T is the target density in nucleons per cm^2, $\eta_\pm$ is the helicity-dependent fractional livetime constructed of the DAQ ($D_\pm$) and electronic ($\epsilon_\pm$) livetime fractions, $\sigma_\pm$ is the physical cross-section assembled of the helicity independent (Σ) and helicity dependent (Δ) terms, and Δ/Σ is the actual asymmetry we wish to extract. The beam and target polarizations are given as P_B and P_T, respectively.

$$\begin{aligned}
A &= \frac{\mathcal{L}_+\sigma_+\eta_+ - \mathcal{L}_-\sigma_-\eta_-}{\mathcal{L}_+\sigma_+\eta_+ + \mathcal{L}_-\sigma_-\eta_-} \\
&= \frac{Q_+\eta_+(\Sigma+\Delta') - Q_-\eta_-(\Sigma-\Delta')}{Q_+\eta_+(\Sigma+\Delta') + Q_-\eta_-(\Sigma-\Delta')} \\
&= \frac{\Sigma(Q_+\eta_+ - Q_-\eta_-) + \Delta'(Q_+\eta_+ + Q_-\eta_-)}{\Sigma(Q_+\eta_+ + Q_-\eta_-) + \Delta'(Q_+\eta_+ - Q_-\eta_-)}
\end{aligned}$$

$$\text{for clarity, assign } f = 1 - \frac{Q_-\eta_-}{Q_+\eta_+}$$

$$\begin{aligned}
A &= \frac{\Sigma f + \Delta'(2-f)}{\Sigma(2-f) + \Delta' f} \\
&= \frac{f + \frac{\Delta'}{\Sigma}(2-f)}{(2-f) + \frac{\Delta'}{\Sigma} f} \\
&\approx \frac{\Delta'}{\Sigma} + f, \text{ since } f \approx .01 \text{ and } \frac{\Delta'}{\Sigma} < .1.
\end{aligned}$$

So the false asymmetry contributions, encapulased in f, contribute directly to the observed asymmetry and must be removed. Rewriting, to first order then in the corrections

$$\begin{aligned}
f &= 1 - \frac{Q_-\eta_-}{Q_+\eta_+} \\
&= 1 - (1 + (\frac{Q_-}{Q_+} - 1))(1 + (\frac{\eta_-}{\eta_+} - 1)) \\
&\approx (1 - (\frac{Q_-}{Q_+}) + (1 - \frac{\eta_-}{\eta_+}) \\
A_{obs} &= A - f \\
&= A + (\frac{Q_-}{Q_+} - 1) + (\frac{\eta_-}{\eta_+} - 1)
\end{aligned}$$

Once A_{obs} is determined, then the physics related backgrounds have to be removed.

Table 1: Contributions to the observed asymmetry and their approximate uncertainties (when known) to kin4, the 1.7GeV^2 dataset.

term	value	uncertainty
$1 - Q_-/Q_+$	< 0.0002	
$1 - D_-/D_+$	0.001	
$1 - \epsilon_-/\epsilon_+$	0.01?	
A	0.04	0.005

2 Physics (background) asymmetries

As will be described elsewhere, the quasi-elastic neutrons and protons are defined by a series of cuts to select quasi-elastic events by matching hits in the ND to the virtual photon direction and momentum as determined by BigBite. To then differentiate the protons and neutrons, cuts are placed upon the veto detectors. This will be detailed elsewhere.

Each of the backgrounds to our desired neutron sample can have an asymmetry as well. Let us write our neutron sample as having contributions from three parts with the notation of the superscript is how the particle is identified by the ND, and the subscript is the identify upon leaving the target: $N_n^{(n)}$ is the number of 'true' neutrons, $N_p^{(n)}$ is the number of protons that are mis-identified as neutrons, and $N_{ACC}^{(n)}$ is number of accidental coincidences (of both neutrons and mis-identified protons) in the neutron sample. A similar set of variables can be defined for proton-candidates: $N_n^{(p)}$, $N_p^{(p)}$, $N_{ACC}^{(p)}$. If each has its own asymmetry of A_n, A_p, $A_{ACC}^{(n)}$ and $A_{ACC}^{(p)}$, then:

$$A_{obs}^{(n)} = \frac{N_n^{(n)}}{T_n} A_n + \frac{N_p^{(n)}}{T_n} A_p + \frac{N_{ACC}^{(n)}}{T_n} A_{ACC}^{(n)} \quad (1)$$

$$A_{obs}^{(p)} = \frac{N_n^{(p)}}{T_p} A_n + \frac{N_p^{(p)}}{T_p} A_p + \frac{N_{ACC}^{(p)}}{T_p} A_{ACC}^{(p)} \quad (2)$$

$$\quad (3)$$

where $T_n = N_n^{(n)} + N_p^{(n)} + N_{ACC}^{(n)}$ and $T_p = N_p^{(p)} + N_n^{(p)} + N_{ACC}^{(p)}$.

To separate the random accidentals from particle identification, lets assemble an intermediate set of asymmetries for only coincident events.

$$\begin{aligned} A^{(n)}_{coinc} &= \frac{N^{(n)}_n}{N^{(n)}_n + N^{(n)}_p} A_n + \frac{N^{(n)}_p}{N^{(n)}_n + N^{(n)}_p} A_p & (4) \\ &= \frac{T_n}{N^{(n)}_n + N^{(n)}_p} \left(A^{(n)}_{obs} - \frac{N^{(n)}_{ACC}}{T_n} A^{(n)}_{ACC} \right) & (5) \\ A^{(p)}_{coinc} &= \frac{N^{(p)}_n}{N^{(p)}_n + N^{(p)}_p} A_n + \frac{N^{(p)}_p}{N^{(p)}_n + N^{(p)}_p} A_p & (6) \\ &= \frac{T_p}{N^{(p)}_n + N^{(p)}_p} \left(A^{(p)}_{obs} - \frac{N^{(p)}_{ACC}}{T_p} A^{(p)}_{ACC} \right) & (7) \end{aligned}$$

Here we define the **dilution parameters** as the fraction of observed coincidence-event neutrons or protons that are properly identified:

$$\begin{aligned} D_n &= \frac{N^{(n)}_n}{N^{(n)}_n + N^{(n)}_p} & (8) \\ D_p &= \frac{N^{(p)}_p}{N^{(p)}_n + N^{(p)}_p} & (9) \end{aligned}$$

The dilution factors can be determined by comparing targets with different proton-to-neutron ratios.

Rewriting eqns 4 and 6 we see a solution to extract the desired A_n and A_p.

$$\begin{aligned} A^{(n)}_{coinc} &= D_n A_n + (1 - D_n) A_p & (10) \\ A^{(p)}_{coinc} &= (1 - D_p) A_n + D_p A_p, \quad \text{so} & (11) \end{aligned}$$

$$\begin{aligned} A_n &= \frac{D_p A^{(n)}_{coinc} - (1 - D_n) A^{(p)}_{coinc}}{D_p + D_n - 1} & (12) \\ A_p &= \frac{D_n A^{(p)}_{coinc} - (1 - D_p) A^{(n)}_{coinc}}{D_p + D_n - 1} & (13) \end{aligned}$$

Alternatively, assuming we know A_p, just go directly to A_n:

$$\begin{aligned} A_n &= \left(A^{(n)}_{coinc} - (1 - D_n) A_p \right) / D_n & (14) \\ & & (15) \end{aligned}$$

3 Proton and Neutron mixing and ID

In the neutron detector, both “proton” and “neutron” candidates are present within the quasi-elastic cuts. However, due to materials before the neutron detector and its intrinsic performance, charged and neutral particles can be converted or leak into the particle ID cuts of the other category. For example, a proton leaving the target could under-go charge-exchange on a Pb nucleus in the shielding in front of the veto detectors resulting in a neutron and low-energy π^+ which might or might-not be detected in the veto planes. Similarly, a neutron could convert in the presence of a nucleus to a $p\,\pi^-$ pair and appear as a charged particle in the neutron detector. This effect can be studied with our data and Monte-Carlo techniques; here I will focus on extracting this from the data.

The observed number of coincident neutron ($N^{(n)}$) and proton ($N^{(p)}$) candidates is

$$N^{(n)} = N_n^{(n)} + N_p^{(n)} \quad (16)$$

$$N^{(p)} = N_n^{(p)} + N_p^{(p)}. \quad (17)$$

All the terms share common scale factors for the luminosity and effective angular acceptance, however they do have differing detection efficiencies and initial cross-sections. Using $\eta_a^{(b)}$ to denote the probability to detect an a as a b, and σ_a as the cross-section for quasi-elastically off an a,

$$N_n^{(n)} \propto (A-Z)\sigma_n \eta_n^{(n)} \quad (18)$$

$$N_p^{(p)} \propto Z\sigma_p \eta_p^{(p)} \quad (19)$$

where A (Z) is the atomic mass (atomic number) of the target nucleus. Taking the ratio of the number of observed neutrons to protons then yields:

$$R_{(A-Z)/Z} = \frac{N^{(n)}}{N^{(p)}} = \frac{(A-Z)\sigma_n\eta_n^{(n)} + Z\sigma_p\eta_p^{(n)}}{(A-Z)\sigma_n\eta_n^{(p)} + Z\sigma_p\eta_p^{(p)}} \quad (20)$$

$$R_{(A-Z)/Z} = \frac{\frac{(A-Z)}{Z}\frac{\sigma_n}{\sigma_p}\left(\eta_n^{(n)}/\eta_p^{(p)}\right) + \left(\eta_p^{(n)}/\eta_p^{(p)}\right)}{\frac{(A-Z)}{Z}\frac{\sigma_n}{\sigma_p}\left(\eta_n^{(p)}/\eta_p^{(p)}\right) + 1} \quad (21)$$

This ratio can be collected for targets with different ratios of nuclear neutrons to protons. The ratio σ_n/σ_p is primarily determined by the known magnetic form factors G_M^p and G_M^n at our kinematics. Assuming that nuclear effects affect the σ_n and σ_p similarly as the target is changed, which can be helped by using targets where the protons and neutrons fill out to the same shell, a collection of $R_{A-Z/Z}$ ratios can be used to determine the detection effiency ratios.

3.1 Dilution factor

Note that the goal is to determine the dilution factors for ^{3}He, which are the fraction of observed neutrons (protons) that originated from the target as a neutron (proton). In this notation, they are given by:

$$D_n = \frac{\frac{\sigma_n}{\sigma_p}\left(\eta_n^{(n)}/\eta_p^{(p)}\right)}{\frac{\sigma_n}{\sigma_p}\left(\eta_n^{(n)}/\eta_p^{(p)}\right) + 2\left(\eta_p^{(n)}/\eta_p^{(p)}\right)} \tag{22}$$

$$D_p = \frac{2}{\frac{\sigma_n}{\sigma_p}\left(\eta_n^{(p)}/\eta_p^{(p)}\right) + 2}. \tag{23}$$

3.2 Different nuclei

During E02-013, we took data on four targets: ^{3}He, N_2, H_2, and a mixed BeO/C foil target. For our purposes, the N_2 and BeO/C targets have the same neutron-to-proton ratios.

$$R_{0/1} = R_H = \eta_p^{(n)}/\eta_p^{(p)} \tag{24}$$

$$R_{1/1} = R_N = \frac{\frac{\sigma_n}{\sigma_p}\eta_n^{(n)}/\eta_p^{(p)} + \eta_p^{(n)}/\eta_p^{(p)}}{\frac{\sigma_n}{\sigma_p}\eta_n^{(p)}/\eta_p^{(p)} + 1} \tag{25}$$

$$R_{1/2} = R_{3He} = \frac{\frac{\sigma_n}{\sigma_p}\eta_n^{(n)}/\eta_p^{(p)} + 2\eta_p^{(n)}/\eta_p^{(p)}}{\frac{\sigma_n}{\sigma_p}\eta_n^{(p)}/\eta_p^{(p)} + 2} \tag{26}$$

This leads to

$$\frac{\eta_p^{(n)}}{\eta_p^{(p)}} = R_H \tag{27}$$

$$\frac{\eta_n^{(n)}}{\eta_p^{(p)}} = \frac{\sigma_p}{\sigma_n}\frac{R_{3He}(R_N + R_H) - 2R_N R_H}{R_N - R_{3He}} \tag{28}$$

$$\frac{\eta_n^{(p)}}{\eta_p^{(p)}} = \frac{\sigma_p}{\sigma_n}\left(\frac{R_{3He} - R_H}{R_N - R_{3He}} - 1\right) \tag{29}$$

and subsequently:

$$D_n = \frac{R_{3He}(R_N + R_H) - 2R_N R_H}{R_{3He}(R_N - R_H)} \tag{30}$$

$$D_p = 2\frac{R_N - R_{3He}}{R_N - R_H} \tag{31}$$

3.3 Approximate Dilution factor

Seamus came up with a different approach to approximate the dilution factor. If one assumes that the neutron-to-proton rate is negligible, then D_n can be approximated by $\widetilde{D_n}$ where

$$\widetilde{D_n} = \frac{R_{3He} - R_H}{R_{3He}}, \text{ since} \tag{32}$$

$$R_{3He} \approx \widetilde{R_{3He}} = \frac{\frac{\sigma_n}{\sigma_p}\frac{\eta_n^{(n)}}{\eta_p^{(p)}} + 2\frac{\eta_p^{(n)}}{\eta_p^{(p)}}}{2}. \tag{33}$$

$$\tag{34}$$

Worked out,

$$\widetilde{D_n} = \frac{\frac{\sigma_n}{\sigma_p}\frac{\eta_n^{(n)}}{\eta_p^{(p)}}\left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}}\frac{\eta_p^{(n)}}{\eta_p^{(p)}}\right)}{2\frac{\eta_p^{(n)}}{\eta_p^{(p)}} + \frac{\sigma_n}{\sigma_p}\frac{\eta_n^{(n)}}{\eta_p^{(p)}}} \tag{35}$$

$$= D_n\left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}}\frac{\eta_p^{(n)}}{\eta_p^{(p)}}\right) \tag{36}$$

In Sergey's studies for kin4, with NO accidentals on the veto detector, he found:

$$\frac{\eta_n^{(p)}}{\eta_n^{(n)}}_{\text{Sergey}} \approx \frac{1820}{3971} = 0.458 \tag{37}$$

$$\frac{\eta_p^{(n)}}{\eta_p^{(p)}}_{\text{Sergey}} \approx \frac{191}{6233} = 0.031 \tag{38}$$

$$\left(1 - \frac{\eta_n^{(p)}}{\eta_n^{(n)}}\frac{\eta_p^{(n)}}{\eta_p^{(p)}}\right) \approx 98.6\% \tag{39}$$

which suggests Seamus's approximation should be very good.

3.4 Rate Dependence

The identification of the protons and neutrons is dependent upon the rate in the veto detectors. Since the neutron ID is performed by requiring a LACK of hits in the veto planes within a reasonably large region and time, changes in the accidental rate can change the observed neutron yield. Thus it is necessary to evaluate the neutron-to-proton ratios for the different targets at the same veto rate.

Protons in this analysis are defined to have AT LEAST ONE veto plane firing in coincidence with the neutron detector hit. Neutrons are required to have NEITHER veto plane fire in a window about the ND hit's time nor in a period matching the electronic dead-time of the veto planes before the ND hit; this prevents protons from leaking into the neutron sample due to the veto plane being "dead".

An approach to handling the rate dependence is to evolve the proton and neutron rates analytically. The number of "observed" protons, p_{obs}, depends upon the electronic deadtime $\epsilon_{Tv} \approx 100$ns of the channels in the veto plane v, the coincidence window $\triangle\tau_P \approx 20$ns for hits in the veto and ND to define a proton, and the anti-coincidence window $\triangle\tau_N \approx 120$ns in which matched hits between the veto and ND reject neutron candidates. The probability to have one or more hits in a time δt with a random rate of r is given by the Poisson distribution to be:

$$\Pr(\delta t, r) = 1 - \exp(-r\delta t) \approx r\delta t \quad (40)$$

Typically, $r\delta t \sim (300\text{kHz})(100\text{ns}) \sim 0.03$ so the function could be well approximated by $\Pr(\delta t, r) \approx r\delta t$.

The "observed" number of protons (neutrons), denoted by p_{obs} (n_{obs}) are:

$$p_{obs}(r) = p_I\left[1 - \Pr(\epsilon_{T1}, k_1 r)\Pr(\epsilon_{T2}, k_2 r)\right] + n_I \Pr(\triangle\tau_P, [k_1 + k_2]r) \quad (41)$$

$$n_{obs}(r) = n_I\left[1 - \Pr(\triangle\tau_N, [k_1 + k_2]r)\right] \quad (42)$$

where k_1 and k_2 scale the overall rate in the vetos to those of the particular planes of interest, and p_I (n_I) is the number of incident protons (neutrons) that trigger the ND.

The first term in p_{obs} which is proportional to p_I slowly drops as the rate increases, due to random hits obscuring the protons' true veto hits; the second term due to neutron mis-identification increases nearly linearly with the random rate and dominates the first term, so the number . In constrast, the number of neutron candidates seen n_{obs} simply drops as the random rate increases.

The ratio of neutrons to protons, useful for determining the dilution factor due to nucleon mis-identification, then has the form:

$$
\begin{aligned}
R(r) &= \frac{n_{obs}}{p_{obs}} && (43)\\
&= \frac{n_I\left[1-\Pr(\triangle\tau_N,[k_1+k_2]r)\right]}{p_I\left(1-\Pr(\epsilon_{T1},k_1 r)\Pr(\epsilon_{T2},k_2 r)\right)+n_I\Pr(\triangle\tau_P,[k_1+k_2]r)} && (44)\\
&= \frac{n_I}{p_I}\,\frac{1-\Pr(\triangle\tau_N,[k_1+k_2]r)}{1-\Pr(\epsilon_{T1},k_1 r)\Pr(\epsilon_{T2},k_2 r)+\frac{n_I}{p_I}\Pr(\triangle\tau_P,[k_1+k_2]r)}. && (45)
\end{aligned}
$$

Functions can be defined to simplify the notation at a random rate r: $N(r)$ is the probability to have a hit in either of the planes within the neutron-candidates' veto time-window, $C(r)$ is the probability to have a hit within the proton-candidates' coincidence time-window, and $B(r)$ is the chance to have hits in both veto planes within the electronic dead-time of the planes such that the "true" veto hits are obscured.

$$
\begin{aligned}
N(r) &\equiv \Pr(\triangle\tau_N,[k_1+k_2]r) && (46)\\
C(r) &\equiv \Pr(\triangle\tau_P,[k_1+k_2]r) && (47)\\
B(r) &\equiv \Pr(\epsilon_{T1},k_1 r)\Pr(\epsilon_{T2},k_2 r) && (48)\\
R_0 &\equiv \frac{n_I}{p_I} && (49)\\
R(r) &= \frac{R_0\left[1-N(r)\right]}{R_0 C(r)+1-B(r)} && (50)
\end{aligned}
$$

This can be manipulated to remove $\frac{n_I}{p_I}$ and put it in terms of the ratio R_f at some arbitrary reference rate f, generally chosen to be the average rate for the production ^{3}He runs.

$$
R(r) = \frac{R_f\left[1-B(f)\right]\left[1-N(r)\right]}{R_f\left[1-B(f)\right]C(r)+\left[1-N(f)-R_f C(f)\right]\left[1-B(r)\right]} \qquad (51)
$$

At this point, the independent parameters are k_1, k_2, and R_f. Since only the small term in the proton yield $B(r)$ corresponding to when both veto detectors fire randomly within the electronic deadtime interval is sensitive to k_1 and k_2 and not their sum, we can make the approximation of setting $k_1 = k_2$. Since r is a measure of the average veto rate itself, and (interestingly) the relative distribution of hits across the veto planes is the same to the 5% level for all targets, k_1 and k_2 are independent of the target. Thus, equation 51 can be fitted for all target simultaneously to get universal values of k_1 and k_2, and independent values of R_f for each target.

The values of R_f can then be used as stated in section 3.2.

4 Application to Kin4

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 *** Summary of Asymmetry ***
Total instrumental (BCA, LT) asymmetry     :  -0.0006
Raw Neutron asymmetry for Pperp < 150 MeV/c:  0.0510 +/- 0.0026 with a total yield of 1.530250e+05 .
Raw random Neutral Background asymmetry    :  0.0144 +/- 0.0074 with a yield of 1.803900e+04  (11.7883 %)
Coincident Neutral-candidate Asymmetry     :  0.0559 +/- 0.0031
Raw Proton  asymmetry for Pperp < 150 MeV/c:  0.0025 +/- 0.0009 with a rel. contr. to neutrons (28.7129 %)
Raw random Proton Background asymmetry     :  0.0032 +/- 0.0137 with a yield of 5.303000e+03  (0.4316 % rel. to protons)
Coincident Proton-candidate Asymmetry      :  0.0025 +/- 0.0009
Extracting Neutron and Proton asymmetries coherently:
Final observed neutron asymmetry is        :  0.0902 +/- 0.0181   (20.0851 %)
Final observed proton  asymmetry is        : -0.0153 +/- 0.0233   (-152.4666 %)
Using 'known' value of proton asymmetry (FW)   GEp/GMp (Q2=1.68606) = 0.292844
  proton APphys =  0.2037 ... has obs asymm of -0.0022 +/- -0.0003
  2nd final observed neutron asymmetry is  :  0.0839 +/- 0.0108   (12.0051 %)
   Effective neutron yield is 91048  (67.45 % of coinc. neutron candidates)
   Effective proton  yield is 1016646  (83.10 % of coinc. proton candidates)
Using a   beam polarization of : 0.8500 +/- 0.0200
  and a target polarization of : 0.4820 +/- 0.0200
  and a neutron/target pol. of : 0.8600 +/- 0.0200
  and a N2 dilution factor  of : 0.9500 +/- 0.0200
Based upon First Final neutron asymmetry:
*** Get Aphys = -0.2694 +/- 0.0563
Get GEn/GMn to be                  -0.2296 +/- 0.07223
Using a value of GMn(1.69 GeV^2) = -0.1677
we find GEn to be 0.03851 +/- 0.01211  ( +/- 31.45 %)

OR, with 2nd Final neutron asymmetry (USING KNOWN PROTON ASYMMETRY)
*** Get Aphys = -0.2506 +/- 0.0354
Apar contributes ~ -0.060
Get GEn/GMn to be                  -0.2061 +/- 0.04347
Using a value of GMn(1.69 GeV^2) = -0.1677
we find GEn to be 0.03457 +/- 0.007289  ( +/- 21.09 %)
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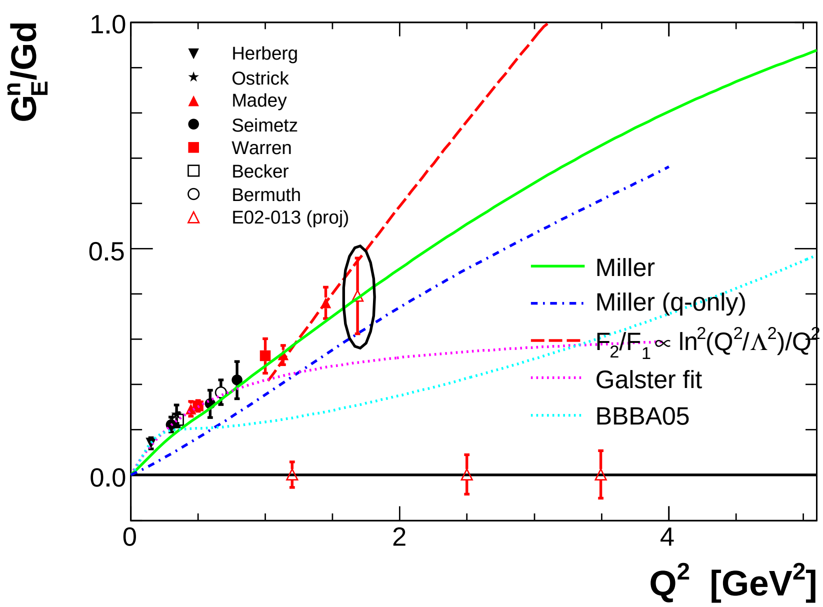

Figure 1: Present understanding of E02-013