

# Form of QED Elastic Cross Sections

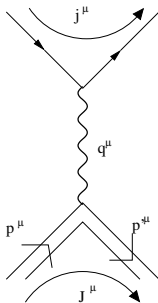
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- ▶ Want to find most general form of cross section with QED considerations
- ▶ Transition amplitude from  $i \rightarrow f$ :

$$T_{fi} = -e \int j_{\mu}^{fi} A^{\mu} d^4x = -e \int j_{\mu}^{fi} \frac{J^{\mu}}{q^2} d^4x \quad (1)$$

- ▶ First, we must describe general e.m. current  $J^{\mu}$  for:



Four vectors available:

$$p^\mu, p'^\mu, \gamma^\mu \quad (2)$$

where

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (3)$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad (4)$$

Tensors:

$$\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \quad (5)$$

Have to consider all linear combinations

For  $4 \times 4$  matrices, 16 linearly independent quantities:

$$\mathbb{I}, \gamma^\mu, \gamma^5, \sigma^{\mu\nu}, \gamma^\mu \gamma^5 \quad (6)$$

where

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (7)$$

Parity operator can be represented  $P = \gamma^0$ , so:

$$\{\gamma^5, P\} = 0 \quad (8)$$

so currents with  $\gamma^5$  ruled out by parity conservation.

- ▶  $p$  and  $p'$  are the only available 4-vectors
- ▶  $q^2$  is the only independent scalar as all combinations of  $p$  and  $p'$  can be reduced to  $m_N$  and  $q$ .
- ▶ The most general form of  $J^\mu$  is then:

$$J^\nu = e\bar{u}(p')\left[K_1(q^2)\gamma^\nu + (p^\nu + p'^\nu)K_2(q^2) + (p^\nu - p'^\nu)K_3(q^2) + i\sigma^{\nu\mu}(p_\mu - p'_\mu)K_4(q^2) + i\sigma^{\nu\mu}(p_\mu + p'_\mu)K_5(q^2)\right]u(p)e^{i(p-p')\cdot x} \quad (9)$$

- Gordon decomposition identity:

$$\bar{u}\gamma^\mu u = \frac{1}{2M}\bar{u}(p^\mu + p'^\mu + i\sigma^{\mu\nu}(p'_\nu - p_\nu))u \quad (10)$$

eliminating  $p + p'$  terms.

- Current conservation says  $\partial_\mu J^\mu = q_\mu J^\mu = 0$ , so terms that do not vanish must have  $K = 0$
- For  $K_3$  term:

$$q^\mu q_\mu K_3 = q^2 K_3 \neq 0 \quad (11)$$

so  $K_3(q^2) = 0$ .

- Rewriting  $J^\mu$  in different terms:

$$J^\nu = e\bar{u}(p')\left[F_1(q^2)\gamma^\nu + \frac{\kappa}{2M}q_\mu\sigma^{\nu\mu}F_2(q^2)\right] \quad (12)$$

- ▶ Transition amplitude from state  $i \rightarrow f$ :

$$T_{fi} = -i \int j^\mu \frac{1}{q^2} J_\mu d^4x \quad (13)$$

- ▶ Transition rate:

$$W_{fi} = \frac{|T_{fi}|^2}{T} \quad (14)$$

- ▶ relates to differential cross section  $d\sigma$ :

$$d\sigma = \frac{W_{fi}}{\Phi} dQ \quad (15)$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[ \left( F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \right) \cos^2 \frac{\theta}{2} \right. \\ \left. - \frac{q^2}{2M^2} (F_1 + \kappa F_2)^2 \sin^2 \frac{\theta}{2} \right] \end{aligned} \quad (16)$$



## References:

- ▶ My thesis
- ▶ Halzen and Martin, *Quarks & Leptons: An Introductory Course in Modern Particle Physics*, Wiley, 1984