Introduction to two-photon exchange corrections

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Overview

- One photon approximation
- Indications of two photon effects from form factors
- Hard Two Photon Effects
- Effects on Experiment
- Upcoming Experiments
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One Photon Approximation

From Seamus's presentation, we know that in the one-photon or Born approximation, the differential cross section can be written in terms of the Born amplitude:

$$\frac{d\sigma_0}{d\Omega} = \left(\frac{\alpha}{4MQ^2}\frac{E'}{E}\right)|\mathcal{M}_0|^2 = \sigma_{\mathrm{Mott}}\frac{1}{\epsilon(1+\tau)}\sigma_R \tag{1}$$

where σ_R is the reduced cross section, convenient for a Rosenbluth separation:

$$\sigma_R = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2) \tag{2}$$

In the one photon approximation, all angular dependence is in ϵ

$$\epsilon = \left[1 + 2(1+\tau)\tan\theta/2^2\right]^{-1} \tag{3}$$

The form factors are functions of only one variable (Q^2).

$$\frac{d\sigma}{d\Omega_{\mathrm{Lab}}} \propto G_M^2 + \frac{\epsilon}{\tau} G_E^2 \tag{4}$$

Using a Rosenbluth separation at high Q^2 to extract G_E requires separating a small term from a very large term.

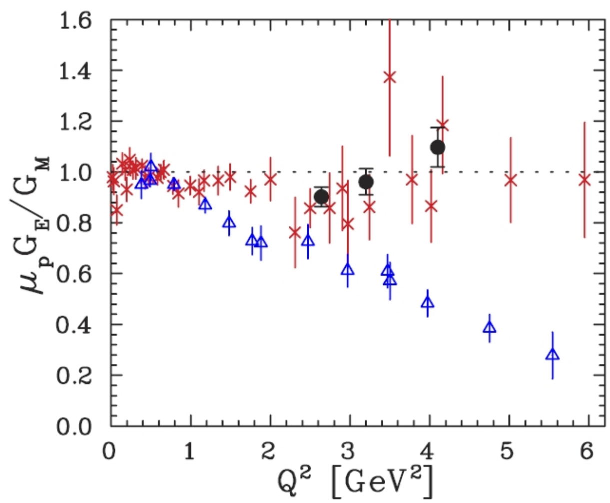
Indications of TPE

Can use another method – double polarized methods (recoil or polarized target)

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\frac{G_E}{G_M} \tag{5}$$

This method is significantly less dependent on the uncertainty in the angle.

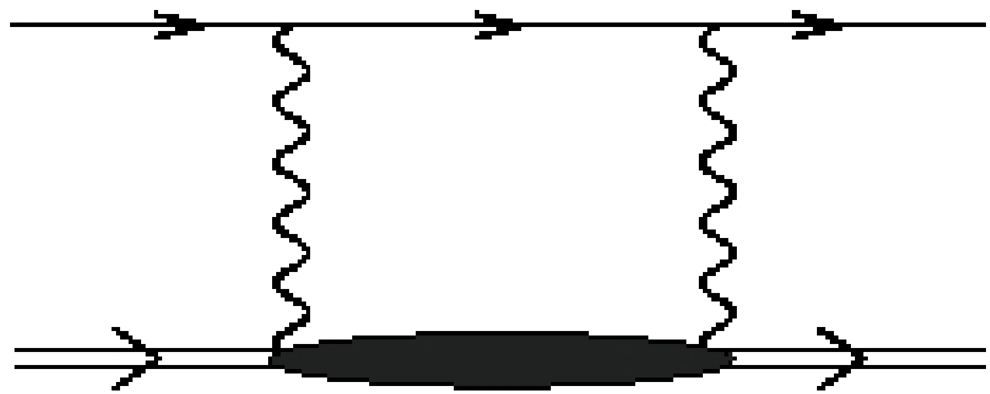
Experimental Discrepancy



Red and black points from Rosenbluth separation. High precision blue points (open triangles) from polarization transfer.

Higher order corrections with an ϵ dependence would have a large effect in Rosenbluth, but not be apparent in polarization transfer.

Hard Two Photon Effects



Hard two photon effects have been studied, but only at low beam energies (< 1 GeV). These effects were determinined to contribute at the 1% level.

Evaluation of the above figure involves the full response of the nucleon, which cannot be calculated in a model independent way.

However, the two-photon exchange graphs can be expressed in an effective current $\times$ current form.

Derivation is in Guichon and Vanderhaeghen, Phys. Rev. Lett. **91** 142303.

Eqn. 2 is modified with the addition of 2-γ terms, $\delta\tilde{G}_M$, $\delta\tilde{G}_E$, and $\tilde{F}_3$

$$\begin{aligned}\sigma_R &= G_M^2 + \frac{\epsilon}{\tau}G_E^2 + 2G_M\mathcal{R}\left(\delta\tilde{G}_M + \epsilon\frac{\nu}{M^2}\tilde{F}_3\right) + 2\frac{\epsilon}{\tau}G_E\mathcal{R}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) \\ &\quad +\mathcal{O}(e^4)\end{aligned}$$

These form factor modifications can be calculated from GPDs

$$\delta\tilde{G}_M^{hard} = C \tag{6}$$

$$\delta\tilde{G}_E^{hard} = -\left(\frac{1+\epsilon}{2\epsilon}\right)(A-C) + \sqrt{\frac{1+\epsilon}{2\epsilon}}B \tag{7}$$

$$\tilde{F}_3 = \frac{M^2}{\nu}\left(\frac{1+\epsilon}{2\epsilon}\right)(A-C) \tag{8}$$

$$A \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 (H^q + E^q) \tag{9}$$

$$B \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q) \tag{10}$$

$$C \equiv \int_{-1}^{1} \frac{dx}{x} \tilde{f}_1^{hard} \mathrm{sgm}(x) \sum_q e_q^2 \tilde{H}^q \tag{11}$$

where s, t, and u are the Mandelstam variables, $\tilde{f}_1$ and $\tilde{f}_3$ are "form factors" of lepton-quark scattering. H^q, E^q, and $\tilde{H}^q$ (along with $\tilde{E}^q$) are the Generalized Parton Distributions.

Effects on Experiment

$$\left(R^{\rm exp}_{\rm Rosenbluth}\right)^2 = \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma} \tag{12}$$

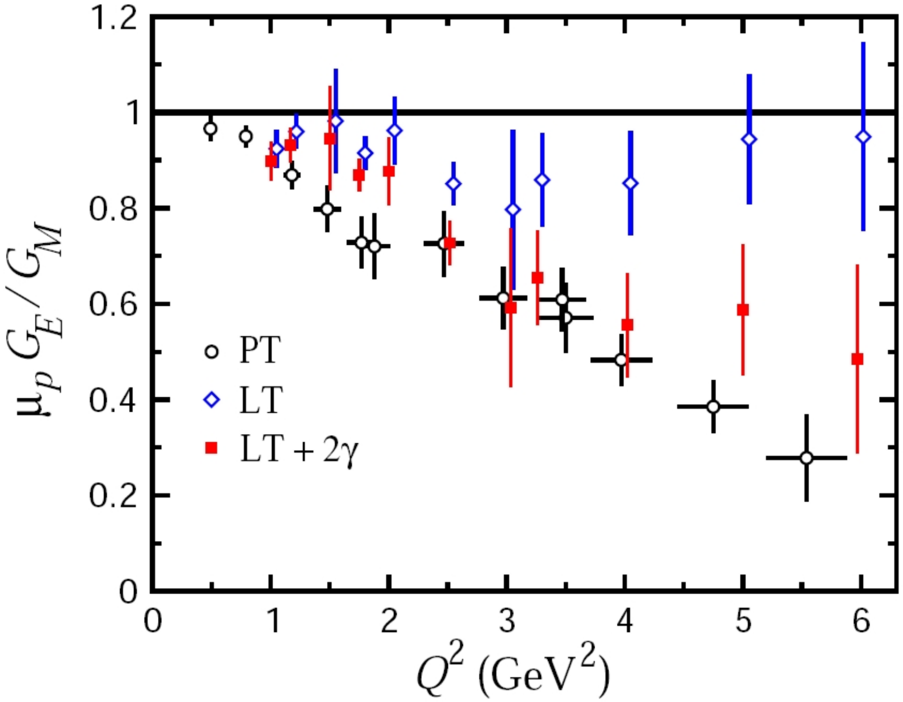
and

$$R^{\rm exp}_{\rm polarization} = \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\epsilon}{1+\epsilon}\frac{|\tilde{G}_E|}{|\tilde{G}_M|}\right) Y_{2\gamma} \tag{13}$$

Where $\tilde{G}_{M,E} = G_{M,E} + \delta\tilde{G}_{M,E}$, and

$$Y_{2\gamma}(\nu, Q^2) = \mathcal{R}\left(\frac{\nu \tilde{F}_3}{M^2|\tilde{G}_M|}\right) \tag{14}$$

Corrected Results



Upcoming Experiments

Target Single Spin Asymmetry – "The imaginary (absorptive) part of the 2γ exchange amplitude can be accessed through a single spin asymmetry (SSA) when either the target or beam spin are polarized normal to the scattering plane, as has been discussed some time ago. As time reversal invariance forces the SSA to vanish for one-photon exchange, it is of order $\alpha = e^2/(4\pi)$."

Measured during G0 in Hall C, and will be measured on the neutron during Transversity in Hall A

Mapping out ϵ dependence – Two-photon exchanges introduce non-linearities in ϵ. Experiments E05-017 and E04-017 will map out this dependence.

References

Two reviews:

Arrington, Melnitchouk and Tjon arxiv:0707.1861[nucl-ex]

Carlson and Vanderhaeghen arxiv:hep-ph/0701272

Form Factor Review:

Perdrisat, Punjabi, and Vanderhaeghen arxiv:hep-ph/0612014

Two photon correction:

Guichon and Vanderhaeghen, *Phys. Rev. Lett*, **91**, 142303