Appendix B

Error Analysis

In the measurement of the electric form factor of the neutron at $Q^2 = 3.5$ GeV^2 statistical uncertainty is the main cmponent of the error. The double polarization method insures small systematic uncertainties (as described in section **??**). Due to teh small asymmetry, large statistics are required. The primary cause of the small asymmetry is the small size of G_E^n.

The error analysis for E02-013 entails propagation of correlated and uncorrelated systematic and statistical uncertainties for the large number of equations used to calculate G_E^n from the measured quantites. Equation 5.43 gives the factors that contribute to G_E^n. The uncertainty in a few of these factors give the most important contributions to the final precision of the G_E^n measurement.

For the propagation of the uncertainties, standard error techniques are used [79]. Assuming the errors are uncorrelated the expression to determine the uncertainty in a function q which depends on the components $x, ..., z$ is

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + ... + \left(\frac{\partial q}{\partial z}\delta z\right)^2} \ . \tag{B.1}$$

When q is proportional to the product or quotient of the components,

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + ... + \left(\frac{\delta z}{z}\right)^2} \ . \tag{B.2}$$

The correlated systematic errors also contribute to the final uncertainty. As an example, there were not independent measurements of polarization for every run, so

the systematic error was defined to be the same fractional error as the polarization after the summation, rather than be included in the summation error calculation (equation 5.47). Explicit discussion of the error analysis using formula B.1 follows.

For statistical error, assuming a poisson distribution, the formula

$$\frac{\delta N}{N} = \frac{1}{\sqrt{N}} \tag{B.3}$$

is used, where N is the pertinant statistics. For small asymmetries and large statistics method of determination of the uncertainty is valid [68]

$$\delta A \approx \frac{1}{\sqrt{N}} - \frac{(N^+ - N^-)^2}{N^{\frac{5}{2}}} \approx \frac{1}{\sqrt{N}} \ . \tag{B.4}$$

The formula B.4 is used to determine the statistical error in emperically measured asymmetries like the raw accidental background asymmetry and raw neutron asymmetry. The statistical uncertainty for quasi-elastic event counts, background counts, and other counts is determined using equation B.3.

The following dilution factors, polarizations, and asymmetry corrections are used in the calculation of the physical asymmetry (equation **??**): D_{back}, D_{N_2}, A_{back}, $D^p_{N_2}$, A_{sum}, D_p, D^p_n, P_n, P_{beam}, $P_{^3He}$ (for description see below). Some of these depend on the same sources of uncertainty. Common sources of uncertainty in these factors and corrections are a systematic uncertainty due to the charge determination in the accidental background technique (see section 5.5.1) and statistical uncertainties due to the raw neutral counts and the raw background counts. While the systematic uncertainty due to the charged to uncharged background ratio is a component of almost all dilutions and corrections, the small number of raw background makes it of minimal importance. This is observed in Figure B where the change

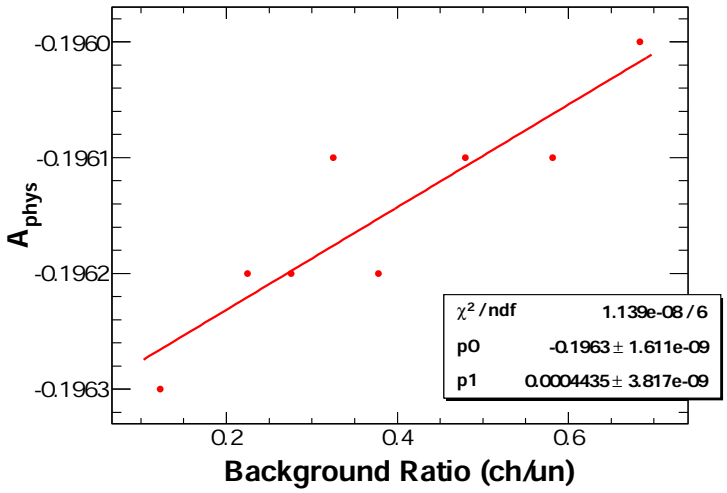

Figure B.1: The physical asymmetry as the accidental background ratio of charged to uncharged used in the calculation is changed. This figure shows that the dependence on this ratio is very small, with an estimated fractional uncertainty on the accidental background charge ratio of 20%. This esimated fractional uncertainty corresponds to a fractional uncertainty of less than .2%. Plotted is a linear fit, showing the small dependence. This is for an analysis of Kinematic 2(b).

of neutron physical asymmetry as a function of the charge ratio of the background is plotted for kinematic 2. This ratio will be treated as a constant for the rest of this error analysis. The value for this ratio and accidental background counts is presented in Table B.3.

As described in section 4.4.2 the polarization changes run to run, and a modified asymmetry is calculated on a run per run basis, and then used in a weighted sum for the entire kinematic (as derived in Appendix C). The only components which change significantly on a per run basis are the target polarization and neutron purity factor. The neutron purity factor and the rest of the dilutions are corrected for after

the summation (see equation 5.48)

$$A_{sum} = \frac{\sum_i \frac{A_{raw,i}}{P_{^3He,i}} N_i P^2_{^3He,i}}{\sum_i N_i P^2_{^3He,i}} \ . \tag{B.5}$$

The factor A_{back} can be found by using equation

$$A_{back} = \frac{N^+_{back} - N^-_{back}}{N \left\langle P_{^3He} \right\rangle} = A_{back,raw} \frac{N_{neut,back}}{N \left\langle P_{^3He} \right\rangle} \ . \tag{B.6}$$

The accidental background asymmetry correction contributes to the statisical uncertainty through the counts in the raw accidental background and quasi-elastic samples and through the accidental background asymmetry. As described in section 5.5.1, the neutral accidental background is related to the raw accidental background by $N_{neut,back} = f_{back} N_{back}$. Additionally, it contributes to the systematic uncertainty through the target polarization.

The polarization of the target and nucleus and beam are presented in sections 4.3 and 4.4.2 and in [32]. The target polarization ($P_{^3He}$) uncertainty is correlated between kinematics 3, 4, and 2(b). As mentioned in 5.7.1, the values for the polarizations of the protons and neutron within the helium nucleus depend on the theoretical model used to calculate the nuclear corrections. Changes in these values will be correlated. The value used for the beam polarization (P_{beam}) and systematic uncertainty is arrived at from inspecting Table 4.2.

The dilution caused by the accidental background (equation 5.38) is written as

$$D_{back} = 1 - \frac{N_{neut,back}}{N} \ . \tag{B.7}$$

The sources statistical uncertainty are the counts of the raw accidental background

and quasi-elastic samples, similar to the accidental background asymmetry correction.

The dilution from the nitrogen in the neutron sample (equation 5.40 gives the method to properly calculate this) can be written as

$$D_{N_2} = 1 - \frac{N_{N_2}}{N - N_{neut,back}} \ . \tag{B.8}$$

with a dependence on the uncertainties in the constants shown in Table B.1. The dominant sources of systematic uncertainty in this factor are from the N_2 and N_2 in the ^{3}He target densities. The dominant source of statistical uncertainty is due to the nitrogen quasi-elastic events and background. The uncertainties for this dilution were propagated using equation B.1. The results for this error analysis are also presented in Table **??**. This uncertainty is treated as independent from the other uncertainties, although there is a small contribution from the ^{3}He quasi-elastic sample.

The dilution of the protons in the neutron sample (equation 5.32) can be expressed as

$$D_n = 1 - \frac{N_{proton}}{N - N_{neut,back} - N_{N_2}} \ . \tag{B.9}$$

Here the sources of uncertainty are statistical from the background, charged quasi-elastic, neutral quasi-elastic in H_2, N_2 and ^{3}He, and systematic from the technique and model. It is assumed that the statistical uncertainty from Minuit and the systematic uncertainty from the technique dominates all other sources of error. By considering other analyses, a fractional systematic error of 5% was arrived at. The cited statistical uncertainty is that given by Minuit fitting [**?**].

Finally, the dilution caused by other final state interactions within the neutron sample can be expressed as

$$D_{FSI} = 1 - \frac{N_{FSI}}{N - N_{neut,back} - N_{N_2} - N_{proton}} \ . \tag{B.10}$$

Because the dilution due to final state interactions is undetermined but guided by theory [**?**][75], an estimation is determined for the systematic uncertainty that accounts for the likely values of this dilution.

The proton asymmetry has similar dilutions as the neutral asymmetry, with a couple of important differences. This formula 5.54 is

$$A_{proton} = \frac{1 - D_n}{D_{back} D_{N2}} P_{beam} P_{prot} A_{p,phys} \ . \tag{B.11}$$

The polarization of protons within the nucleus, P_{prot} is provided by theory []. The proton physical asymmetry is calculated from the proton form factor ratio as described in equation 5.49:

$$A_{p,phys} = \frac{B\lambda_{proton} + C}{\lambda^2_{proton} + D} \ . \tag{B.12}$$

Here the uncertainty in λ_{proton} is determined from the parameterization [48]. The calculation and uncertainties for the quantities used in the determination of A_{proton} are in Table B.2.

A reminder that the expression for the physical asymmetry is (equation 5.43)

$$A_{phys} = \frac{A_{sum} - A_{back} - A_{proton}}{P_n P_{beam} D_{back} D_{N_2} D_p D_{fsi}} \ . \tag{B.13}$$

By applying equation B.1 to equation 5.43, the statistical and systematic uncertainty of the physical asymmetry is calculated. These results, in addition to the

	D_{N_2}			$D^c_{N_2}$		
$Q^2 =$	2.5 GeV^2	3.5 GeV^2	1.5 GeV^2	2.5 GeV^2	3.5 GeV^2	1.5 GeV^2
$\rho_{^3He}$	0.164	0.164	0.164	0.164	0.164	0.164
ρ_{N_2}	4.4	3.6	9.1	4.4	3.6	9.1
$Q_{^3He}$	184399	1441010	–	184399	1441010	–
Q_{N_2}	14020	37920	–	14020	37920	–
$N_{^3He}$	2089.1	2686.2	–	19076.7	30331	–
N_{N_2}	220.5	28.5	–	1123.2	263.5	–
D_{N_2}	0.9493	0.9818	–	0.9714	0.9851	–
Statistical	0.0036	0.0035	–	0.0009	0.0009	–
Systematic	0.0047	0.0015	–	0.0027	0.0013	–

Table B.1: A table showing the measurements and error calculations for the nitrogen dilution of the uncharged sample, for all three kinematics. $N_{^3He}$ is the number of background corrected counts for the selected ^3He runs, ρ_{N_2} is the nitrogen density, $Q_{^3He}$ is the total charge of the selected ^3He runs. This calculation is as described in section 5.5.2.

components contribution to the uncertainties, are presented in Table **??** for kinematic 2a, Table B.5 for kinematic 2b, Table B.6 for kinematic 3, and Table B.7 for kinematic 4.

The uncertainty in the ratio of neutron form factors, or λ, is determined using equation B.1 and 5.55. This is a straight forward calculation [31], but the value for λ is only determined for the central value of Q^2. To determine the value for the electric form factor of the neutron, equations B.2 and 5.58 were used. Results are

parameter	$Q^2 = 2.5$ GeV^2	$Q^2 = 3.5$ GeV^2	$Q^2 = 1.5$ GeV^2
G_E^p	0.0330 ± 0.0051	0.018 ± 0.004	0.077 ± 0.007
G_M^p	0.1378 ± 0.0049	0.089 ± 0.004	0.262 ± 0.006
$A_{p,phys}$	0.177 ± 0.028	0.182 ± 0.030	0.0031
P_{prot}	-0.028 ± 0.004	-0.028 ± 0.004	-0.028 ± 0.004
P_{beam}	$0.835 \pm (0.03)$	$0.835 \pm (0.03)$	0.835 ± 0.03
D_{N2}	$0.949 \pm 0.004 \pm (0.004)$	$0.982 \pm 0.004 \pm (0.002)$	0.946 ± 0.002
D_{back}	0.9792 ± 0.0003	0.9732 ± 0.0005	0.993
D_n	$0.8 \pm 0.023 \pm (0.04)$	$0.865 \pm 0.046 \pm (0.043)$	0.677 ± 0.025
A_{proton}	-0.0009 ± 0.0002	-0.0006 ± 0.0001	-0.0005 ± 0.0002

Table B.2: A table of the errors for the calculation of A_{proton}. The calculation is calculated according to equation 5.54. The uncertainty in G_M^p and G_E^p is determined using a parameterization [48] and the propagation of the error is according to equation B.1.

displayed in Table B.4.

parameter	$Q^2 = 2.5$ GeV^2	$Q^2 = 3.5$ GeV^2	$Q^2 = 1.5$ GeV^2
Hadron Ratio	0.325 ± 0.065	0.26 ± 0.04	0.33 ± 0.05
Hadons	2495	4201	2762
Neutral	1883.2	1092.3	911.5
Charged	611.8	3108.7	1850.5

Table B.3: A table for all three kinematics showing the measurements for the charged and uncharged background and the uncertainty in the background ratio.

parameter	$Q^2 = 2.5$ GeV^2	$Q^2 = 3.5$ GeV^2	$Q^2 = 1.5$ GeV^2
GEn/GMn	-0.232	-0.250	0.0031
Error	$\pm 0.0333 \pm (0.031)$	$\pm 0.047 \pm (0.021)$	$\pm 0.0004 \pm 0.0004$
GMn	-0.0955 ± 0.0018	-0.0557 ± 0.0011	0.00356
GEn	0.0221	0.0139	0.01552
Error	$\pm 0.0032 \pm (0.0030)$	$\pm 0.0026 \pm (0.0012)$	$\pm 0.0058 \pm 0.0030$

Table B.4: A table of the errors for the calculation of G_E^n. Uncertainty in the ratio of form factors was calculated from the physical asymmetry as expressed in section 5.6 and Table 5.9 and Table 5.8. The uncertainty in G_M^n is determined from a parameterization [48] and propagation of the error is according to equation B.1.

quantity	value	σ_{stat}	contrib. A_{phys}	σ_{syst}	contrib. A_{phys}
A_{meas}	-0.0449	± 0.0055	0.024	–	-
$P_{^3He}$	0.4362	–	-	± 0.02	0.018
$A_{back,raw}$	0.0279	± 0.0004	0.00004	–	-
D_{FSI}	0.95	–	-	± 0.01	0.002
D_n	0.80	± 0.023	0.006	± 0.04	0.010
$A_{p,phys}$	0.177	–	-	± 0.028	0.0003
P_{beam}	0.835	–	-	± 0.03	0.007
N_{total}	33001	± 182	0.0001	–	-
N_{back}	4201	± 65	0.0003	–	-
P_n	0.86	–	-	± 0.02	0.005
P_{prot}	-0.028	–	-	± 0.004	0.0003
D_{N_2}	0.949	± 0.0036	0.00074	± 0.0036	0.00074
A_{phys}	-0.198	-	± 0.024	-	± 0.022

Table B.5: A table of the uncertainties that contribute to the physical asymmetry (A_{phys}). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation **??**. This table shows the errors for $Q^2 = 2.5$ GeV^2.

quantity	value	σ_{stat}	contrib.	σ_{syst}	contrib.
A_{meas}	-0.0356	± 0.0081	0.030	–	-
$P_{^3He}$	0.477	–	-	± 0.02	0.011
$A_{back,raw}$	0.0225	± 0.0004	0.00004	–	-
D_{FSI}	0.95	–	-	± 0.01	0.001
D_n	0.865	± 0.046	0.006	± 0.043	0.006
$A_{p,phys}$	0.182	–	-	± 0.030	0.0002
P_{beam}	0.835	–	-	± 0.03	0.005
N_{total}	15381	± 124	0.0001	–	-
N_{back}	2762	± 53	0.0003	–	-
P_n	0.86	–	-	± 0.02	0.003
P_{prot}	-0.028	–	-	± 0.004	0.0002
D_{N_2}	0.982	± 0.0035	0.0004	± 0.0015	0.0002
A_{phys}	-0.130	-	± 0.031	-	± 0.014

Table B.6: A table of the uncertainties that contribute to the physical asymmetry (A_{phys}). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation **??**. This table shows the errors for $Q^2 = 3.5$ GeV^2.

quantity	value	σ_{stat}	contrib.	σ_{syst}	contrib.
A_{meas}	-0.05118	± 0.00677	0.02596	–	-
$P_{^3He}$	0.49368	–	-	± 0.02	0.016078
$A_{back,raw}$	0.05411	± 0.00008	0.00108	–	-
D_{FSI}	0.95	–	-	± 0.05	0.01047
D_n	0.80	± 0.02332	0.00554	± 0.07287	0.01730
$A_{p,phys}$	0.17667	–	-	± 0.02755	0.00024
P_{beam}	0.835	–	-	± 0.03	0.00720
N_{total}	21804	± 147.662	0.00014	–	-
N_{back}	2495	± 49.95	0.00042	–	-
P_n	0.86	–	-	± 0.02	0.00463
P_{prot}	-0.028	–	-	± 0.004	0.00022
D_{N_2}	0.975	± 0.0015	0.00030	± 0.0023	0.00047
A_{phys}	-0.19888	± 0.02655	-	± 0.02722	-

Table B.7: A table of the uncertainties that contribute to the physical asymmetry (A_{phys}). The contributions are calculated and contribute to the physical asymmetry precision as determined by equation **??**. This table shows the errors for kinematic 4.